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VDE
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FIELD ENGINEERING.

A HAND-BOOK OF THE

THEORY AND PRACTICE OF RAILWAY SURVEYING,
LOCATION, AND CONSTRUCTION,

DESIGNED FOR THE

CLASS-ROOM, FIELD, AND OFFICE,

AND CONTAINING

A LARGE NUMBER OF USEFUL TABLES,

ORIGINAL AND SELECTED.

BY

WILLIAM H. SEARLES, C.E.,

MEMBER AMERICAN SOCIETY OF CIVIL ENGINEERS.

EIGHTH REVISED EDITION.

NEW YORK:
JOHN WILEY & SONS,
15 ASTOR PLACE.
1887.

THE NEW YORK
PUBLIC LIBRARY
439571 A
ASTOR, LENOX AND
TILDEN FOUNDATIONS
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PREFACE.

ALTHOUGH the modern railway system is but about fifty years old, yet its growth has been so rapid, and the progress in the science of railway construction so great, as to render the earlier technical books on this subject inadequate to the needs of the engineer of to-day.

In the course of his practical experience as a railway engineer, the author was strongly impressed with the want of a more complete hand-book for field use, and finally concluded, at the solicitation of his friends, to undertake the preparation of the present volume.

The aim in this work has been:

First—To present the general subject of railway field work in a progressive and logical order, for the benefit of beginners.

Second—To classify the various problems presented, so that they may be readily referred to.

Third—To embrace discussions of all the more important practical questions while avoiding matters non-essential.

Fourth—To employ throughout the work a uniform and systematic notation, easily understood and remembered, so that after one perusal the formulæ may be intelligible at a glance wherever referred to.

Fifth—To express the resulting formula of every problem in the shape best adapted to convenient numerical computation.

Sixth—To furnish a large variety of useful tables, more complete and extended than any heretofore published, especially adapted to the wants of the field engineer.

An elementary knowledge of algebra, geometry and trigono-etry on the part of the reader has been taken for granted, as command of these instrumentalities is deemed essential ne education of the civil engineer. The few referenc *echanics, analytical* geometry, optics and the calculu *e assumed correct* by those not conversant wit *ranches.*

Many of the problems in curves are new, yet there is hardly one that has not presented itself to the author in the course of his practice. The investigation of the valvoid curve is original, and though the mathematical discussion is somewhat difficult, yet the resulting formulæ, taken in connection with Table X, are exceedingly simple and convenient for the solution of a certain class of problems.

The treatment of compound curves is novel and exhaustive. A few general equations are established, which, by slight modifications, solve all the problems that can occur.

No discussion of reversed curves is given, because these are inconsistent with good practice, except in turnouts, under which head they are noticed.

The chapter on levelling includes a discussion of stadia measurements, with practical formulæ. The chapter on earthwork contains a review of several methods for calculating quantities, and states the conditions under which these succeed or fail in giving correct results.

Among the tables, numbers 3, 5, 6, 10, 18, 19, 26 and 29 are original. The adoption of versed sines and external secants throughout the work, wherever these would simplify the formulæ, rendered necessary the preparation of tables of these functions. The table of logarithmic versed sines and external secants has been computed from ten-place logarithmic tables of sines and tangents, so that the last decimal is to be relied on, and no pains have been spared to make the table thoroughly accurate.

Tables numbers 4, 7, 8, 9, 11, 12, 13, 14 and 30 have been recalculated, enlarged, and some of them carried to more decimal places than similar tables heretofore published. The intention has been to give one more decimal than usual, so that in any combination of figures the result of calculation might be reliable to the last figure usually required.

The tables which have been compiled and rearranged are numbers 1, 2, 15, 16, 17, 24, 25 and 31. The tables of log. sines and tangents here given are the only six-place which give the differences correctly for seconds. of logarithms of numbers is accompanied by a con *of proportional parts,* which greatly facilitates int *for the fifth and sixth figures.*

ll the tables, whether new or old, scrupulous been taken to make the last figure correct, and the greatest diligence has been exercised by various checks and comparisons to eliminate every error. It is, therefore, hoped and believed that a very high degree of accuracy has been obtained, and that these tables will be found to stand second to none in this respect.

The preparation of this work has extended over several years, as time could be spared to it from other engagements. It is, therefore, the expression of deliberate thought, based on experience, and as such is submitted to the judgment of brother engineers. If it shall prove to have even partially met the aim herein announced, and so shall serve to smooth the way of the ambitious student, or to assist the expert in his responsible duties, the labors of the author will not have been in vain.

WM. H. SEARLES, C.E.

NEW YORK, March 1st, 1880.

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FIELD ENGINEERING.

CHAPTER I.

RECONNOISSANCE.

1. The engineering operations requisite to and preceding the construction of a railroad are in general:

THE RECONNOISSANCE,
THE PRELIMINARY SURVEY, and
THE LOCATION.

2. *The Reconnoissance* is a general and somewhat hasty examination of the country through which the proposed road is to pass, for the purpose of noting its more prominent features, and acquiring a general knowledge of its topography with reference to the selection of a suitable route. The judicious selection of a route may be a very simple or complex problem, depending on the character of the topography, and more especially on the direction of the streams and ridges as compared with the general direction of the proposed road.

3. A road running along a water-course is most easily located. In this case the choice is to be made merely between the two banks of the stream, or between keeping one bank continuously and making occasional crossings. When the stream is small it will usually be found best to cross it at intervals, the advantage of direct alignement outweighing the cost of bridging; but when the stream is of considerable size the solution of the problem is not so obvious, requiring patient comparison of results in the two cases to determine whether to cross or not, while in the case of the larger rivers crossing may be out of the question.

When there is a choice of sides, both banks should be traversed by the engineer on reconnoissance, and while examining [illegible]tail the one side he should take a general and compre[illegible] view of the other. Only thus can he gain a complete kno[illegible] of either side. The points to be considered are the rela[illegible]lue of the property on either side, the number and size of tributary streams, and probable cost of crossing them, the cost of graduation as affected by the amount and character of the material to be removed, and the liability to land slides, the amount and degree of curvature required, and the probable revenues which the road can command. If, in respect to these points, one bank of the stream gives the more favorable result all the way, the question is decided at once; but in case the greater inducements are found on either bank alternately, as usually happens, the propriety of bridging the stream, with the costs and advantages, must be considered as an additional element in the problem.

4. When no water-course offers along which the road may be located, the difficulties of selecting a route are increased, and these usually become greatest when the streams are found to run about at right angles to the direction of the road. Valleys and ridges are to be crossed alternately, involving the necessity of ascending and descending grades, diverting the road from a straight line, and increasing the distance and curvature. The engineer must now seek the lowest points on the ridges, and the highest banks at the stream crossings, in order to reduce as much as possible the total rise and fall, but these points must be so chosen relatively to each other as to admit of their being connected by a grade not exceeding the maximum which may be allowable. The intervening country between summit and stream must usually be carefully examined, even on reconnoissance, to determine where the assumed grade will find sustaining ground at a reasonable expense for graduation and right of way.

In selecting stream crossings, regard should be had not only to the height of the bank, but also to the character of the bottom, its suitability for foundations, and its liability to be washed by the current. The direction and force of the current should be observed, and its behavior during freshets, and the extremes of high and low water ascertained, if possible. An approximate estimate of the cost of bridging may be made.

5. The engineer should not only seek the best gl on the route first assumed, but should have an eye to allr possible routes, holding them in consideration pendi s accumulation of evidence, and being ready, finally, t pt that one which promises the greatest ultimate economy. should be able to read the face of the country like a and to carry in his mind a continuous idea or image of any line he is examining, so as to judge with tolerable accuracy of the influence any one portion of the line may have on another as to alignment and grade, even though many miles apart. In the successful prosecution of a reconnoissance he must depend mainly on his own natural tact and a judgment matured by experience.

6. The engineer will bring to his aid in the first place the most reliable maps, and those drawn on the largest scale. The sectional maps of United States surveys will be found very useful when they exist. In addition to these it is often desirable to prepare a map on a scale of one or two inches to a mile, on which will be drawn the principal features of the country to be traversed, such as streams, roads, towns, and the principal ridges, if known, but leaving the further details to be filled in by the engineer as he progresses. Such a map furnishes a correct scale for his sketches, and saves much valuable time, as he has only to sketch what the map does not contain, and occasionally to make corrections when he finds the map to be in error. He also notes on the map the *governing points* of the route, such as the best crossings of streams, ridges, or other roads, and any point where the line will evidently be compelled to pass. He may then indicate the route by a dotted line on the map drawn through the governing points. Having traversed the route in one direction he should retrace his steps, verifying or correcting his observations, and making such further notes as seem important. When in a densely wooded country, with but few openings, it may be impossible for him to get a commanding view from any point that will afford him the necessary information as to the general topography. He must then depend largely upon instrumental observations, taking these more frequently, and noting carefully all details likely to prove useful in future surveys.

7. The instruments required on an extended reconnoissance are the barometer and thermometer, the hand or Locke level, a pocket or prismatic compass, and a telescope or strong field-glass. To these may be added a telemeter for measuring distances at sight, but when good maps are to be had this instrument is seldom needed. So also some portable astronomical instruments are necessary in a new country, for determining latitude and longitude, but would only be a useless incumbrance in a settled district.

8. The mercurial barometer has generally been relied upon for the determination of heights, but owing to its inconvenient dimensions and the danger of breaking, it is now discarded by railroad engineers in favor of the more portable aneroid barometer, except in the case of trans-continental surveys, and when astronomical instruments are to be used also.

9. The best aneroids are designed to be self compensating for temperature, so that with a constant atmospheric pressure the reading shall be the same at all temperatures of the instrument. This, however, being a very delicate adjustment, is not always successfully made, so that each instrument is liable to have a small error due to temperature peculiar to itself. This error will be found rarely to exceed one hundredth of an inch, *plus* or *minus*, per change of ten degrees Fah., and is frequently much less than this. Just what the error is in a particular instrument may be determined by careful comparison with a standard mercurial barometer at the extremes of temperature, assuming the error found as proportional to the difference of temperature for all intermediate degrees of heat. The error having been determined for any aneroid, it should be applied, with its proper sign, to every reading to obtain the true reading.

The sizes generally used are 1¾ and 2½ inches in diameter, respectively, and experience seems to prove that there is no advantage in using larger sizes, but rather the contrary.

10. The ordinary barometric formulæ and tables have been prepared with reference to the mercurial barometer. In order that they may apply to the aneroid, it is necessary that the latter should be adjusted to read inches of mercury identically with the mercurial column at the sea level at a temperature of 32° Fah. But as the aneroid, unlike the mercurial column, requires no correction for latitude, nor for the variation in the force of gravity due to elevation, that portion of the formula which provides for such corrections, as well as that which provides for a correction due to the temperature of the instrument itself, may be omitted when using an aneroid. Thus the general formula is very much simplified, and becomes

$$z = \log \frac{h_{,}}{h'} \, 60384.3 \left(1 + \frac{t_{,} + t' - 64^\circ}{900}\right)$$

in which $h_{,}$ and h' are the readings of the aneroid in inches, and $t_{,}$ and t' the readings of a Fahrenheit thermometer at the lower and upper of any two stations respectively, and z is the difference in elevation in English feet of those stations.

To facilitate the calculation of heights by this formula, we may write

$$\text{Log} \frac{h_{,}}{h'} 60384.3 = [\log h_{,} - \log h'] \, 60384.3$$

and since only the *difference* of the logs. is required, this will not be affected, if we subtract unity from each. The quantities in Table XV. are prepared, therefore, by the formula

$$(\log h - 1) \, 60384.3$$

for every $\frac{2}{100}$ths of an inch from 19 inches to 31 inches.

Table XVI. contains values of $\frac{t_{,} + t' - 64°}{900}$ for every degree of $(t_{,} + t')$ from 20° to 200° Fah.

11. *To find the difference in elevation of any two stations by the tables:*

Take the difference of the quantities corresponding to $h_{,}$ and h' in Table XV. as an approximation, and for a correction multiply this difference by the coefficient corresponding to $(t_{,} + t)$, in Table XVI., adding or subtracting the product according to the sign of the coefficient.

Example.—

	Lower Sta. in.	Upper Sta. in.
Aneroid	$h_{,} = 29.92$	$h' = 23.57$
Thermometer	$t_{,} = 77°.6$	$t' = 70°.4$

By Table XV. for 29.92 we have 28741
for 23.57 22485

Difference 6256

By Table XVI. for $77.6 + 70.4 = 148$ we have $+ .0933$

Then $6256 \times .0933 = 583.6848$

and $6256 + 584 = 6840$ ft. $= z$.—*Ans.*

12. Certain precautions are to be observed in the use of the aneroid. When the index has been adjusted to a correct reading by means of the screw at its back, it should not be *meddled with until it* can again be compared with a standard *nercurial barometer,* and even then some engineers prefer t *ke note of its* error, if any, rather than disturb the anero

Again, since the principle of compensation supposes the aneroid to have a uniform temperature throughout its parts, it must be guarded against sudden changes, as otherwise the metallic case will be considerably heated or cooled before the change can affect the inner chamber, thus inducing very erroneous results. The aneroid, therefore, should seldom be taken from its leather case, nor exposed to any radiant heat of sun or fire, nor worn so near the person as to increase its temperature above that of the surrounding atmosphere. If removed to an atmosphere of decidedly different temperature, time must be allowed for the aneroid to be thoroughly permeated by the new degree of heat. The aneroid should be held with the face horizontal while being read; it should be handled carefully, and all concussions avoided, and it should be compared with a standard as often as practicable to make sure that it has suffered no derangement. Observing these precautions, and having a really good aneroid, the engineer should obtain excellent results in the estimation of heights. It has been found that the slight error in compensation, previously alluded to, is subject to a change during the first year or two after the instrument is made, but subsequently it becomes quite permanent.

13. For the purpose of obtaining approximate elevations by a simple inspection of the dial, the modern aneroid is provided with a secondary scale reading hundreds of feet, which is placed outside the scale of inches. It is divided according to the following formula prepared by Prof. Airy:

$$z = 55032\,\frac{h_{,} - h'}{h_{,} + h'}\left(1 + \frac{t_{,} + t' - 100^\circ}{1000}\right)$$

in which it is evident that no correction for temperature is required when the *average* temperature of the two stations is 50°. When the two scales are engraved on the same plate the zero of the scale of feet is coincident with 31 on the scale of inches; but in some aneroids the scales are on two concentric plates, so that the zero of one may be made to coincide with any division of the other, which is in some respects an advantage.

14. The theory of the barometer, as expressed in the above formulæ, assumes the atmosphere to be at rest, and its pres-ure affected only by temperature, whereas, in fact, the pressure at any point is liable to sudden changes due to variations in the force of the wind, the amount of humidity, etc. The best way to eliminate errors due to these causes is to take readings simultaneously at the points the elevations of which are to be compared. For this purpose an assistant should be stationed at some point of known elevation contiguous to the route to be surveyed, and provided with an aneroid similar to that carried by the engineer. The aneroids, time-pieces, and thermometers having been compared at this point, the assistant should record the readings every ten minutes, with the time, temperature, and state of the weather. The engineer will thus have a standard with which to compare his own observations. If the survey is so extended that the same conditions of atmosphere are not likely to be experienced by the two observers, the assistant should be instructed to move forward to a new station at a designated time; or two assistants may be employed, one at each of two stations between which the engineer intends to make a reconnoissance. Even with these precautions no attempt should be made to obtain the elevation of important points during, or just before, or after a storm of wind or rain.

15. When but one aneroid is used the observations at the several stations should be taken as nearly together as possible in point of time, and then repeated in inverse order, taking the mean of the observations at each station, and repeating the whole operation if necessary. Only approximate results can be hoped for, however, with a single instrument, unless the atmospheric conditions are very favorable.

16. The Locke Level is an instrument in which the bubble and the observed object may be seen at the same instant, enabling the operator to keep the instrument horizontal, while holding it in the hand, like an ordinary spy-glass. While very portable, it enables the observer to define rapidly all visible points of the same elevation as his own, and to estimate from these the relative heights of other points. It may be made useful in a variety of ways which easily suggest themselves to the engineer in cases where no great precision is required, and where a more elaborate level is not at hand.

17. The Prismatic Compass is a portable instrument with folding sights, in using which the bearing to an object may be read at the same instant that the object is observed.

The bearings are read upon a floating card, graduated and numbered from zero to 360°, so that no error can be made in substituting one quadrant for another. The instrument may be held freely in the hand during an observation, though better results are obtained by giving it a firm rest.

CHAPTER II.

PRELIMINARY SURVEY.

18. A preliminary survey consists in an instrumental examination of the country along the proposed route, for the purpose of obtaining such details of distances, elevations, topography, etc., as may be necessary to prepare a map and profile of the route, make an approximate estimate of the cost of constructing the road, and furnish the data from which to definitely locate the line should the route be adopted. The survey is more or less elaborate, according to circumstances. In case the country is new, or the reconnoissance has been incomplete, or if several routes seem to offer almost equal inducements, the survey will partake somewhat of the nature of a reconnoissance, and will be made more hastily than if but one route is to be examined, and that, perhaps, presenting serious engineering difficulties. The survey is made as expeditiously as possible, consistent with general accuracy, but should not usually be delayed for the sake of precision in matters of minor detail.

19. For preliminary survey the **Corps of engineers** is organized as follows:

A chief engineer, an assistant engineer, two chainmen, one or two axemen, a stakeman, and a topographer, these forming the compass (or transit) party, to which a flagman is sometimes added; a leveller and one or two rodmen, forming the level party; and to these is sometimes added a cross-level party of two or three assistant rodmen.

20. The chief engineer takes command of the corps, and directs the survey. He ascertains or estimates the value of the lands passed over, the owners' names, and the boundary lines crossed by the line of survey. He examines all streams, and estimates the size and character of the culverts and bridges which they will require; he notices existing bridges, and inquires concerning their liability to be carried away by freshet; he selects suitable sites for bridges, examines the character of the foundations, the direction of the current relatively to that of the line, and considers any probable change in the direction of the current during freshets; he inspects the various soils, rocks, and kinds of timber as they are met with, and takes full notes of all these and kindred items in his field book. He not unfrequently assumes in addition the duties of topographer. He should run his line as nearly as may be over the ground likely to be chosen for location, so that the information obtained may be pertinent, and so that the length of the line, the shape of the profile, and the estimate based on the survey may approximate to those of the proposed location. To this end he has due regard to the levels taken, and when they show that the line as run fails to be consistent with allowable grades, he either orders the corps back to some proper point to begin a new line, or makes an offset at once to a better position, or continues the same line with some deflection, simply noting the position and probable elevation of better ground, as in his judgment he thinks best. He should at all times maintain a friendly attitude toward proprietors, and by his polite bearing endeavor to secure their cordial support of the new enterprise. If he is tolerably certain that the location will follow nearly the line of the preliminary survey, he should have with him some blank deeds of right of way, and let these be signed by land-owners while they are favorably disposed. When this cannot be done, a blank form of agreement to allow the surveys and construction of the road to proceed until such time as the terms of right of way may be agreed upon may be made very useful. The chief also selects quarters for his men, and in case of camping out he directs the movements of the camp equipage.

21. The assistant engineer takes the bearings of the courses run, and makes a minute of them, with their lengths, or the numbers of the stations where they terminate. He sees that the axemen keep in line while clearing, and the chainmen while measuring; he takes the bearings of the principal roads and streams, and of property lines when met with. In an open country he may save time by selecting some prominent distant object toward which the chainmen measure without his assistance, while he goes forward and prepares to take the bearing of the course beyond. In traversing a forest with not too dense undergrowth, when the line is being run to suit the ground according to a given grade, it is a good plan for the assistant to go ahead of the chainmen as far as he can be seen, select his ground, take his bearing by backsight on the last station, and then have the chainmen measure toward him. In this case both he and the head chainman should be provided with a good sized red and white flag, mounted on a straight pole, to be waved at first to call attention, and afterward held vertically for alignement. Otherwise a flagman must be added to the party, who will select the ground ahead, under the instructions of the chief, and toward whom the survey will proceed in the usual manner.

22. The head chainman drags the chain, and carries a flag which is put into line at the end of each chain length by the assistant engineer or the rear chainman. It is his duty to know that his flag is in line and that his chain is straight and horizontal before making any measurement, and to show the stakeman where each stake is to be driven. A stake is usually driven at the end of each measured chain length, called a station, though in an open and level country the stakes at the odd stations may be omitted, in which case marking pins are used to indicate the odd stations temporarily. In case there is much clearing to be done the head chainman plants his flag in line, and ranging past it, indicates to the axemen what is to be cut, going a little in advance through the bushes so that they may work toward him. The head chainman should be a quick, active and strong man, with a good eye and a taste for his work, as very much of the real progress of the survey depends upon him.

23. The rear chainman holds his end of the chain firmly at the last stake or pin by his own strength, *not* by means of the stake. He keeps the tally by the pins when they are used, and watches the numbers on the stakes to see that they are correct. The end of a course should always be chosen at the end of a chain, if possible, and if not, then at a brass tag indicating tens of feet, as thus the labor of plotting the map will be much lessened. The numbering of stations is not recommenced with each new course, but is continued from the beginning to he end of the survey, through all its courses, and if one course ends with a portion of a chain the next course begins with the remainder of it. It is the rear chainman's duty to attend to this, holding the proper link at the compass station. Any fraction of a chain measured on the line is called a *plus*, and is counted in feet from the previous station. The length of an offset in the line is never included in the length of the line, but if the line should change its course by a right angle, or more, or less, the numbering would go on as usual.

24. The axemen should be accustomed to chopping and clearing, and are, therefore, to be selected in the country rather than the city. They will cut out so much of the underbrush and overhanging branches as may interfere with the sight of the assistant or leveller; but care must be taken not to cut unnecessarily wide, and no tree of considerable size should be felled, except in rare instances. When running by compass, if the assistant goes ahead of the chain, he can always select a position so that no large tree will interfere; or, if the line must be produced and strikes a tree, the compass may be brought up and set close to the tree on the forward side as nearly in line as can be estimated, the slight error in offset being neglected, since the line will be produced parallel to itself by the needle.

25. The stakeman prepares and marks the stakes, and drives them at the points indicated by the head chainman. When no clearing is needed, the axemen keep him supplied with stakes, as the rapid progress of the chain will only give him time to drive them. The stakes should be two feet long and pointed evenly so as to drive straight, and are blazed or faced on two opposite sides, one of which is marked in red chalk with the number of the station. The stake must be driven vertically, and with the marked face to the *rear*, so that it may be read by the rodman as he follows the line.

26. The topographer makes accurate sketches of all features of the country immediately on the line, and extends the sketches as far each side of the line as he can, in a book prepared for the purpose. He must never sketch in advance of the chain, nor in advance of his own position. His work should be done to scale as nearly as possible, using the same scale for distances on the line and at right angles to it. The scale adopted *should never be less than that of the map to be made from the sketches.* The ruled lines of a field book are usually one quarter of an inch apart, so that a scale of one line to a station equals about four hundred feet to an inch. This is the smallest scale ever used. The scale of two lines to a station is most convenient for general use. Four lines to a station are needed in special cases to show details, as in passing through villages. The scale may be changed from time to time as found necessary, but no two scales should ever be used on the same page. The numbering of the stations up the page indicates the scale of the sketch.

27. When the contours of the surface are required, the topographer may join the level party in order that his estimates of heights and slopes may be corrected by the instrument. He should never draw a mass of contours indiscriminately, but should sketch them as they exist at a uniform vertical interval. This interval may be assumed at five feet in a gently rolling country, and at twenty feet in a mountainous one, but an interval of ten feet will be found most convenient generally. If the topographer accompanies the level he can assume the contours at the even tens of feet in elevation, and mark them so, noting where a contour crosses the surveyed line, and sketching its direction and shape both ways from that point. He will estimate the rate of slope of the ground at right angles to the line as so many feet per hundred, and record it from time to time, noting ascent from the line on either side by "A," and descent by "D." If the slope changes within the limit of the page, the line of change may be sketched and the next slope recorded. When little banks or terraces occur, or bluffs and rocks, which cannot be sufficiently indicated by contours, they should be shown by hatchings, and the height noted. Special care should be taken to sketch roads and houses in their correct positions and dimensions, the latter to be either measured, paced or estimated. The dimensions should also be recorded in numbers. The outline of forests may be shown by a scalloped line, and the kind of timber, and whether dense or scattered, written within the inclosed space. Correct outlines are essential, but no time should be given to shading up a sketch with conventional signs. A single sign, or the name of the thing intended, is all sufficient. Land-owners' and residents' names should be recorded whenever they can be obtained, as well as the names of roads, streams and public buildings.

28. The leveller takes charge of the level party and keeps the notes of his work. He reads the rod on benches and at turning points to hundredths of a foot and to tenths at other points. He should direct a bench to be made at least once every half mile, and in a very rough country every quarter of a mile. The benches need not be far from the line, and, if well chosen, may be used as turning points, thus saving time. The elevation of turning points must be computed when taken, so that the elevation of any one of them may be instantly given when called for, and the other elevations will be filled in as far as may be without delaying the survey. As the levels are usually the most essential part of the survey, much care should be taken to have the instrument well adjusted and truly level, and the rod held vertically and correctly read on turning points, but the intermediate work should not be so done as to delay the party unnecessarily. The leveller should use every endeavor to follow closely after the surveying party, so that the chief and topographer may have the advantage of his notes.

29. The rodman's first duty is to hold the rod vertically, and he must learn to do this in calm or windy weather, in level field or on side hill. He may carry a small *disk-level,* which applied to the edge of the rod will show when it is vertical. The turning points are to be selected for firmness and definiteness, and so that they will afford a clear view from beyond for a backsight. The rod is held for a reading on the ground at every stake, the number of which is called out to the leveller as soon as the rodman arrives at it; the rod is also to be held at every prominent change of slope on the line, as the crest and foot of every bank, the rodman calling out its distance from the last stake as *plus* so many feet, but all gentle undulations and minor irregularities are to be neglected. The rod will always be read at the surface of a stream or pond, and also at its deepest part on the line, when possible; otherwise the depth of the water may be found by sounding, and so recorded. Should the line run along a stream the surface will be taken occasionally, opposite certain stations, and in case of a canal, the *elevation* of surface above and below each *lock must be noted.* The rodman makes inquiry for high-*water marks or seeks traces* of them himself in an uninhabited *strict, and holds the* rod upon them that their elevation may be determined. The rodman carries a small axe or hatchet with which to make benches and to trim out any stray branch that may intercept the leveller's view.

30. The assistant rodmen take the slope and elevations of the ground at right angles to the line, using vertical and horizontal rods and a pocket level, or a tape line and clinometer. The cross levels are not taken throughout the whole survey, if at all, but only where the roughness of the country seems to demand it. They may be extended to any distance from the centre line required by the chief—not less, however, than fifty feet as a rule. They may be taken at the stations only, or oftener, if necessary, depending upon the roughness of the surface, the object being to define accurately the contours, and so the shape of the ground. The assistant rodmen will also take soundings when they are needed, either on the line or at right angles to it.

31. In defining the duties of the members of the corps, the instruments used have been incidentally noticed.

32. The compass is preferable to the transit on preliminary surveys, because it can be operated more rapidly, is lighter, and usually has a better needle. It may have either plain sights or telescope, and be mounted on tripod or jacob staff. The simpler forms are preferred for forest work. Not unfrequently the engineer's transit is employed, but using the needle. A preliminary line should not be run by backsights and deflections, unless local attraction is found to exist to such an extent as to destroy confidence in the needle; or, in special cases, where the natural obstacles to a survey are very great. In the latter case the survey partakes of the nature of a location, and should be conducted with similar care and fidelity.

33. The chain is 100 feet long, and composed of 100 links. It should be of steel for lightness, durability, and greater accuracy. Those having rings of hard steel, unbrazed, are least apt to wear. Five marking pins are needed, each having a piece of red flannel attached, for temporary stations, or for keeping points temporarily while measuring by parts of a chain up or down a slope. A pointed plumb bob, with several yards of small cord wound on a carpenter's spool, is useful in chaining over steep declivities or bluffs.

34. The axes should be of best quality, with hand-made *handles, and not too* heavy. The axe of the stakeman should have a fine edge for dressing and a broad head for driving the stakes. When the stakes are not required to be over two feet long, a stout *basket*, having a square, flat bottom, 26x14 inches, should be furnished the stakeman. He will then prepare a basketful of stakes, ready marked, and place them in the basket regularly, in the reverse order of their numbers, so that they will come to hand as wanted. *A small hand-saw* no larger than the basket, with rather coarse teeth, wide set, will be found extremely useful in cutting stakes with square heads and of uniform length, and much more rapidly than can be done with an axe. When not in use, it is to be strapped to the inside of the basket, to prevent its being lost by the way. When the basket is about empty, the stakeman, with the assistance of the axemen, can soon replenish it, and the stakes being all numbered at once, there is less danger of a mistake being made in the tally than when they are marked only as wanted.

35. The level should be the regular engineer's level, the same as used on location.

36. The rod should be self-reading, *i.e.*, to be read by the leveller, as too much time would be consumed in the constant adjustment of a target by the rodman. It should be as long as can be conveniently handled in order to reduce the number of turning points on hill sides. A very convenient rod may be made of thoroughly seasoned clear white pine, sixteen feet long and two inches wide, with a thickness of one inch at the bottom, increasing to one and a quarter inches at six feet from the bottom, and then gradually diminishing to three eighths of an inch at the top. The rod is shod with a stout strap of steel, extending five inches up the edges, and secured by screws. The top is protected for a few inches by a plate of sheet brass on the back. The face of the rod is a plain surface throughout, and is graduated from the lower edge of the steel shoe as zero. The divisions are fine cuts made with the point of a knife. At the foot and half-foot points the cuts extend across the face. For the tenths and half tenths they extend three quarters of an inch from the right hand edge, terminating in a line scribed parallel to the edge of the rod, thus forming rectangular blocks half a tenth wide, every other one of which is painted black, *the body of the* rod being white. The feet are *indicated by numerals* painted red on the blank part of the face, each figure standing exactly on its foot mark, and being exactly one tenth high. For the figure 5 the Roman V. is substituted and for 9 the Roman IX., so that in case a dumpy level is used the 5 may not be mistaken for a 3, nor the 9 for a 6. At the half-foot points a red diamond is painted, so that the graduated line bisects it. No other figures nor graduations are required. With this rod the leveller can read quite accurately to hundredths of a foot, and after some practice can estimate the half hundredths.

37. The horizontal rod for cross-levels may be made of white pine, ten feet long and one inch thick by three wide, tipped with brass, painted white, and graduated to feet and tenths. It must be a straight edge, and is levelled by a pocket level placed upon it when needed, or by a small level embedded permanently in one edge. The vertical rod to be used with it is made of pine eight feet long and one and a quarter inches square, and graduated to feet, tenths, and half tenths. All rods when not in use should be laid on a flat surface to prevent their being sprung. Leaning them in a corner soon ruins them for use.

38. The clinometer is any small instrument which will measure the slope angle of the surface. The angle is always estimated from the horizon, a vertical being 90°. The rise per 100 feet is found by multiplying the nat. tangent of the slope angle by 100. It may often be found more easily by the leveller reading the rod at a station and then 100 feet left or right of the line. If surface measures are taken in connection with a slope angle they are reduced to horizontal measures by multiplying them by the cosine of the slope angle.

39. The plane-table is rarely if ever used on preliminary surveys in the United States. Occasional bearings taken to prominent objects by the assistant engineer, or the use of a prismatic compass by the topographer in connection with his sketches, is found to answer every purpose.

40. In case a survey is to be made with a transit, it is necessary to add a back flagman to the party, who will hold his flag or rod on the point last occupied by the transit, so that the assistant may take a backsight upon it. The direction of a new course in each case is determined by the deflection angle to the right or left of the preceding course produced. The bearing of one long course near the beginning of the survey having been carefully ascertained, the bearing of each succeeding course is calculated from the deflections, and entered in a column of the field book headed *Calculated Bearings*, from which the line is afterwards plotted. The magnetic bearing of each course should also be taken from the needle, and recorded as such, but is used only as a check on the transit work. The deflections should be made in degrees, halves, or quarters, if possible, to facilitate the calculation of bearings, and to admit of using a traverse table.

41. The attached level and vertical arc of the transit are useful in determining approximately the grade of the line run in advance of the level party, or in seeking for one assumed grade to which it is desired that the line shall conform. For this purpose it is only necessary to set the vertical arc to the angle corresponding to the grade as given in Table XIV., and let the head chainman move about until a point on his rod at the same height from the ground as the telescope is covered by the horizontal cross-hair.

42. The point on the ground where a transit is set up is marked by a good-sized plug, flat headed, and driven down flush with the ground, with a tack set in the head to show the exact point or centre. This is called a *transit point*. When a transit point occurs at a regular station, the stake bearing the number of that station is set three feet to the left of the line opposite the plug and facing it. When a transit point occurs between stations the stake is driven three feet to the left of it, marked with the number of the preceding station + the distance from that station in feet.

43. As a transit is capable of giving a line with great precision, it is important that the flags used in connection with it should be equally precise in giving points. An excellent flag for this purpose is made of well-seasoned clear white pine ten feet long, two and a half inches wide, and one inch thick. It is tapered for the last four inches to an edge at one end, the edge being formed at the middle of the width. The tapered end is shod with a band of steel covering the edge of the rod, and secured by screws, and the steel is brought to a sharp edge at the point of the rod. The rod is then painted white and tipped with brass at the square or upper end. A centre line on the face is then struck from the point of the steel to the middle of the brass tip by means of a piece of sewing silk, and a fine cut made with a knife and steel straight edge. The centre line must *not* be scribed parallel to one edge of the rod, as this is rarely ever straight. The face of the rod is then divided into one-foot spaces, measured from the head of the rod, and these are painted red on either side of the centre line in alternate blocks. On the back of the rod at three and a half feet from the point is placed a small ground-glass bubble-tube, mounted very simply, and attached to the rod by a brass plate and screws, and guarded by two blocks of wood for protection. The centre line of the rod is made vertical by a plumb-line while the level tube is being attached, which ever after secures a vertical rod. If only two feet of this rod can be seen over any obstruction, a point can be set with great precision, provided the level tube is in adjustment. This flag can also be used as a plumb in chaining with much more satisfaction than a cord and weight, especially in windy weather.

44. A transit survey usually requires more clearing than one made by compass. When a given course is to be produced in a forest, some large trees will inevitably be encountered, but the labor and delay of felling them may be avoided by the use of *auxiliary lines*. These may be classified as running *parallel* to the main line, at a *small angle* with it, or at a *large angle* with it.

45. The parallel line is established by means of two short perpendicular offsets measured with care before reaching the obstacle, and the main line is established beyond the obstacle by means of two more equal offsets. But since short back-sights are to be avoided, these offsets should be at least 100 feet apart, so that it may be difficult to find a parallel line of sufficient length which does not strike some other obstacle, or at least require considerable extra clearing.

46. The auxiliary lines making a small angle with the main line are more convenient, not only on this account, but because they require a less number of transit points. By them an isosceles triangle is formed on the ground, having the intercepted portion of the main line as base, and the vertex near the obstacle. The deflections at the points where the lines leave and join the main line are similar and equal, and flection at the vertex is double in amount and opposite ection. No calculation is necessary, for the error in rement due to the deviation is too small to be noticed, nce the main line is immediately resumed, the calculated gs of the auxiliary lines are unnecessary. Should the where the second line joins the main line prove unsuit or a transit point, the second line may be produced to nvenient point beyond, and so go to form an isosceles le on the opposite side of the main line, the triangle completed by running a third line parallel to the first, qual to the difference of the first and second. Again, cond line may encounter a serious obstacle before reache main line. To avoid this a parallel to the main line e run from the end of the first line for a contt distance, and there the second line be put in el and equal to its first position, as before ded, thus forming a trapezoid.

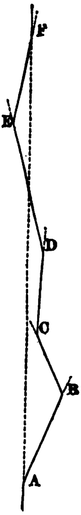

FIG. 1.

. The following **general solution** of this m allows the engineer to make use of any er of auxiliary lines, provided that none of make an angle of much more than one degree he main line, with a certainty of resuming the line in position and direction at the extremity y course desired, and without necessitating igonometrical calculation. It is based on the ption, practically true for small angles, that ies are proportional to their angles, and is exd by the following **rule:**

all deflections to the right *plus*, and all to ft *minus;* multiply the length of each course t by the algebraic sum *in minutes* of all the ry deflections made to reach that course; he algebraic sum of these products, and the sum equals zero the extremity of the last will be on the main line. The deflection ed at that point to give the *direction* of the main line al to the algebraic sum of all the preceding deflections, ken with the contrary sign.

s, if we have left the main line at A, and run by these *(Fig. 1.)*

Sta.	Defl.	Dist.		Factors.	Products.	
A	16′ R	190	we shall have these products:	$+16 \times 190 =$	$+3040$	
B	31′ L	120		$-15 \times 120 =$		-1800
C	18′ R	175		$+\ 3 \times 175 =$	$+\ 525$	
D	13′ L	265		$-10 \times 265 =$		-2650
E	15′ R			$+\ 5 \times$ (?)		
					3565	-4450

and their algebraic sum is $-\ 885$

Therefore to render the sum zero we must add 885 as the product of the last course. But 5′ is already given as one factor, so that the other factor must be $\frac{885}{5} = 177$, which is the length of the last course, giving some point *F* on the main line. The deflection at *F* from the last course to give the direction of the main line is

$$16 - 31 + 18 - 13 + 15 = 5'$$

and changing the sign we have $-5'$; that is, the deflection is to the *left*.

The distance on the main line from *A* to *F* equals the sum of the courses, or 927 feet, but this we have by the stations, which have been kept by stakes in the ordinary way. All the stakes on the auxiliary lines will be more or less off the main line, but as these offsets are usually very small, they are considered of no consequence on a preliminary survey through a forest. In Fig. 1 the offsets are very much magnified. **The field notes** of such auxiliary courses should be kept, not as regular notes, but on the margin or opposite page, and in such a way that, while the line may be retraced by them on the ground, the draughtsman may see that it is not necessary to plot them, when a straight line ruled and measured through is sufficient. It is obvious that in selecting a closing course either the deflection may be assumed and the length calculated, or *vice versa;* but care should be taken to assume such values as do not involve a fraction in either factor, if possible.

48. The method of passing an obstacle on the line by ***auxiliary lines*** **at a large angle** with the main line will *only be resorted to* when circumstances are such that the other *methods mentioned* cannot be employed, as in passing a building, *pond, or densely* wooded swamp. In such a case we ma turn a right angle with the transit, and measure accurately one offset, putting a transit point at its extremity, where another right angle will give a parallel line. If the offset prove too short for an accurate backsight, a temporary point at a sufficient distance may be established for that purpose on the offset line produced before the instrument is removed from the main line. If any other angle than 90° is used it should be selected, when circumstances permit, so that the distances on the intercepted part of the main line may be in some **simple ratio** to the distances measured on the auxiliary line. Thus a deflection of 60° gives a distance on the main line equal to half the length of the auxiliary course, that is,

60°	gives	a ratio	of ½ =	0.5	
53° 08′	"	"	"	0.6	nearly
45° 34½′	"	"	"	0.7	"
36° 52″	"	"	"	0.8	"
25° 50½′	"	"	"	0.9	"

the angles being taken to the nearest half minute.

49. If it be desired that the stakes on the auxiliary line should stand on **perpendiculars** through the true stations on the main line, a certain correction must be added to each chain length depending on the angle which the auxiliary makes with the main line. If there is a fraction of the chain at either end of the course, a proportional addition must be made for this. Thus, by referring to the table of external **secants,** we find that we must add a correction as follows:

2° 33½′	...0.1 ft.	per chain.	6° 45½′	...0.7 ft.	per chain.
3° 37′	...0.2 "	"	7° 13½′	...0.8 "	"
4° 26′	...0.3 "	"	7° 39½′	...0.9 "	"
5° 07′	...0.4 "	"	8° 04′	...1.0 "	"
5° 43′	...0.5 "	"	9° 52′	...1.5 "	"
6° 15½′	...0.6 "	"	11° 22′	...2.0 "	"

These methods of suiting the angle to an even measure are much superior to assuming an even number of degrees deflection, and *then calculating* the distance by trigonometry. The *ast table, which may* be extended indefinitely by reference to *e table of Ex. secants,* is perfectly adapted to **chaining** b *rface measure* **on regular slopes** when the slope angle known, the chain being lengthened by the correction corresponding to the slope angle.

50. If the chain is lengthened as per above table on auxiliary lines, the numbering of the stakes goes on as usual, but they should have an additional mark as × to show that they are off the main line; and they may stand facing the true stations which they represent, and the length of offset, if known, may also be recorded on them. The leveller will then understand that he is to read the rod not only at the stakes as they stand, but also at the true stations, as nearly as may be. The assistant engineer will always make **a diagram** in his field book, showing exactly the method pursued in reference to auxiliary lines. Having passed the obstacle, it is advisable to return to the main line by a course equal in length to the first auxiliary, and making an equal angle with the main line. If this cannot be done from the end of the first course, a parallel to the main line may be run any convenient distance, and the return line then put in, forming a trapezoid.

51. When there is no **obstruction** to sight on the main line, but only **to measurement,** a transit point should be set in line beyond the obstacle before the transit leaves the main line, as a check on the other operations, and the main line should be afterward produced from this point by backsight on the main line, rather than by deflection from an auxiliary line.

52. The main line should always be resumed as soon as practicable, making the auxiliary lines the mere exception. When a number of courses at a large angle are likely to be required before the main line can again be reached, it may be better to consider these as regular courses of the survey, and to note them as such. The *simplest* method is always the *best*, because least likely to involve mistakes.

53. When the **natural obstacles** are so **numerous** and of such magnitude as to render any continuous line of survey or location extremely difficult, if not impossible, as in the case of a bold rocky shore, all the data necessary to a location should be gathered with precision on the preliminary survey, *the measurements* and angles being taken with the greatest care, *and as many checks* as possible should be introduced to verify *the work.* In meandering such a shore it is probable that a large *number of short* courses will be used, which may be measured correctly, but there is liability to error in the angles. To verify the latter the more conspicuous transit stations on prominent points of the shore are selected, and these being named by the letters of the alphabet, the deflections between them are taken by careful observations repeated a number of times, as for a triangulation. These points, joined by tie-lines, then form a survey of themselves, much simpler than the full traverse. To obtain the length of these tie-lines, the angles between them and the courses meeting at the same station are measured. Then since each tie-line forms the closing side of a field, in which all the bearings are known, and all the distances, save one, that one may be calculated by latitude and departures. But the angles should first be tested for error in each complete field, and if the error be large the angles must all be remeasured until the error is found and corrected, but if very small it may be distributed among the angles, or among those most probably inaccurate. Before calculating the traverse of any of these fields, it will be advantageous to assume, for an artificial meridian, a line parallel to the average direction of the shore for several miles, and to refer all courses to this meridian for their bearing. This meridian is called the *axis* of the survey, and all bearings referred to it are called *axial bearings*, as distinguished from magnetic bearings. The magnetic bearing of the axis should be some exact number of degrees, so as to facilitate the reduction from one system to the other.

54. In plotting the map, the axis is first laid down, and then the lettered stations in their respective positions, after which the meandering surveys can be filled in. The map being drawn on a scale of one hundred feet to an inch, and the contours constructed from the notes of the level and cross-level parties, the engineer may project the location upon it with great certainty and economy of result. But he should calculate the traverse of the location as projected, and compare it with the traverse of the preliminary, to eliminate all errors in drafting, before taking his notes to the field to reproduce the location on the ground. Any point where the location crosses the preliminary should have the same latitude and longitude by the traverse of either line. This system, though laborious, is the *only one that will* ensure a successful location under the *circumstances supposed*. Advantage may sometimes be taken of cold weather to cross bays and inlets on the ice, but there is great liability to error in angles taken upon the ice, due both to its motion and to the sinking of the feet of the tripod into the ice as soon as exposed to the rays of the sun.

CHAPTER III.

THEORY OF MAXIMUM ECONOMY IN GRADES AND CURVES.

55. Before commencing the field work of location it devolves upon the engineer to decide as to which of the surveyed routes shall be adopted as being most advantageous in all respects, and also to establish the maximum grade in each direction and the minimum radius of curve on that route.

The general considerations which guide the engineer in the selection of one of several routes for location are such as were hinted at in the chapter on reconnoissance, but upon the completion of the preliminary surveys he has at hand a large amount of information which enables him to consider this important question much more in detail. Unless his instructions are explicitly to the contrary, he may assume it to be his duty to find the *best line*, or that one which, for a series of years following the completion of the road, will require the least annual expense, including interest on first cost. The finances of the company may be so limited as not to permit the construction of the best line at once, and it may then be the duty of the engineer to select the *cheapest line*, or that of least first cost, as a temporary expedient, with the expectation of building the road at its best when the improved credit of the company will permit. But generally he will be able to build the cheaper portions of the best line at once, only making deviations and introducing heavier grades at the expensive points to avoid a cost beyond the present means at his command. The selection of the best line may be a question as between different routes or as between different grades and curves on the same route. We will consider the latter case first.

56. *To solve the problem of true economy we must determine the actual expense both of building and operating* the line at a given maximum grade, and also what changes will be made in these expenses by a change in that maximum. We have then, on one hand, the annual interest upon the original cost, and, on the other, the annual expense of operating the road. *The best grade is that which will render the sum of these two a minimum.* Both forms of expense consist of two parts: one that is affected by a change in grade, and the other that is not. Clearly the former is the only one we have to consider in either, since when the sum of the variable portions is a minimum, the sum total will be a minimum also. The varying portions then are functions of the grade, though independent of each other. If, therefore, we let z' represent the maximum grade in feet per mile, and let x represent the corresponding value of that portion of the annual expense which varies with the grade, and establish the relation existing between the two, we shall have $x = f(z')$. Similarly if we let y represent the interest on so much of the first cost as is affected by grade, we shall have $y = f'(z')$. The problem then is to find that value of z' which shall render

$$x + y = a\ minimum.$$

Let us now seek the complete expression represented by $x = f(z')$.

The elements that enter into this expression are numerous, and will be considered in succession.

57. The traction of an engine is the force with which it pulls a train, and is limited by the reaction of the drivers against the rails. It depends on the weight upon each driver, the number of drivers, and the coefficient of friction. The weight on one driver should not exceed 12,000 lbs., and is usually less than this. If the exact proportions of engine that will be used on the road are not known, the weight per driver may be assumed at 10,000 lbs., with 4 drivers for ordinary grades and traffic, or at 11,000 lbs. with 6 drivers, if the grades are steep and the tonnage large. For extraordinary grades special engines are required, having 8 or 10 drivers. The coefficient of friction, called also the adhesion, varies from .09 to .37, these being the extremes on record. The lowest is due to extremely unfavorable circumstances, as sleet and frost; the highest doubtless to the use of sand, though not so stated in the record. The more common range of values is from .15

.25. For our present purpose it will be assumed at .20, so that if a 4-driver engine has 10,000 lbs. on each driver, its traction is 40,000 × .20 = 8000 lbs. when hauling its maximum train.

58. The expense of running an engine one mile, hauling a train, on the proposed road, can only be estimated from the experience on other roads similarly situated. The expense is composed of the items of fuel, water, oil and waste, repairs (including renewals), wages of conductor, engineer, and fireman, engine-house expenses, and interest on first cost of engine and engine-stall. The range and approximate average of these items is here given:

ITEMS.	4-DRIVER ENGINE.		4-DRIVER	6-DRIVER	8-DRIVER
	Lowest.	Highest.	Average.	Average.	Average.
Fuel....	$0.050	$0.210	$0.100	$0.165	$0.213
Water...............	.001	.010	.004	.006	.008
Oil and waste..	.004	.030	.006	.008	.010
Repairs and renewals	.050	.150	.080	.104	.133
Wages........	050	.100	.075	.075	.075
Engine-house.	025	.060	.035	.050	.060
Interest..............	.025	.038	.030	.038	.047
Totals...	.205	.598	.330	.446	.546

In a given case the probable value of each item should be estimated separately, and the sum taken afterwards. In the above averages each engine is supposed to haul its maximum train. The relative expense of the several classes of engines has not been established conclusively.

59. The resistance offered to the motion of a railway train is occasioned by a variety of causes, concerning which a great deal of uncertainty exists as to their relative effect. An investigation which should seek to determine the exact amount of each partial resistance, and then by a summation derive the total, would be tedious, and, in the present state of our knowledge, unsatisfactory. We shall therefore simply *group* the resistances under three general heads, namely:

Resistance due to uniform motion on a straight, level track;

Resistance due to grade;

Resistance due to curvature.

60. The first of these, considered as an aggregate of the various items of friction in engine and train, of oscillations and impacts, and of resistance of the atmosphere, is found to vary nearly or quite as the square of the velocity. The friction of an engine is greater in proportion to its weight than that of a car, owing to its many moving parts, so that the resistance of a short train is greater in proportion to its total weight than that of a long train. The resistance of the atmosphere is greater also in proportion to the weight of a short train than of a long one. An empty train will offer more resistance in proportion to its weight than a loaded one. A formula which shall express the resistance of a train to uniform motion must include at least the velocity and the weight of the train and engine.

The following empirical formula is based upon a careful investigation of all such records of experiments on the subject, several hundred in number, as have come to the author's notice, and is believed to give results agreeing closely with the *average* experience and practice of the present day. It is designed to give the resistance per ton for all trains, whether freight or passenger, and at any velocity, under ordinary circumstances. Accidental circumstances, such as the state of the weather, and the condition of the road-bed, rails, and rolling stock, may largely modify the resistance, but these, of course, are not taken account of in the formula.

Let V = velocity of train in miles per hour,
" E = weight of engine and tender in tons,
" W = weight of cars in tons,
" T = weight of freight in tons,
" q = resistance to uniform motion in lbs. per ton.

We then have the formula

$$q = 5.4 + \left(.006 + \frac{.0006E^2}{E + W + T}\right) V^2 \qquad (1)$$

61. The second resistance considered is that due to gravity in grades. It varies in the exact ratio of the rise to the length of the grade.

Let G_s = *rise of grade* in feet per station.
" G_m = *rise of grade* in feet per mile.
" q' = *resistance* in pounds per ton due to grade.

Then,

$$\left.\begin{aligned} q' &= 2240\,\frac{G_s}{100} = 22.4\,G_s \\ q' &= 2240\,\frac{G_m}{5280} = \frac{14}{33}G_m \end{aligned}\right\} \qquad (2)$$

62. The third resistance considered is that due to curvature of the track. This resistance is due to the friction of the wheels upon the top of the rail, and of their flanges upon the side of the rail. The top friction is lateral, due to the oblique position of the wheel on the rail, and longitudinal, due to the greater length of the outer rail, since both wheels are rigidly attached to the axle. The flange friction is due to the reaction of the top friction, which, combined with the parallelism of the axles, throws the truck into an oblique position on the track. A forward flange presses the outer rail, while a rear flange is usually in contact with the inner rail. The centrifugal force of the car will increase the pressure on the outer rail, unless the ties are inclined at an angle sufficient to counterbalance this force. But if the ties are inclined too much, or the velocity is less, the pressure on the inner rail will be increased. An uneven track will cause the truck to pursue a zigzag course, increasing the resistance considerably.

Experiments for determining the amount of curve resistance have been neither numerous nor very satisfactory, but the generally accepted conclusion is that the resistance is a little less than half a pound per ton on a one-degree curve, and that it varies as the degree of curve. On European roads, however, it is estimated at about one pound per ton per degree of curve, owing largely to the form of rolling stock used.

63. Let q'' = curve resistance in pounds per ton on any curve,

and D = degree of curve.

Then, assuming the resistance per ton on a one-degree curve at 0.56(, we have for any other curve

$$q'' = 0.56D \qquad (3)$$

To ascertain what grade upon a straight line will offer the same resistance as a given curve; substitute the value of q'' r q' *in eq. (2)* and solve for G; whence

$$\left.\begin{aligned} G_s &= 0.025\,D \\ G_m &= 1.32D \end{aligned}\right\} \qquad (4)$$

For definition of degree of curve, see Art. 84.

64. It is evident that grades and curves, by their resistances, fix a limit to the weight of a train which a given engine can haul over them.

A locomotive is usually built with such a surplus of boiler and cylinder capacity that its power, at ordinary velocities, is limited by the adhesion of the drivers, so that the adhesion is the proper measure of the tractive force.

To find an expression for the maximum train which a given engine can haul over a given grade and curve:

Let P = tractive force of engine in pounds,
" T' = weight of paying load in tons per maximum train,
" W' = weight in tons of cars carrying the load T'.

Then for uniform motion, at a given velocity,

$$(E + W' + T')(q + q' + q'') = P \qquad (5)$$

Let t = average load of one car in tons
" w = average weight of one car and load in tons.

Then $W' + T' = \frac{w}{t}T'$, substituting which in eq. (5) we derive

$$T' = \frac{t}{w}\left(\frac{P}{q + q' + q''} - E\right) \qquad (6)$$

In this equation q = the resistance per ton due to uniform motion, q' = the resistance per ton due to the *maximum grade* opposed to the direction of the train, and q'' = the resistance per ton due to the *sharpest curve on that grade.*

For accelerated motion the reaction of inertia of the train must be added to the above resistances. This is estimated at $\frac{1}{2}q$, in order that a train starting from rest may acquire the requisite maximum velocity, even on a maximum grade, in a reasonable time, say from 3 to 6 minutes. Therefore, for accelerated motion,

$$T' = \frac{t}{w}\left(\frac{P}{\frac{3}{2}q + q' + q''} - E\right) \qquad (7)$$

Now, the *values of* T and q involve each other, but if we *ccent* W *and* T *in eq.* (1) the value of q becomes that used in eq. (7), and we may eliminate q between these equations, and derive the value of T'; whence

$$T' = \frac{\frac{t}{w}(P - .0009E^2V^2)}{q' + q'' + 8.1 + .009V^2} - \frac{t}{w}E \qquad (8)$$

Also, for the weight of maximum train and load,

$$W' + T' = \frac{P - .0009E^2V^2}{q' + q'' + 8.1 + .009V^2} - E \qquad (9)$$

which is the expression required.

When there is no curve on the maximum grade, q'' is zero; and when there is no grade, q' is zero; hence for a straight level track eq. (7) becomes

$$\left.\begin{aligned} T_0' &= \frac{t}{w}\left(\frac{2P}{3q} - E\right) \\ \text{and eq. (8)} \qquad T_0' &= \frac{\frac{t}{w}(P - .0009E^2V^2)}{8.1 + .009V^2} - \frac{t}{w}E \end{aligned}\right\} \qquad (10)$$

65. An engine-stage is a division of the road to which an engine is limited, and over which it regularly hauls a train. Its length varies, on existing roads, from 50 to 200 miles or more, depending on the grades, on the length of the whole line, and on the distance between points favorable for the location of shops, etc. The average engine-stage on American roads is not far from 75 miles. If there are to be several engine-stages on the proposed line, the problem of maximum economy of grade must be solved with reference to each of them separately.

Let L = length of engine-stage in miles,
" e = expense per engine-mile in dollars,
" A = average annual paying freight in tons moving in one direction, and
" a = average annual paying freight in tons, moving in the opposite direction; and if these are not equal, let A be *greater* than a. Now T' eq. (8) is the maximum train-load *which, at a velocity* V, should be hauled up steepest grade z', *opposed to the direction* of the tonnage A; hence $\frac{A}{T'}$ = the

number of trains per annum; and since each train must go and return, $\therefore \frac{2LA}{T'}$ = the total train-mileage per annum.

If there were no return tonnage, the annual expense chargeable to A would be $\frac{2ALe}{T'}$, but since some of the cars return loaded with the freight a, these are not chargeable to A, and must be deducted from the above expression. Hence if we denote the annual expense of engine-mileage by x,

$$x = \frac{(2A - a)\,Le}{T'} \tag{11}$$

in which the value of the maximum grade z' is involved in the value of T'.

But we may obtain an expression for x in terms of z'; for, at any given velocity, the resistance, q_0, on a level is equal to the resistance due to a certain grade z_0, the value of which is, by eq. (2), for uniform motion,

$$z_0 = \frac{33}{14} q_0$$

So the resistance, q, to motion up a grade z' is equal to the resistance due to some grade $z = \frac{33}{14}\, q$, the total resistance being that due to the combined grades $z + z'$. Now, since the gross weight of a maximum train, under a constant engine power, is inversely as the resistances, we have, for conditions of accelerated motion:

$$\frac{w}{t} T' + E : \frac{w}{t} T_0' + E :: \tfrac{3}{2} z_0 : \tfrac{3}{2} z + z'$$

whence

$$T' = \frac{\tfrac{3}{2} T'_0 z_0 - \frac{t}{w} E\left(z' + \tfrac{3}{2}(z - z_0)\right)}{\tfrac{3}{2} z + z'} \tag{12}$$

in which T'_0 = maximum train-load on a level line. Substituting this value of T' in eq. (11) we have

$$x = \frac{\tfrac{3}{2} z + z'}{\tfrac{3}{2} T'_0 z_0 - \frac{t}{w} E\left(z' + \tfrac{3}{2}(z - z_0)\right)} (2A - a)\,Le \tag{13}$$

which is the complete expression for $x = f(z')$ required.

66. Could we also find a complete expression for $y = f'(z')$, we might then proceed to find, by analysis, that value of z' which would render $x + y =$ a minimum. But the value of y cannot be formulated, since it depends on the accidental features of the country through which the line passes; it can only be determined for any given value of z' by an estimate based on the survey. We therefore resort to a graphical solution.

Equation (13) is the equation of a curve in the plane ZX, Fig. 2. If we assume several values of z', and calculate the corresponding values of x, we may lay these off by scale on the axes of Z and X respectively, and so obtain several points through which the curve of annual expense may be drawn. We then make estimates of the cost of constructing the road at the same values of z', and taking the annual interest of each estimate as an ordinate y to OZ in the plane ZY, we lay it off to scale at the proper height, thus obtaining a series of points in the plane ZY, through which the curve of annual interest on first cost may be drawn. If now we suppose the plane ZY to be revolved to the left about the axis OZ until it coincides with the plane OX, as in Fig. 2, we shall see that the two curves are convex to OZ and to each other. The shortest horizontal line intercepted by them indicates the minimum value of $(x + y)$, and the point where this line cuts the axis OZ indicates the corresponding value of z', which is the one required. If tangents be drawn to the curves at the points where the shortest horizontal line intersects them, the tangents will be parallel to each other. Any convenient scales may be used to lay off the values of z' and x, provided that the values of x and y be laid off to the same scale. It is well

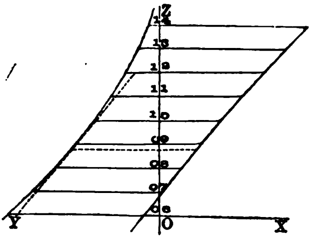

FIG. 2.

to reduce all the values of x by an amount common to them all, and the same with respect to values of y, before laying them off to scale. This will bring the two curves nearer together without altering their form.

67. To facilitate the calculation of x, we give on the next page a table of values of $\frac{1}{T'}$ for several engines, using eq. (8) for this purpose. The value of x is therefore found, eq. (11) or (13), by multiplying $(2A - a)\ Le$ by the proper tabular number, under conditions assumed as follows:

t = 10 tons of freight per car-load;
w = 18 tons per car and load;
V = 12 miles per hour.
For a 4-driver engine, $E = 42$ tons; $P = 8100$ lbs.
For a 6-driver engine, $E = 49.5$ " $P = 12600$ "
For an 8-driver engine, $E = 59.4$ " $P = 17280$ "

Substituting these values in eq. (8), and making $q'' = 0$, we find the maximum loads of freight which the several engines can haul up the grade whose resistance is q'. The reciprocals of these loads are given in the table opposite the grades noted in the first and last columns.

68. Since q'' is made zero, the grades in the table are assumed to be on straight lines. In locating a road, the *maximum* grade should be reduced on a curve by the amount of the equivalent-grade of the curve, eq. (4), so that the resistance may be no greater on the curve than elsewhere. But grades less than the maximum need not necessarily be reduced for the curves upon them, unless the sum of the grade and the curve-equivalent exceeds the maximum.

69. For an example, let us suppose that a certain engine-stage is to be 80 miles long, and that an estimate of the cost of construction has been made, based on a ruling or maximum grade of 52.8 ft. per mile against the heavier traffic, and that the annual interest on the estimate amounts to $168,000.

Let us further suppose that the average traffic in one direction is estimated at 375 000 tons per annum, and in the other direction at 125 000 tons, that it is decided to use 6-driver engines, and that the expense per engine-mile is estimated at 40 cents; hence $(2A - a)\ Le = 20\,000\,000$. We are now required o find the most economical maximum grade.

We first select at least two other maximum grades, and havir

$V = 12$. TABLE OF RECIPROCALS OF T'. $t = 10$, $w = 18$.

G_s.	$E = 42$ $P = 8100$	Diff.	$E = 49.5$ $P = 12600$	Diff.	$E = 59.4$ $P = 17280$	Diff.	z' ft. per mile.
4.0	.0479 844	22 445	.0241 385	8 954	.0162 847	5 597	211.20
3.9	.0457 399	21 363	.0232 431	8 692	.0157 250	5 464	205.92
3.8	.0436 036	20 357	.0223 739	8 442	.0151 786	5 336	200.64
3.7	.0415 679	19 420	.0215 297	8 203	.0146 450	5 212	195.36
3.6	.0396 259	18 547	.0207 094	7 974	.0141 238	5 092	190.08
3.5	.0377 712	17 732	.0199 120	7 753	.0136 146	4 978	184.80
3.4	.0359 980	16 968	.0191 367	7 543	.0131 168	4 866	179.52
3.3	.0343 012	16 253	.0183 824	7 341	.0126 302	4 757	174.24
3.2	.0326 759	15 583	.0176 483	7 147	.0121 545	4 653	168.96
3.1	.0311 176	14 952	.0169 336	6 960	.0116 892	4 553	163.68
3.0	.0296 224	14 360	.0162 376	6 780	.0112 339	4 455	158.40
2.9	.0281 864	13 803	.0155 596	6 608	.0107 884	4 360	153.12
2.8	.0268 061	13 277	.0148 988	6 442	.0103 524	4 269	147.84
2.7	.0254 784	12 779	.0142 546	6 282	.0099 255	4 180	142.56
2.6	.0242 005	12 310	.0136 264	6 128	.0095 075	4 094	137.28
2.5	.0229 695	11 867	.0130 136	5 979	.0090 981	4 011	132.00
2.4	.0217 828	11 447	.0124 157	5 836	.0086 970	3 930	126.72
2.3	.0206 381	11 048	.0118 321	5 699	.0083 040	3 852	121.44
2.2	.0195 333	10 670	.0112 622	5 566	.0079 188	3 775	116.16
2.1	.0184 663	10 311	.0107 056	5 436	.0075 413	3 701	110.88
2.0	.0174 352	9 970	.0101 620	5 312	.0071 712	3 630	105.60
1.9	.0164 382	9 646	.0096 308	5 193	.0068 082	3 560	100.32
1.8	.0154 736	9 337	.0091 115	5 076	.0064 522	3 493	95.04
1.7	.0145 399	9 043	.0086 039	4 965	.0061 029	3 427	89.76
1.6	.0136 356	8 763	.0081 074	4 856	.0057 602	3 363	84.48
1.5	.0127 593	8 494	.0076 218	4 751	.0054 239	3 301	79.20
1.4	.0119 099	8 239	.0071 467	4 649	.0050 938	3 240	73.92
1.3	.0110 860	7 995	.0066 818	4 551	.0047 698	3 181	68.64
1.2	.0102 865	7 761	.0062 267	4 457	.0044 517	3 124	63.36
1.1	.0095 104	7 538	.0057 810	4 365	.0041 393	3 069	58.08
1.0	.0087 566	7 324	.0053 445	4 274	.0038 324	3 015	52.80
.9	.0080 242	7 119	.0049 171	4 187	.0035 309	2 962	47.52
.8	.0073 123	6 923	.0044 984	4 104	.0032 347	2 910	42.24
.7	.0066 200	6 734	.0040 880	4 022	.0029 437	2 860	36.96
.6	.0059 466	6 553	.0036 858	3 943	.0026 577	2 811	31.68
.5	.0052 913	6 380	.0032 915	3 865	.0023 766	2 764	26.40
.4	.0046 533	6 213	.0029 050	3 791	.0021 002	2 718	21.12
.3	.0040 320	6 052	.0025 259	3 719	.0018 284	2 672	15.84
.2	.0034 268	5 898	.0021 540	3 648	.0015 612	2 628	10.56
.1	.0028 370	5 750	.0017 892	3 580	.0012 984	2 585	5.28
0.0	.0022 620		.0014 312		.0010 399		0.00

In this table T′ = tons of freight for a maximum train of fully loaded cars hauled up any grade z' at a velocity of 12 *miles per hour; the ratio of dead to paying load being assumed at 8 to 10. Hence gross load of train behind engine =* $\frac{18}{10}$T′. *The track is assumed straight, hence* $q'' = 0$ *in eq. (8) fo this table.*

made an estimate of the cost of constructing the road upon each, take the annual interest of each, as in the first case. Let us suppose the two ruling grades thus selected to be 73.92 ft. and 31.68 ft. per mile, or 1.4 ft. per station and 0.6 ft. per station, and the interest on the estimates to be $145 596 and $204 388 respectively, giving the following statement:

G_s.	y.	1st diff.	2d diff.
1.4	145 596	22 404	
1.0	168 000	36 388	13 984
0.6	204 388		

Interpolating by second differences, we have the complete statement:

G_s.	y.	diff, y.	diff. x.	x.	$x + y$.	z'.
1.4	145 596	4 290	9298	142 934		73.92
1.3	149 886	5 164	9102	133 636		
1.2	155 050	6 038	8914	124 534		
1.1	161 088	6 912	8730	115 620		
1.0	168 000	7 786	8548	106 890	274 890	52.80
0.9	175 786	8 660	8374	98 342	274 128	47.52
0.8	184 446	9 534	8208	89 968	274 414	42.24
0.7	193 980	10 408	8044	81 760		
0.6	204 388			73 716		31.68

The numbers in the fourth and fifth columns are obtained as follows: the values assumed above give us $(2A - a)\ Lc =$ $20 000 000, and this multiplied by the tabular differences in the preceding table for a 6-driver engine, gives the numbers in the fourth column. We now observe that the differences of x and of y increase in opposite directions, therefore at some point they will be equal; and a simple inspection shows us that this point is at or near the grade of 0.9, which is therefore the grade required. We now multiply the tabular number for 0.9, and a 6-driver engine by $20 000 000, for the number in the fifth column, and this added to the value of y on the same line gives the sum of $(x + y)$ for the most economical grade. This of course *is not* the total annual outlay of the *road, or engine-stage,* because many items of expense which *are independent of* a maximum grade have not been con*sidered.*

If an 8-driver engine were to be used, and the expense per engine-mile estimated at 50 cts., then $(2A - a)\,Le = \$25\,000\,000$; hence

G_s.	y.	diff. y.	diff. x.	x.	$x+y$.	z'.
1.1	161 088	6 912	7 673			
1.0	168 000	7 786	7 538	95 810	263 810	52.80
0.9	175 786					

indicating a saving of $10 318 per annum in the case supposed by using 8-driver engines, although on a steeper ruling grade. On the other hand, should we adopt 4-driver engines, and estimate the expense per engine-mile at 30 cents, we should find the most economical grade to be 0.7 per station and $(x + y) = \$293\,280$, showing a loss in this case of $19 152 per annum, as compared with the results of 6-driver engines.

It should be remembered that the table § 67 is prepared on the assumption that the ratio $\frac{t}{w} = \frac{10}{18}$. If cars are to be used giving for full loads any other ratio, $\frac{t'}{w'}$, a new table may be prepared by multiplying each tabular number by $\frac{10}{18} \times \frac{w'}{t'}$. The velocity adopted of 12 miles per hour is sufficient for ordinary grades. When the maximum grade is very low, it would be better to use 15 or 18 miles an hour in calculating the value of x.

70. Since x, eq. (11), varies directly as L, it is important that an engine-stage having heavy grades should be short. Its length, however, must be consistent with the economical length of the adjoining engine-stages, and with the amount of work which an engine ought to perform daily. The most favorable condition for a road would be that in which all the engine-stages were operated at equal expense. But if, to secure this result, the engine-stage of heavy grades must be unreasonably reduced in length, it will be better to adapt the grades to the use of two engines per train.

71. The maximum grade z', opposed to the heavier tonnage A, having been determined, we have now to consider what is the limit to grades in the opposite direction. The engines are supposed to haul their maximum loads in moving the tonnage A, and since the return tonnage, a, is less than A, the engines, in returning, will not be worked to their full capacity if they encounter no grades steeper than z'. We therefore have a margin of power in the returning engines which may be taken advantage of to cheapen the cost of construction, or to shorten the line, by introducing grades, steeper than z', against the lighter traffic.

The weight of a maximum train moving up the grade z' is, eq. (9), $W' + T'$; the weight of the train returning will be

$$W' + \frac{a}{A}T' = \left(\frac{w-t}{t} + \frac{a}{A}\right)T'$$

Substituting this in place of $(W' + T')$, eq. (9), and solving for q', we find the resistance due to a maximum grade opposed to the returning train. Whence, by eq. (2), if we let $Z =$ the maximum return grade, and make $q'' = 0$,

$$Z = \frac{33}{14} \cdot \frac{P - .0009E^2V^2}{E + \left(\frac{w-t}{t} + \frac{a}{A}\right)T'} - \frac{33}{14}(8.1 + .009V^2) \qquad (14)$$

Inasmuch as the value of Z varies with every change made in z', the engineer, when estimating the cost of construction upon the basis of any maximum grade z', should take care that the return grade Z nowhere exceeds its limit as given by the last equation (14). In the example, §69, $z' = 47.52$; hence $T' = 203.37$, eq. (8). Substituting these values, in eq. (14), we find $Z = 81.25$, which is therefore the limit for return grades in this case. With regard to curves on the maximum grade, see §68.

72. If in eq. (1) we let $z = \frac{33}{14}q$ be the grade per mile which offers a resistance equal to the resistance to uniform motion on a level, we have

$$z = 12.73 + \left(.01414 + \frac{.001414E^2}{E + W + T}\right)V^2 \qquad (15)$$

When $V = 20$ this becomes

$$z = 18.386 + \frac{.5657E^2}{E + W + T} \qquad (15\tfrac{1}{2})$$

which is the grade down which a train, whose weight is $(E + W + T)$, if started at 20 miles an hour, will continue to move at that speed without steam or brakes. As that speed is not objectionable, so the grade z, which induces it is not, provided it does not exceed the values of z' or Z respectively, determined with reference to economy. For the extra work done by the engine in ascending one grade z is utilized in descending the next; and the net result is the same as though the two were replaced by a uniform grade. The engineer therefore is not warranted by economic considerations in reducing undulating grades which do not exceed z to a uniform grade, when to do this would cause any increase in the cost of construction, unless z exceeds the grades z' or Z of maximum economy.

73. But when grades exceed z, eq. (15½), the resulting speeds of the maximum train become too great, and the necessary application of the brakes absorbs a portion of the power previously expended in gaining the summit, which is thus worse than wasted, since it increases the wear and tear of machinery and track. Therefore the engineer is justified in spending a certain sum of money in reducing grades which exceed z to that limit. A calculation of the loss of power due to the use of brakes on a grade, and of the cost of that lost power, together with the resulting wear and tear per annum, will give the interest on the sum that may be justifiably spent in reducing the grade from its position of cheapest construction.

74. The limit z is not constant, but depends on the weight of the maximum train, which in turn depends on z'. It will not be the same in both directions unless $A = a$, giving $z' = Z$. In the example §69, $E = 49.5$ and $W' + T' = 366.07$; hence, eq. (15½), $z = 21.72$ descending in the direction of the traffic A. Also $W' + \frac{a}{A} T' = 230.49$, whence $z = 23.34$ descending in the opposite direction. These are the limits in this case at which undulating grades cease to be profitable.

75. We have finally to consider the method for selecting the best line from several proposed routes. For this purpose we determine the most economical grade on each route thought worthy of consideration, and calculate the interest on the *entire cost of constructing* the line with that ruling grade, and also the *annual expense* of operating the line, and take the sum of the two. That route is best in respect to which this *sum* is the least.

76. The value of saving one mile in distance on any route is found by dividing the sum of the annual operating expense and the interest on the cost of construction by the rate of interest, and the quotient by the length of the line in miles.

77. We have now fully discussed the theory and developed the formulæ necessary to the determination of the most economical grades; but the value of the results in a given case depend upon the correctness of the engineer's estimates which enter into the formulæ. These may seldom prove precisely accurate, yet, if he can bring them within definite limits, he may determine the grades of maximum economy within corresponding limits. In the case of a finished road and in full operation, however, the elements of first cost, of traffic, and of operating expenses being known, an investigation by means of the foregoing formulæ becomes a critical test as to the economy of the location and grades; and should the road fail to pay dividends, or be forced to charge high rates of toll, we can determine, though perhaps too late, to what extent the location is chargeable with these results.

CHAPTER IV.

LOCATION.

78. A railroad is said to be located when its centre line is established on the ground in the position which it is intended finally to occupy. The location is made by an engineer corps similar in its organization to that employed on preliminary surveys. The instruments used are also the same, except that the transit is substituted for the compass, and usually the target rod for the self-reading rod. The magnetic needle is never used upon the centre line, except as a rough check on the transit work. It is used, however, to obtain the direction of property lines, roads, and other topographical data.

79. The remarks upon transit work in the preceding chapter apply to the running of straight lines on location. All field-work on location should be done with accuracy and fidelity. No guesswork, nor rude approximations, are to be tolerated. All transit points are made as secure and permanent as possible, and the more important ones are guarded by other transit points set in safe positions near by, their distances and directions from the main point being recorded.

The stakes for the stations are made neatly, and somewhat uniform in size, and they are firmly driven. Sometimes a small plug is driven down flush with the surface of the ground to indicate the station point, and the stake is then set near by as a witness.

In locating a very **long tangent** the greatest care is required to make it *straight*. If the tangent is produced from point to point by backsights and foresights, the observation should be repeated in every instance with reversed instrument, to eliminate any possible lack of adjustment, and to check any accidental error. (Indeed it is proper to observe this rule on curves, as well as on tangents.) When some object in the horizon can be used as a foresight, it is preferable to set the instrument by this rather than by a backsight. For final location, the line should be cleared to give as continuous a line of sight as possible, but in case of an obstacle which cannot be removed at the time, at least two independent methods of passing it should be employed, so that there may be a check upon the alignment beyond.

80. The leveller selects his benches far enough from the line to prevent their being disturbed during the construction of the road. They should be nearly at grade, as a rule, though it is well to leave a bench near a water-course for reference in laying out masonry or trestle-work. The rodman holds the rod at every station, and at every point on the centre line where the slope changes direction, so that these points may be accurately defined on the profile. When he uses a target rod, he sets the target as directed by the leveller, and after clamping it, takes the reading. He reads to thousandths upon turning points and benches, but only to tenths of a foot elsewhere, and announces the readings to the leveller for record. He also records the readings upon turning points and benches in his own book as a check. At the close of each day the leveller and rodman compare notes, and draw a profile of the line surveyed. *(See also §§ 28, 29, 30.)*

81. The **fixing** of the **grade-lines** upon the profile is one of the most important operations connected with the location. It is usually performed by the engineer in charge of the locating party, as being most conversant with the general character and detailed requirements of the line. The maximum gradients will have generally been determined in advance from the preliminary data by the principles laid down in the preceding chapter, but the position of each grade-line, relative to the profile of the surface, must be left to the judgment and skill of the engineer. In general, the grade-line is so placed as to equalize the amounts of excavation and embankment, but there are various exceptions to this rule. Thus, the excavation may be in excess: *first*, when it is necessary to pass under some other road or highway, the grade of which cannot be changed; *second*, when valuable property is to be avoided, the appropriation of which would cost more than the excavation; *third*, when the grade is at the maximum near a summit, and cannot be raised parallel to itself without incurring too great an expense for masonry, etc., at some other part of the line. The embankment may be in excess, *first*, when the country is flat and wet, in order to keep the road-bed well drained; (the grade-line should be at least two feet above the average level of the surface, or above high-water mark, if the district is subject to overflow;) *second*, in approaching a stream, where it is necessary to raise the grade above the requirements of navigation; *third*, when the cuttings on the line are largely in solid rock, and a cheaper material for embankments may be conveniently had at other points; *fourth*, in a district subject to heavy drifts of snow, by which deep cuts would be liable to be obstructed; *fifth*, in side-hill work, where there is danger of land-slips; *sixth*, when it is determined to supply the place of a portion of an embankment by a timber trestle-work or other viaduct.

The apparent equality of cut and fill on the profile does not represent an equality in fact, owing to the different bases and slopes of the sections adopted, and to the various inclinations of the natural surface transversely to the line. This is especially true in side-hill work, where there are both cut and fill at every point, while the profile shows very little of either. In the latter case it is an excellent plan to combine with the profile of the centre line the profiles of parallel lines ten or twenty feet either side of the centre, and drawn with different colored inks, as these will indicate tolerably well the relative amount of cut and fill required. But after the grade has been thus chosen, the only safe method in side-hill work is to actually compute the amounts of excavation and embankment from cross-sections, mark the amount for each cut and fill on the profile, and compare the results. Any changes required in the grade or alignment may then be discovered and effected before the work of construction has begun.

CHAPTER V.

SIMPLE CURVES.

A. Elementary Relations.

82. The centre line of a located road is composed alternately of straight lines and curves.

The straight lines are called **tangents** because they are laid exactly tangent to the curves. A tangent may be indefinitely long, but should never, as a rule, be shorter than 200 feet between two curves which deflect in opposite directions, nor shorter than 500 feet between curves which deflect in the same direction. A curve should not be less than 200 feet long. When a tangent is said to be *straight*, the meaning simply is that it has no deflections to the right or left; for since it follows the surface of the ground, it evidently has as many undulations as the ground. But if we conceive a vertical plane to be passed through the line, a horizontal trace of this plane will accurately represent the line; and so, if we conceive a vertical cylinder to be passed through a curve on the surface of the ground, a horizontal trace of that cylinder will accurately represent the curve, since all distances and angles are measured horizontally, whatever be the irregularities of the surface. In all problems, therefore, relating to this subject, we may consider the ground to be an absolutely level plain.

***83.* A Simple curve** is a circular arc joining two tangents. *It is always* considered as limited by the two tangent oints, and any part of it beyond these points is called the urve produced. The first tangent point, or the point where ıe curve begins, is called the *Point of Curve*, and is indicated y the initials *P. C.* The point where the curve ends, and the ext tangent begins, is called the *Point of Tangent*, and is indiıted by the initials *P. T.* When accessible, these points are lways occupied by the transit in the course of the survey, nd the plug driven to fix the point is guarded, not only by ıe usual stake bearing the number of the station, but also by nother bearing the proper initials, the "degree" of the curve, nd an "*R*" or "*L*" to indicate whether the deflection is to ıe Right or Left.

84. A simple curve is designated either by the radius, *R*, r the degree of curve, *D*.

The Degree of Curve, *D*, is an angle at the centre, subended by a chord of 100 feet. It is expressed by the number f degrees and minutes in that angle, or in the arc of the curve limited by the chord of 100 feet. Therefore *D equals the number of degrees of arc per station.*

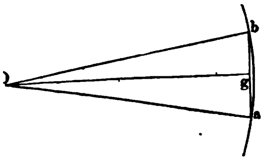

FIG. 3.

The radius *R* and degree of curve *D* can be expressed in terms of each other.

Let *ab*, Fig. 3, be a chord of 100 feet subtending an arc deıribed with a radius $ao = R$ from the centre *o*. Then, by efinition the angle $boa = D$. Bisect the angle *boa* by a line *ı*, and this line will also bisect the chord *ab* and be perpenicular to it; and in the right-angled triangle *bgo* we have

$$bg = ob \times \sin bog$$

r

$$\frac{100}{2} = R \sin \tfrac{1}{2}D$$

ence, to find Radius in terms of Degree of Curve:

$$R = \frac{50}{\sin \tfrac{1}{2}D} \qquad (16)$$

nd *to find Degree of Curve in terms of* **Radius**:

$$\sin \tfrac{1}{2}D = \frac{50}{R} \qquad (17)$$

It is the practice of English engineers to assume the radius at some round number of feet and calculate the degree of curve, which is therefore fractional. In America, on the contrary, the degree of curve is assumed at some integral number of degrees or minutes, and the radius deduced from this.

Example.—What is the radius of a 3° 20′ curve?

$$\begin{array}{llll} & 50 & \log & 1.698970 \\ & \tfrac{1}{2}D = 1^\circ\ 40' & \log\sin & 8.463665 \\ \cline{4-4} \textit{Ans.} & R = 1719.12 & \log & 3.235305 \end{array}$$

Thus the second and third columns of Table IV. have been calculated.

Example.—What is the degree of curve when the radius is 600 feet?

$$\begin{array}{llll} & 50 & \log & 1.698970 \\ & R = 600 & \log & 2.778151 \\ \cline{4-4} & \tfrac{1}{2}D = 4^\circ\ 46'\ 48''.73 & \log\sin & 8.920819 \\ \textit{Ans.} & D = 9^\circ\ 33'\ 37''.46 & & \end{array}$$

Measurement of Curves.

85. A railroad curve is always assumed to be measured with a 100-foot chain, and as the chain is stretched straight between stations it cannot coincide with the arc of the curve, but forms a chord to the arc, as in Fig. 3. Consequently the curve as measured from one tangent point to the other is an inscribed polygon of equal sides, each side being 100 feet. The sum of these sides (with any fraction of a side at either end of the curve) is called the *Length of curve, L.* This length L is evidently a little less than the length of the actual arc between the same points, but the latter we very seldom have occasion to consider.

86. If the chain lengths were taken on the arc instead of as chords of the curve, the degree of curve would be inversely proportional to the radius, and since the arc whose length is equal to radius contains 57.3 degrees nearly, we should have

$$D : 57^\circ.3 :: 100 : R.$$

or

$$R = \frac{5730}{D}$$

nvenient formula, but only approximately true when D is
ll, and seriously at fault when D is large; the error in-
ved being proportional to the difference in length of a
-foot chord, and the arc which it subtends.

37. The Central Angle of a simple curve is the angle
he centre included between the radii which pass through the
gent points (*P.C.*) and (*P.T.*). It is therefore equal to the
mber of degrees contained in the entire arc of the curve
tween those points. The central angle will be designated
the Greek letter Δ (delta).

From the definitions of the *length* and *degree* of curve we
ve the proportion,

$$D : \Delta :: 100 : L.$$

ence, to find the **Length of curve** *in terms of the central*
gle:

$$L = 100\,\frac{\Delta}{D} \tag{18}$$

Example.—What is the length of a 4° curve when the cen-
al angle is 29°?

$D = 4°$ and $\Delta = 29°$ $\left\{ \begin{array}{l} 4)\underline{2900} \\ \quad 725 \text{ feet.} \end{array} \right.$

Ans. $L = 7$ stations $+ 25$ feet

To find the **Central angle** *in terms of the length and degree*
curve:

$$\Delta = \frac{DL}{100} \tag{19}$$

Example.—What is the central angle of a 5° curve 730 feet
ng?

$D = 5°$, $L = 730$, $\frac{5 \times 730}{100} = 36°.5$

Ans. $\Delta = 36°\ 30'$

To find the **Degree of curve** *in terms of the length and*
ntral angle:

$$D = 100\,\frac{\Delta}{L} \tag{20}$$

Example.—What is the degree of a curve 8 stations long,
d having a central angle of 26° 40'?

$L = 800$, $\Delta = 26°.666$, $100\,\frac{26°.666}{800} = 3°.333$

Ans. $D = 3°\ 20'$

88. If two tangents, joined by a simple curve, are produced (one forward and the other backward) until they intersect, the point of intersection, V (Fig. 4), is called the **vertex,** and the exterior or deflection angle which they make with each other is equal to the central angle, $\triangle$

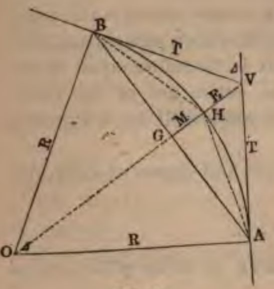

FIG. 4.

The Tangent-distance, T, is the distance from the vertex to either tangent point; thus in Fig. 4, $T = AV = VB$.

The Long Chord, C, is the line AB joining the two tangent points.

The Middle-ordinate, M, is the line GH, joining the middle point of the long chord with the middle point of the curve.

The External distance, E, is the line HV, joining the middle point of the curve with the vertex.

We observe that both the middle-ordinate, M, and the external distance, E, are on the radial line joining the centre, O, with the vertex, V, and that this line is perpendicular to the long chord, C; also, that it bisects the central angle $AOB = \triangle$, and its supplement AVB. (Tab. I. 14.) We also observe that the angle $VAB = VBA = \frac{1}{2}\triangle$ (Tab. I. 20); and if in the figure we draw the two chords AH and BH, the angle BAH equals one half the angle BOH, or $BAH = ABH = \frac{1}{4}\triangle$ (Tab. I. 18); also the angle $VAH = VBH = \frac{1}{4}\triangle$.

89. If we have laid out two tangents on the ground, intersecting at V, and have measured the angle, $\triangle$, between them, we may then assume any other one of the elements of a simple curve before mentioned, and calculate the rest. If we assume D, for instance, we then find R by eq. (16) or by Table IV.

Then, having $\triangle$ and R, we may proceed to calculate the other elements as they are needed.

***90.** To find the* **Tangent-distance** *in terms of the Radius and Central Angle:*

In the right-angled triangle VOA, Fig. 4, we have

$$VA = OA \times \tan VOA$$
$$\therefore T = R \tan \tfrac{1}{2}\Delta \qquad (21)$$

Otherwise, approximately: In Table VI., opposite the central angle, take the value of T for a 1° curve and divide it by the degree of curve D. If desirable, add the correction taken from Table V., corresponding to D.

Example.—What is the tangent distance of a 4° curve with a central angle of 30°?

$D = 4°$	R (Table IV.)	log	3.156151
$\Delta = 30°$,	$\tfrac{1}{2}\Delta = 15°$	log tan	9.428052
Ans. $T = 383.89$ feet		log	2.584203

Otherwise:

By Table VI.	4)1535.3
Approximate ans.	383.82
Correction from Table V.	.08
Ans. $T =$	383.90 feet.

91. *To find the* **Long Chord** *C, in terms of Radius and 'entral Angle:*

In the right-angled triangle BOG, Fig. 4, we have

$$BG = BO \times \sin BOG$$
$$\tfrac{1}{2}C = R \sin \tfrac{1}{2}\Delta$$
$$. \ C = 2 R \sin \tfrac{1}{2}\Delta \qquad (22)$$

But in case Δ can be divided by D without a remainder, at is, if the curve contains an exact number of stations (not ceeding 12), we may take the long chord at once from ble VII.

Example.—What is the long chord of a 3° 20′ curve with a ntral angle of 36° 40′?

2		log	0.301030
$D = 3° 20'$,	R (Tab. IV.)	log	3.235305
$\Delta = 36° 40'$,	$\tfrac{1}{2}\Delta = 18° 20'$	log sin	9.497682
Ans. $C = 1081.48$ feet		log	3.034017

Otherwise:

$$\frac{\Delta}{D} = \frac{36\tfrac{2}{3}^\circ}{3\tfrac{1}{3}^\circ} = 11 \text{ stations}$$

And by Table VII. $C = 1081.48$.

92. *To find the* **Middle-ordinate** ***M*, *in terms of Radius and Central Angle:***

It is evident from the figure that if the radius OH were unity, the line GH would be the nat. versed sine of the arc BH. But the arc BH measures the angle $BOH = \frac{1}{2}\Delta$, and $OH = R$;

$$\therefore\ M = R \text{ vers } \tfrac{1}{2}\Delta \tag{23}$$

But in case Δ can be divided by D without a remainder, that is, if the curve contains an exact number of stations (not exceeding 12), we may take the middle-ordinate at once from Table VIII.

Example.—What is the middle-ordinate of a 4° 30′ curve with a central angle of 40° 30′?

$$\begin{array}{llr}
D = \ 4^\circ\ 30', & R\ (\text{Tab. IV.})\ \log & 3.105022 \\
\Delta = 40^\circ\ 30', & \tfrac{1}{2}\Delta = 20^\circ\ 15'\ \log \text{ vers} & \underline{8.791049} \\
\textit{Ans. } M = 78.717 & & 1.896071
\end{array}$$

Otherwise:

$$\frac{\Delta}{D} = \frac{40.5}{4.5} = 9 \text{ stations}$$

and by Tab. VIII. $M = 78.717$

93. *To find the* **External Distance** ***E in terms of Radius and Central Angle.***

It is evident from the figure that if the radius OA were unity, the portion HV of the secant line OV would be the external secant of the arc AH. But the arc AH measures the angle $AOH = \frac{1}{2}\Delta$, and $OA = R$;

$$\therefore\ E = R \text{ ex sec } \tfrac{1}{2}\Delta \tag{24}$$

Otherwise, approximately:

In Table VI., opposite the central angle, take the value of E for a 1° curve, and divide it by the degree of curve D. If desirable, add the proper correction corresponding to D, taken from Table V.

Example.—What is the external distance E of a $7° 30'$ curve when the central angle is $60°$?

$D = 7° 30'$, R (Tab. IV.) log 2.883371
$\Delta = 60°$, $\frac{1}{2}\Delta = 30°$ log ex sec 9.189492

Ans. $E = 118.27$ feet log 2.072863

Otherwise:

By Tab. VI. 7.5)886.38
Approximate ans. 118.184
Correction for $D = 7° 30'$ (Tab. V.) .084

Ans. $E =$ 118.268

94. But, instead of assuming D or R, we may prefer, or may find it necessary to assume, some other element of the curve, the central angle being given.

If we assume the tangent distance, then:

95. *To find the* Radius *and* Degree of Curve *in terms of the Tangent-distance and Central Angle.*

From eq. (21), and by Table II. 40, we have

$$R = T \cot \tfrac{1}{2}\Delta \tag{25}$$

Otherwise, approximately:

Divide the tangent of a 1° curve found opposite the value of Δ in Table VI., by the assumed tangent distance; the quotient will be the degree of curve in degrees and decimals.

Example.—The exterior angle at the vertex is 54°, and the tangent distance must be about 700 feet. What shall be the degree of curve?

$\Delta = 54°$, $\frac{1}{2}\Delta = 27°$ log cot 0.292834
$T = 700$ 2.845098

log $R =$ 3.137932
Ans. By Table IV. $D = 4° 10' +$

Otherwise:

By Table VI. 700)2919.4
Ans. $D = 4° 10' 15''$ 4.1706

But as it is difficult to lay out a curve when D is fractional, *we discard the fraction* and assume $4° 10'$ as the value of D.

This may require us to recalculate the value of T, which we do by eq. (21) and find $T = 700.8$ feet log 2.845596. If the other elements are required, they may be calculated by eqs. (22), (23), (24), or directly from T and Δ, as follows:

96. *To find the* **External distance** *E, in terms of the Tangent-distance and Central Angle.*

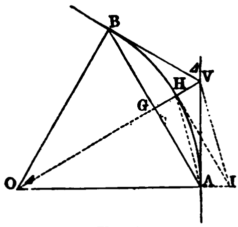

FIG. 5.

In Fig. 5 we have given $AOB = \Delta$ and $AV = T$, to find $HV = E$. In the diagram draw the chord AH, and through H draw a tangent line to intersect OA produced in I, and join VI.

Then HI is parallel to BA, and since $HI = AV = T$, and $AI = HV = E$, VI is parallel to HA, and $VIH = HAB = \frac{1}{4}\Delta$. (Tab. I. 18.) In the right-angled triangle VHI we have

$$HV = HI \times \tan VIH$$

or

$$E = T \tan \tfrac{1}{4}\Delta \qquad (26)$$

Example.—The angle at the vertex being 54° and the tangent-distance 700.80 feet, how far will the curve pass from the vertex?

$$\begin{array}{lll} & T = 700.80 \text{ (from last example)} & 2.845596 \\ & \Delta = 54^\circ,\ \tfrac{1}{4}\Delta = 13^\circ\ 30' \quad \text{log tan} & 9.380354 \\ \hline \textit{Ans.} & E = 168.25 \text{ feet} \qquad\qquad \text{log} & 2.225950 \end{array}$$

(For the formulæ by which to find the long chord and middle-ordinate in terms of the tangent-distance and central angle, see Table III. 12 and 13.)

97. Again, it may be necessary to assume the *external distance* in order to determine the proper degree of curve.

To find the **Radius** *and* **Degree of Curve** *in terms of the External distance and Central Angle:*

By eq. (24)

$$R = \frac{E}{\text{ex sec} \tfrac{1}{2}\Delta} \qquad (27)$$

Otherwise:

In Table VI. divide the external distance of a 1° curve, opposite the given value of Δ, by the assumed external distance; the quotient is the degree of curve required.

Example.—The angle at the vertex being 24° 30′, the curve is desired to pass at about 65 feet from the vertex. What is the proper degree of curve?

$$\begin{array}{lll} & E = 65 \quad \log & 1.812913 \\ \Delta = 24^\circ\ 30', & \tfrac{1}{2}\Delta = 12^\circ\ 15' \ \log \text{ex sec} & \underline{8.367345} \\ & \log R = & 3.445568 \end{array}$$

Ans. By Table IV. $D = 2^\circ\ 03' +$

Otherwise:

$$\begin{array}{ll} & \text{By Table VI.} \quad 65)133.50 \\ \textit{Ans.}\ D = 2^\circ\ 03'\ 14'' & \qquad\qquad 2^\circ.0538 \end{array}$$

We may therefore assume a 2° curve, unless required by the circumstances to be more exact, when we might use a 2° 03′ curve. Assuming a 2° curve, we have by eq. (24)

$$E = 66.75 \quad \log 1.824460$$

Having decided on the degree of curve, we may calculate the remaining elements by eqs. (21), (22), (23), which is always the better way, but we may calculate them directly from E and Δ.

98. *To find the* **Tangent-distance** *in terms of the External distance and Central Angle:*

From eq. (26), and by Table II. 40,

$$T = E \cot \tfrac{1}{2}\Delta \qquad (28)$$

Example.—The angle at the vertex is 24° 30′, and the curve passes 66.75 feet from the vertex. How far are the tangent points from the vertex?

$$\begin{array}{lll} E = 66.75 \text{ (from last example)} & \log & 1.824460 \\ \Delta = 24^\circ\ 30', \quad \tfrac{1}{2}\Delta = 6^\circ\ 07'\ 30'' & \log\cot & \underline{0.969358} \\ \textit{Ans.}\ T = 622.04 \text{ feet} & & 2.793818 \end{array}$$

99. *Remark.*—Eqs. (27) and (28) are particularly useful in defining *the curve of a* railroad track where all original

points are **lost.** Produce the centre lines of the tangents of the curve to an intersection *V*, and there measure the angle Δ. Bisect its supplement *AVB*, and measure the distance on the bisecting line from *V* to the centre line of the track. This will give $VH = E$. Then *R* and *T* may be calculated, and the distance *T* laid off from *V* on the tangents, giving the tangent points *A* and *B*.

(For the formulæ by which to find the long chord and middle-ordinate in terms of *E* and Δ, see Table III. 16 and 17.)

100. Again, having only the central angle given, we may assume the long chord, or the middle-ordinate, and from either of these and the central angle calculate the remaining elements. Or, finally, the central angle being *unknown*, we may suppose any two of the linear elements given, and from these calculate the rest. As such problems have little practical value, their discussion is omitted. The requisite formulæ for their solution are given in Table III., and the verification of them is suggested as a profitable exercise to the student.

B. Location of Curves by Deflection Angles.

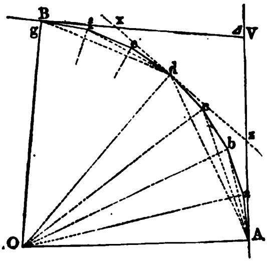

FIG. 6.

101. In order that the stakes at the extremities of the 100-foot chords, by which the curve is measured, shall be set exactly on the arc of the curve by transit observation, it is necessary at the point of curve, *A*, to deflect certain definite angles from the tangent *AV*. Let us suppose that in the curve *AB*, Fig. 6, the points *A*, *a*, *b*, *c*, *d*, etc., indicate the proper positions of the stakes 100 feet apart, and that *OA* is the radius of the curve. In the diagram join *Oa*, *Ob*, etc., and also *Aa*, *ab*, *bc*, etc. Then, by definition, the angle $AOa = D$, and by Geom. (Tab. I. 20 and 11) the angle $VAa = \frac{1}{2}D$. Therefore if we set the transit at *A*, and deflect from *AV* the angle $\frac{1}{2}D$, we shall get the direction of the chord *Aa*, on which by measuring 100 feet from *A* we fix the stake, *a*, in its true *position on* the curve. So again, since the angle *aOb*, *at the centre*, $= D$, the angle *aAb*, at the circumference, $= \frac{1}{2}D$.

If therefore, with the transit at *A*, we deflect the angle $\frac{1}{2}D$ from the chord *Aa*, we shall get the direction of the chord *Ab*; and when the stake *b* is on this chord it will also be on the curve, if *b* is 100 feet distant from *a*. Thus, in general, we may fix the position of any stake on the curve, by deflecting an angle $\frac{1}{2}D$ from the preceding stake, and at the same time measuring a chain's length from it,—the chain giving the distance, while the instrument at *A* gives the direction of the point.

$\frac{1}{2}D$ is called the Deflection-angle of the curve; so that in any curve, *the deflection-angle is equal to one half the degree of curve.*

102. Since each additional station on the curve requires an additional deflection-angle, the *proper deflection* to be made at the tangent point *from the tangent* to any stake on the curve is equal to the deflection-angle of the curve multiplied by the number of stations in the curve up to that stake; or it is equal to one half the angle at the centre subtended by the included arc of the curve.

103. It may happen that all the stations of a curve are not visible from the tangent point, *A*. When this is the case a new transit-point must be prepared at some point on the curve, by driving a plug and centre in the usual manner, and the transit moved up to it. Let us suppose that the point *d*, Fig. 6, has been selected for a transit-point, and that the transit has been set up over it. Before the curve can be run any farther, it is necessary to find the direction of a tangent to the curve at the point *d*. For this purpose we deflect from chord *dA* an angle *Adz* equal to the angle *VAd* previously deflected to fix the point *d*. (Tab. I. 16.) Or we may adopt the following

Rule: *To find the* **direction of the tangent** *to a curve at the extremity of a given chord, deflect from the chord an angle equal to one half the angle at the centre subtended by the chord.* (Tab. I. 20.)

Having thus found the direction of the auxiliary tangent *zdx*, we proceed to deflect from *dx*, ($\frac{1}{2}D$) for the next station *e*, 2 ($\frac{1}{2}D$) for station *f*, 3($\frac{1}{2}$D) for station *g*, etc., as before. When the end of the curve is reached, a transit-point is set at the Point of Tangent, after which it only remains to find the *direction of the tangent*, by the above rule. Thus if g is to be the point of tangent, we obtain the direction of the tangent by deflecting from the chord *gd* an angle equal to *xdg*, or to $\frac{1}{2}$ *dOg*. If this tangent *VB* was already established, the line *gx* thus obtained should coincide with it; and if it does so, the correctness of our work is proved.

104. The centre line is measured, and the stations numbered regularly and continuously through tangents and curves from the starting point fo the end of the work. It therefore frequently happens that a curve will neither begin nor end at an even station, but at some intermediate point, or *plus* distance.

If the Point of Curve occurs a certain number of feet beyond a station, the first chord on the curve is composed of the remaining number of feet required to make 100.

Any chord less than 100 feet is called a **subchord.**

If a curve ends with a subchord, the remainder of the 100 feet must be laid off on the tangent from the Point of Tangent to give the position of the next station, so that the stations may everywhere be 100 feet apart.

105. *The deflection to be made for a subchord is equal to one half the arc it subtends.*

Let c = length of any subchord in feet.

" d = angle at centre subtended by subchord.

Then, from eq. (22), by analogy

$$c = 2R \sin \tfrac{1}{2}d \qquad (29)$$

But by eq. (16) $$2R = \frac{100}{\sin \frac{1}{2}D}$$

$$\therefore c = 100 \frac{\sin \frac{1}{2}d}{\sin \frac{1}{2}D} \qquad (30)$$

$$\therefore \sin \tfrac{1}{2}d = \frac{c}{100} \sin \tfrac{1}{2}D \qquad (31)$$

When D does not exceed 8° or 10°, we may assume without serious error that the angles are to each other as their sines, and the last two equations become

(approx.) $$c = 100 \frac{d}{D} \qquad (32)$$

and $$\tfrac{1}{2}d = \frac{c}{100}\left(\tfrac{1}{2}D\right) \qquad (33)$$

In curves sharper than 10° per station, the error involved in this assumption becomes apparent and must be corrected.

106. If curves were measured on the actual arc, then eqs. (32) and (33) would be true in all cases; but since a curve is measured by 100-ft. chords, it is evident that if a 100-ft. chord between any two stations were replaced by two or more subchords, these taken together would be longer than 100 feet, since they are not in the same straight line. Let us conceive the actual arc of one station to be divided into 100 equal parts; since the arc is longer than the chord, each part will be slightly longer than one foot. Now if we take an arc containing any number of these parts (less than 100), the *nominal* length of the corresponding subchord in feet will *equal* the number of parts, and the deflection for the subchord will be *proportional* to the number of parts which the arc contains. The deflection therefore will be exactly given by eq. (33) if in that equation we let c equal the number of parts in the arc, or the nominal length of the subchord in feet. Having thus obtained the correct value of ($\frac{1}{2}d$), we may introduce it into eq. (29) or (30), and obtain the *true* value of the subchord, which will always be a little greater than its nominal value.

Suppose, for instance, that the arc of one station is to be divided into four equal portions; then each subchord will be *nominally* 25 feet long; and by eq. (33)

$$\tfrac{1}{2}d = \frac{25}{100}\left(\tfrac{1}{2}D\right) = \tfrac{1}{4}\left(\tfrac{1}{2}D\right)$$

which is the correct value of the deflection, whatever be the degree of curve. Substituting this value in eq. (29) or (30) we obtain the *true value* of the subchord, c, a little greater than 25; the *excess* is called the *correction* of the nominal length.

107. This correction for any given subchord bears an almost constant ratio to the excess of arc per station, whatever be the degree of curve. These ratios are shown in the following table for a series of subchords, and Table VII. gives the length of actual arc per station for various degrees of curve. *Subtracting 100* we have the excess of arc per station, *and multiplying this excess* by the *ratio* corresponding to the nominal length of subchord we obtain as a product the proper correction for the subchord.

TABLE OF THE RATIOS OF CORRECTIONS OF SUBCHORDS TO THE EXCESS OF ARC PER STATION.

Nominal Length of Subchord.	Ratio.	Nominal Length of Subchord.	Ratio.	Nominal Length of Subchord.	Ratio.
0	.000	35	.307	70	.356
5	.050	40	.336	75	.327
10	.099	45	.358	80	.287
15	.147	50	.374	85	.235
20	.192	55	.383	90	.169
25	.234	60	.383	95	.092
30	.273	65	.374	100	.000

We observe that the largest correction is required by a subchord between 55 and 60 feet in length.

Example.—It is proposed to run a 14° curve with a 50-ft. chain. What correction must be added to the chain?

$$D = 14^\circ \qquad \tfrac{1}{2}D = 7^\circ \qquad \tfrac{1}{2}d = \frac{50}{100} \times 7^\circ = 3^\circ.5 = 3^\circ\ 30'$$

By eq. (30)

$$c = 100\,\frac{\sin 3^\circ\ 30'}{\sin 7^\circ} = 50.093$$

Ans. Correction = .093

$$\begin{array}{lrr} \text{Or, by Table VII.,} & \text{length of arc} = & 100.249 \\ & \text{excess of arc} = & .249 \\ \text{and by above table,} & \text{ratio for 50 feet} = & .374 \\ \hline \textit{Ans. } \text{Correction} = \text{product} & = & .093 \end{array}$$

Example.—The *P.C.* of an 18° curve is fixed at + 55 feet beyond a station. What are the nominal and true values of the first subchord, and what the proper deflection?

$$\text{Nominal value} = 100 - 55 = 45 \text{ feet}$$

$$\text{Deflection} = \tfrac{1}{2}d = \frac{45}{100} \times 9^\circ = 4^\circ.05 = 4^\circ\ 03'$$

and by eq. (30)

$$\textit{True value} = c = 100\,\frac{\sin 4^\circ\ 03'}{\sin 9^\circ} = 45.148$$

Or, by Table VII.,	excess of arc =	.412
by above table,	ratio for 45 feet =	.358
	. Correction = product =	.147
Ans. True value of subchord		= 45.147

Example.—The last deflection at the end of a 40° curve is found to be 6° 30′. What are the nominal and true values of the last subchord?

Here $\frac{1}{2}d = 6° \; 30'$, and by eq. (32)

$$\text{Nominal value, } c = 100 \frac{6.5}{20} = 32.5 \text{ feet}$$

By eq. (30)

$$\text{True value, } c = 100 \frac{\sin 6° \; 30'}{\sin 20°} = 33.098 \text{ feet}$$

Or by Table VII.,	excess of arc 40° =	2.060
by above table,	ratio for 32.5 feet =	.290
	Correction = product =	.597
	Nominal value of subchord =	32.5
	True value =	33.097

108. For convenience in making deflections, the zeros of the instrument should always be together when the line of collimation coincides with a tangent to the curve. Thus, in beginning a curve, **the transit** being set at the *P. C.* zeros together, and line of collimation on the tangent, the reading of the limb for any station on the curve has simply to be made equal to the proper deflection from the tangent for that station. After the transit is moved forward from the *P. C.* and set at another point of the curve, the vernier is set to a reading equal to the reading used to establish that point, *but on the opposite side of the zero of the limb*, and the line of collimation is set on the *P. C.* just left. Then by simply turning the zeros together again, the line of collimation will be made to coincide with a tangent to the curve through the new point, and the deflections for the succeeding stations can be read off directly, as before. Thus any number of transit points may be used in locating a curve by finding the direction of the tangent through each by a deflection from the preceding point, until finally the *P. T.* is reached, where another *deflection gives the direction* of the located tangent.

109. The assistant engineer keeps neat and systematic **field-notes** of all his operations with the transit in running curves. The numbers of the stations are written in regular order *up* the first column of the left-hand page of the field-book, using every line, or every other line, as may be preferred. The second column contains the initials of each transit point on the same line as the number of its station, or between lines, if the point occurs between two stations. In the third column, and opposite the initials in the second, is recorded the station and plus distance, if any, of each transit point. The fourth column contains, opposite the "*P.C.*," the degree of curve used, and an *R* or *L*, showing whether the curve deflects to the right or left; the fifth column contains the readings or deflections made from a tangent to set each station or point, written on the same line as the number of that station or point; and the sixth column contains the central angle of the whole curve, △, written opposite the "*P.T.*" The plus distances recorded in the third column are always the *nominal* lengths of subchords, but if the true lengths have been calculated and laid off on the ground, these should also be recorded in parenthesis. On the right-hand page are recorded the calculated bearings of the tangents and their magnetic bearings; and on the centre line of the page, opposite the record of each transit point, a dot is made with a small circle around it, to show the relative position of the several points on the ground. Some slight topographical sketches may be made, indicating the more prominent objects, but the full sketches should be taken by the topographer in a separate book.

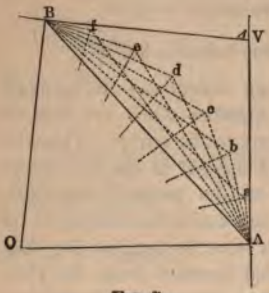

FIG. 7.

110. Since the deflections start from *zero* at each new transit point, *the* **sum** *of the deflections by which the transit points are located will be equal to* **one half the central angle** *of the curve.*

111. *The stations on a curve may be located* **by deflections only,** without linear measurements. For this purpose *two transits* are set at two transit points on the curve, as and *B*, Fig. 7, and the proper deflections for any station are made with both instruments, the station being located by finding the intersection of the two lines of collimation.

This method requires that the two transit points shall have been previously established, that their distance from each other shall be known, that they shall be visible from each other, and that they shall both command a view of the stations to be located. It is not therefore generally useful, but may be resorted to to set stations which fall where chaining cannot be accurately done, as in water or swamps. The chord joining the two transit points becomes, in fact, a base-line, and the deflections form a series of triangulations.

C. Location of Curves by Offsets.

112. A curve may be located by linear measurement only, without angular deflections. There are four general methods, viz.:

By offsets from the chords produced,
By middle-ordinates,
By offsets from the tangents, and
By ordinates from a long chord.

To locate a curve by offsets from the chords produced.

When the curve begins and ends at a station.

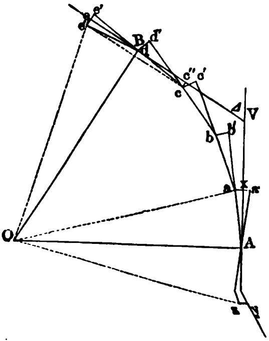

FIG. 8.

113. Let *A*, Fig. 8, be the *P. C.* of a curve taken at a station, to locate the other stations, *a*, *b*, *c*, etc. The chords *Aa*, *ab*, *bc*, etc., each equal 100 feet, and since the angle $AOa = D$, the angle $VAa = \frac{1}{2}D$. (Tab. I. 20.) Taking an offset $ax = t$, perpendicular to the tangent, we have in the right-angled triangle *Axa*.

$$ax = Aa \times \sin \tfrac{1}{2}D$$

or

$$t = 100 \sin \tfrac{1}{2}D \qquad (34)$$

The offset *t* is called the tangent offset, *and its value is given for all degrees of curve in Tab. IV. col. 4.*

If the curve were produced backward from A, 100 feet to station *z*, the offset *zy* wo equal t; and if the chord zA were produced 100 feet from A to a', the offset $a'x$ would also equal t. Therefore the distance $aa' = 2t$, and the angle $aAa' = D$. So if we produce the chord Aa 100 feet to b', the distance $bb' = 2t$.

To lay out the curve, stretch the chain from A, keeping the forward end at a perpendicular distance, t, from the line of the tangent to locate station a. Then find the point b' by stretching the chain from a in line with a and A, and then stretching the chain again from a, fix its forward end at a distance from b' equal to $2t$. This gives station b. In the same way find other stations.

When the last station, as d, of the curve is reached, produce the curve one station farther to e''. Then the tangent through d is parallel to the chord ce'', and laying off t from c and e'' perpendicular to this chord, the tangent $c''e$ is found. If the work has been correctly done the tangent $c''e$ will coincide with the given tangent VB.

When the curve begins or ends with a subchord.

114. Let A, Fig. 9, be the $P.C.$ and Aa the first subchord $= c$, and the angle $VAa = \frac{1}{2}d$, and let the offset $ax = t_1$. Then

$$t_1 = c \sin \tfrac{1}{2}d \qquad (35)$$

Producing the curve backward to the nearest station z, we have another subchord $Az = (100 - c)$, and the angle $yAz = \frac{1}{2}(D - d)$, and putting the offset $yz = t_z$

$$t_z = (100 - c) \sin \tfrac{1}{2}(D - d) \qquad (36)$$

Laying off the two subchords on the ground, and making the proper offsets, t_1 and t_z, at the same time, we fix the position of the two stations a and z on the curve; after which we may produce the chord za 100 feet to b', and proceed as before until the curve is finished.

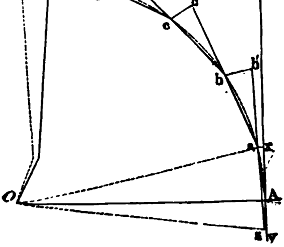

Fig. 9.

If the curve ends with a subchord, as dB, produce the curve to the first station beyond B, as e'', then calculate the two offsets for the two subchords Bd and Be'', and lay them off from d and perpendicular to the supposed direction of the tangent. If the line $d''e$ so obtained coincides with the given tangent, *VB*, the work is correct.

115. We may find the values of $t_{,}$ and $t_{,,}$ otherwise than by the formulæ above, for in Fig. 8 we have shown that the angle $aAa' = aOA$, and since these triangles are isosceles, they are similar; therefore

Fig. 8, $OA : Aa :: Aa : aa'$

or $R : 100 :: 100 : 2t$

$$\therefore \quad t = \frac{(100)^2}{2R} \qquad (37)$$

and similarly, Fig. 9,

$$t_{,} = \frac{c^2}{2R} \qquad (38)$$

Hence

$$t_{,} : t :: c^2 : (100)^2$$

$$\therefore \quad t_{,} = \frac{c^2 t}{(100)^2} \qquad (39)$$

Thus $t_{,}$ may be found by multiplying the square of the sub-chord by the value of t given in Tab. IV., and dividing the product by 10000. As c is always less than 100, so $t_{,}$ is always less than t.

116. In eqs. (35), (38), and (39) it is customary to use the *nominal* values of c, and this can produce no error in t or $t_{,}$ exceeding ·005, when the degree of curve does not exceed ten degrees. In the case of a very sharp curve, the formulæ eqs. (40) and (41) are preferable.

To locate a curve by middle-ordinates.

When the curve begins and ends at a station.

117. In Fig. 10, let A be the *P.C.* at a station, and let a and z be the next stations on the curve either way from A. Then, since $zy = ax = t$, the chord za is parallel to the tangent AV, and $Ag = t$. Hence, having any two consecutive stations on the curve, as z and A, we may lay off the tangent offset t from A *to g on the radius*, and find the next station, a, 100 feet *from A on the line zg* produced. Then laying off $ah = t$ on *the radius aO*, a point on the line Ah produced and 100 fe *from a* will be the next station b.

On reaching the end of the curve, the tangent is found precisely as described in the method by chords produced, § 113.

In Fig. 10, we observe that if the radius OA were unity, gA would be the versed sine of the angle $aOA = D$. But $gA = t$,

$$\therefore t = R \text{ vers } D \qquad (40)$$

When the curve begins or ends with a subchord.

118. Let A, Fig. 11, be the *P.C.*, and a and z the nearest

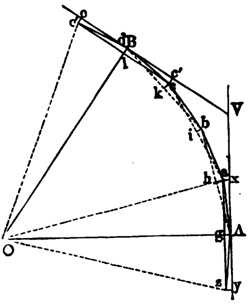

FIG. 10.

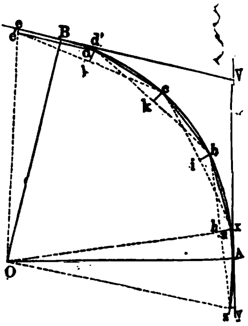

FIG. 11.

stations. Then $Aa = c$, the first subchord, and $aOA = d$, and by analogy, we have from the last equation, if $ax = t_{\prime}$ and $zy = t_{\prime\prime}$

$$\left.\begin{aligned} t_{\prime} &= R \text{ vers } d \\ t_{\prime\prime} &= R \text{ vers } (D-d) \end{aligned}\right\} \qquad (41)$$

or eq. (39) may be used if preferred.

Having found the two stations, a and z, on the curve, lay off from the forward station a, $ah = t$ on the radius, and so continue the curve as described above.

When the end of the curve is reached, produce the curve to the next station beyond, and find the tangent by offsets as described in the previous method, § 114.

To locate a curve by offsets from the tangents.

When the curve begins at a station.

19. *Let A, Fig.* 12, be the *P.C.* at a station. Then the *tation a is* located by the tangent offset t, taken from ab. IV., or calculated by eq. (40). To calculate the distances nd offsets for the following stations, b, c, etc., in the diagram raw lines through the points b, c, etc., parallel to the tangent AV, intersecting the radius AO in g', g'', etc., and draw the ines bx', cx'', etc., perpendicular to the tangent.

Then

$$Ax' = g'b = Ob \sin bOA$$

or

$$\left.\begin{aligned} Ax' &= R \sin 2D \\ Ax'' &= R \sin 3D \\ \text{etc.} & \quad \text{etc.} \end{aligned}\right\} \qquad (42)$$

and

Also,

$$bx' = g'A = Ob \text{ vers. } bOA$$

or

$$\left.\begin{aligned} t' &= R \text{ vers } 2D \\ t'' &= R \text{ vers } 3D \\ \text{etc.} & \quad \text{etc.} \end{aligned}\right\} \qquad (43)$$

and

But these calculations may be avoided, for as twice ag equals the chord of two stations, so twice bg' equals the chord of four stations, and twice cg'' the chord of six stations, etc. So also as Ag is the middle-ordinate of two station, Ag' is the middle-ordinate of four, and Ag'' the middle-ordinate of six stations, etc. Hence the rule:

The distance on the tangent from the tangent point to the perpendicular offset for the extremity of any arc is equal to one half the long chord for twice that arc; and the offset from the tangent to the extremity of any arc is equal to the middle-ordinate of twice that arc.

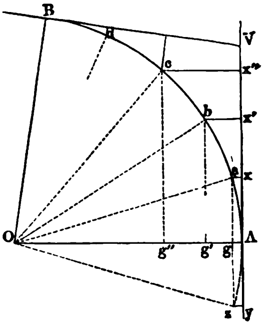

FIG. 12.

The long chords and middle-ordinates may be taken from Tables VII. and VIII. for 2, 4, 6, 8, etc., stations, when the *P.C.* is at a station, or for 1, 3, 5, 7, etc., stations, when the *P.C.* is at + 50, or half a station.

If the offsets from the first tangent AV prove inconveniently *long, the second half of the* curve may be located from the *ther tangent BV,* beginning at the point of tangent B, ar *osing on a station* located from the first tangent.

When the curve begins with a subchord.

120. If $d =$ the angle at centre, subtended by the first subchord, we have for the distances on the tangent (Fig. 13)

$$\left.\begin{aligned} Ax &= R \sin d \\ Ax' &= R \sin (d + D) \\ Ax'' &= R \sin (d + 2D) \\ \text{etc.} & \qquad \text{etc.} \end{aligned}\right\} \qquad (44)$$

and for the offsets (Fig. 11)

$$\left.\begin{aligned} t_{,} &= R \text{ vers } d \\ t' &= R \text{ vers } (d + D) \\ t'' &= R \text{ vers } (d + 2D) \\ \text{etc.} & \qquad \text{etc.} \end{aligned}\right\} \qquad (45)$$

If the first subchord equals 50 feet (nominal), then $d = \frac{1}{2}D$, and the Tables VII. and VIII. may be used as explained

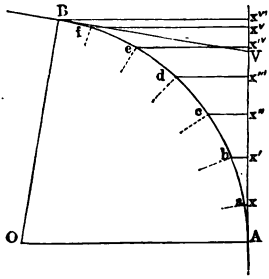

FIG. 13.

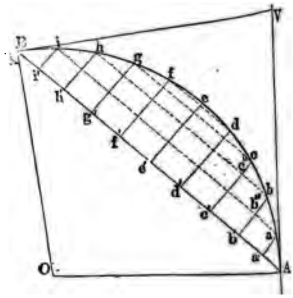

FIG. 14.

above. These tables may be used in any case, by adopting a temporary tangent through any station, and laying off the distances on this, and making the offsets from it.

When a curve is located by offsets the chain should be carried around the curve, if possible, to prove that the stations are 100 feet apart.

To locate a curve by ordinates from a long chord.

the curve begins and ends at a station.

In Fig. 14 draw the long chord AB, joining the tan- ts, and from this draw ordinates to all the stations on the curve. We then require to know the several distances the long chord Aa', $a'b'$, $b'c'$, etc., and the length of ordinate at each point.

Let $C =$ the long chord AB, then eq. (22)

$$C = 2R \sin \tfrac{1}{2} \triangle$$

If a is the second station and i next to the last on the curve, join ai, and let the chord $ai = C'$. Then since the arc $Aa = ik = D$, the angle at the centre subtended by C' is $(\triangle - 2D)$.

$$\therefore \; C' = 2R \sin \tfrac{1}{2} (\triangle - 2D)$$

Again, if we join b and h the next stations and let $bh = C''$

$$C'' = 2R \sin \tfrac{1}{2} (\triangle - 4D)$$

and so on for other chords.

Since $Aa' = ki$, $C = C' + 2Aa'$

$$\therefore \; Aa' = \frac{C - C'}{2}$$

Similarly,

$$a'b' = \frac{C' - C''}{2}$$

Thus we continue to find the distances up to the middle of the curve, after which they repeat themselves in inverse order.

122. When the long chord C, subtends an *even number of stations* (as 10 in Fig. 14), the middle ordinate of the chord is the ordinate of the middle station, as e. Since the chords AB and ai are parallel, the ordinate $a'a$ or $i'i$ is evidently equal to the difference of the middle ordinates of these chords.

Let M, M', M'', etc., be the middle-ordinates of the chords C, C', C'', etc. Then eq. (23)

$$\begin{aligned} M &= R \operatorname{vers} \tfrac{1}{2} \triangle \\ M' &= R \operatorname{vers} \tfrac{1}{2} (\triangle - 2D) \\ M'' &= R \operatorname{vers} \tfrac{1}{2} (\triangle - 4D) \\ &\text{etc.,} \qquad \text{etc.} \end{aligned}$$

And

$$\begin{aligned} a'a &= i'i = M - M' \\ b'b &= h'h = M - M'' \\ &\text{etc.} \quad \text{etc.} \quad \text{etc.} \end{aligned}$$

The values of the chords and middle-ordinates may be *at once from Tables* VII. and VIII.

Example.—It is required to locate a 4 degree curve of ten stations by offsets from the long chord.

By Table VII.:

		Diff.	½Diff.
10 sta.	$C = 980.014$		
8 "	$C^{i} = 789.803$	190.211	$95.105 = Aa' = ki'$
6 "	$C^{ii} = 595.744$	194.059	$97.030 = a'b' = i'h'$
4 "	$C^{iii} = 398.782$	196.962	$98.481 = b'c' = h'g'$
2 "	$C^{iv} = 199.878$	198.904	$99.452 = c'd' = g'f'$
0 "	$C^{v} = 000.000$	199.878	$99.939 = d'e' = f'e'$

From Table VIII.:

		Diff.	
10 sta.	$M = 86.402$		
8 "	$M^{i} = 55.500$	30.902	$= a'a = i'i$
6 "	$M^{ii} = 31.308$	55.094	$= b'b = h'h$
4 "	$M^{iii} = 13.943$	72.459	$= c'c = g'g$
2 "	$M^{iv} = 3.490$	82.912	$= d'd = f'f$
0 "	$M^{v} = 0.000$	86.402	$= e'e$

123. When the long chord C subtends an *odd number of stations*, the middle ordinate will fall half-way between two stations, and need not be laid off.

If the ordinates near the middle of the curve prove inconveniently long, we may subtract $M - M'$, $M' - M''$, etc., and so obtain in Fig. 14 $a'a$, $b''b$, $c''c$, etc. We then lay off Aa', $a'a$, ab'', $b''b$, bc'', etc., turning a right angle at every point. The chain should be carried along the curve at the same time to make the stations 100 feet apart.

Example.—It is required to locate a 10-degree curve of nine stations by offsets from the long chord.

By Table VII.:

		Diff.	½Diff.
9 sta.	811.314		
7 "	658.105	153.209	$76.604 = Aa'$
5 "	484.900	173.205	$86.603 = a'b'$
3 "	296.962	187.938	93.969 etc.
1 "	100.000	196.962	98.481
0 "	*0.000*	100.000	50.000

By Table VIII.:

		Diff.	
9 sta.	168.029	64.279	$= a'a$
7 "	103.750	50.000	$= b''b$
5 "	53.750	34.202	$= c''c$
3 "	19.548	17.365	etc.
1 "	2.183	2.183	
0 "	0.000		

124. The tables can be used equally well when the curve th begins and ends with **a half station**; also to locate lf-station points throughout the curve, but in the latter case e numbers are taken from *consecutive* columns of the tables stead of from *alternate* columns, as in the above examples.

When the curve begins or ends th any subchord.

125. Let A, Fig. 15, be the $C.$ and $Aa = c$ the first subord, and d the angle it subnds at the centre. In the diaam draw the long chord AB, d the ordinates to each stan, and through each station aw a line parallel to AB, and t $AOB = \triangle$.

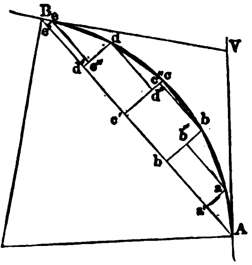

FIG. 15.

Since the angle $VAB = \frac{1}{2}\triangle$ and $Aa = \frac{1}{2}d$, the angle $aAB = \frac{1}{2}(\triangle - d)$. The deflection angle m the subchord Aa produced to the chord ab is $\frac{1}{2}(d + D)$, e deflection angle between any two consecutive chords of 0 feet is $\frac{1}{2}(D + D) = D$. Therefore the angle

$$bab'' = \tfrac{1}{2}(\triangle - d) - \tfrac{1}{2}(d + D) = \tfrac{1}{2}(\triangle - 2d - D)$$

$$cbc'' = \tfrac{1}{2}(\triangle - 2d - D) - \tfrac{1}{2}(2D) = \tfrac{1}{2}(\triangle - 2d - 3D)$$

$$cdd'' = \tfrac{1}{2}(\triangle - 2d - 3D) - \tfrac{1}{2}(2D) = \tfrac{1}{2}(\triangle - 2d - 5D)$$

etc. etc. etc.

Solving the several right-angled triangles we have, Fig. 15.

$$\left.\begin{array}{l}
Aa' = c.\ \cos\frac{1}{2}(\triangle - d) \\
ab'' = 100\cos\frac{1}{2}(\triangle - 2d - D) \\
bc'' = 100\cos\frac{1}{2}(\triangle - 2d - 3D) \\
dd'' = 100\cos\frac{1}{2}(\triangle - 2d - 5D) \\
\text{etc.,} \qquad\qquad \text{etc.,}
\end{array}\right\} \qquad (46)$$

And also

$$\left.\begin{array}{l}
a'a = c.\ \sin\frac{1}{2}(\triangle - d) \\
b''b = 100\sin\frac{1}{2}(\triangle - 2d - D) \\
c''c = 100\sin\frac{1}{2}(\triangle - 2d - 3D) \\
d''c = 100\sin\frac{1}{2}(\triangle - 2d - 5D) \\
\text{etc.,} \qquad\qquad \text{etc.,}
\end{array}\right\} \qquad (47)$$

When the middle point of the curve is passed the minus quantities in the parentheses become greater than △, making the parentheses negative, and, therefore, the sines negative, and indicating that such values as are determined by them must be laid off *toward* the long chord AB.

By a proper summation of the quantities determined by eqs. (46) and (47) we obtain the distances Aa', Ab', Ac', etc., and the ordinates $a'a$, $b'b$, $c'c$, etc., and the curve may be located accordingly. It is well to make all the necessary calculations before beginning to lay down the lines on the ground, thus avoiding confusion and mistakes.

Example.—The *P.C.* of a 3° 20′ curve is fixed at + 25 feet beyond a station, and the central angle is 16° 24′ = △. It is required to locate the curve by ordinates from the long chord.

We have $c = 100 - 25 = 75$ and $d = 2^\circ\ 30'$ and $D = 3^\circ\ 20'$. Hence, eqs. (46)

$$\begin{array}{llr|r}
Aa' = & 75\cos & 6^\circ\ 57' = 74.449 & 74.449 = Aa' \\
ab'' = & 100\cos & 4^\circ\ 02' = 99.752 & 174.201 = Ab' \\
bc'' = & 100\cos & 0^\circ\ 42' = 99.993 & 274.194 = Ac' \\
d''d = & 100\cos & (-2^\circ\ 38') = 99.894 & 374.088 = Ad' \\
e''e = & 100\cos & (-5^\circ\ 58') = 99.458 & 473.546 = Ae' \\
e'B = & 17\cos & (-7^\circ\ 55') = 16.838 & 490.384 = AB
\end{array}$$

By eqs. (47)

$$\begin{array}{llr|r}
a'a = & 75\sin & 6^\circ\ 57' = \quad 9.075 & 9.075 = a'a \\
b''b = & 100\sin & 4^\circ\ 02' = \quad 7.034 & 16.109 = b'b \\
c''c = & 100\sin & 0^\circ\ 42' = \quad 1.222 & 17.331 = c'c \\
cd'' = & 100\sin & (-2^\circ\ 38') = -\ 4.594 & 12.737 = d'd \\
de'' = & 100\sin & (-5^\circ\ 58') = -10.395 & 2.342 = e'e \\
ee' = & 17\sin & (-7^\circ\ 55') = -\ 2.341 & 0.000\ \ldots\ldots
\end{array}$$

The same formulæ can be used when the curve begins at a station by making $c = 100$ and $d = D$.

126. The methods of locating curves by linear measurements do not require the use of a transit, although one may be used to advantage for giving true lines, turning right angles, etc. When a transit is not used the alignments should be made across plumb-lines suspended over the exact points previously marked on top of the stakes. **A right angle** may easily be obtained, without an instrument, by laying off on the ground the three sides of either of the right-angled triangles represented in the following table (or any multiples of them), always making the *base* coincide with the given line.

TABLE OF RIGHT-ANGLED TRIANGLES.

Base.	Hypothenuse.	Perpendicular.
4	5	3
12	13	5
24	25	7
40	41	9
60	61	11
84	85	13
96	100	28

D. Obstacles to the Location of Curves.

127. *To locate a curve joining two tangents when the* **intersection** ***V*** **is inaccessible.** Fig. 16.

From any transit point p on one tangent run a line pq to intersect the other tangent; measure pq and the angles it makes with the tangents. Then the sum of the deflections at p and q equals the central angle Δ. Solve the triangle pqV and find Vp. Having decided on the radius R of the curve, calculate the tangent distance VA by eq. (21), and lay off from p the distance $pA = VA - Vp$ to locate the point of curve. The point p being assumed at random, Vp may exceed VA, in which case the difference pA is to be laid off *toward* V.

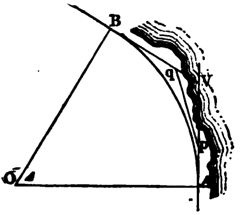

FIG. 16.

In case obstacles prevent the direct alignment of any line pq, *a line of several* courses may be substituted for it (as explained in §§ 46, 47, 48,) from which the length of *pq* will be deduced. The algebraic sum of the several deflections will equal Δ.

128. *To locate a curve when the* **point of curve** *is* **inaccessible.** Fig. 17.

Assume any distance Ap on the curve which will reach to an accessible point p. Then by eq. (19) the angle

$$pOA = \frac{D \times Ap}{100}$$

$$Ap' = R \sin pOA$$

$$p'p = R \operatorname{vers} pOA$$

$$Vp' = VA - Ap'$$

Measure Vp' and $p'p$ to locate a transit point at p; and measure an equal offset from some transit point on the tangent, as qq'. This gives a line pq', parallel to the tangent, from which deflect at p an angle equal to pOA for the direction of a tangent through the point p.

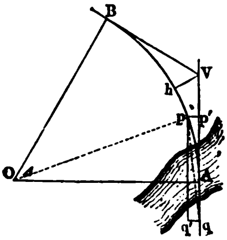

Fig. 17.

Instead of measuring the second offset qq' we may deflect from pq an angle found by $\tan qpq' = \frac{pp'}{qp'}$ and so obtain the line pq' parallel to the tangent. Or we may deflect from pV the angle found by $\tan pVp' = \frac{pp'}{Vp'}$ to obtain the line $q'p$ produced, from which the tangent to the curve at p is found as above.

Again, we may lay off from V, the external distance Vh found by eq. (24) or Tab. VI on a line bisecting the angle AVB. This gives us h, the middle point of the curve, and a line at right angles to hV is tangent to the curve at h, from which the curve may be located in either direction.

To locate a curve when both the **Vertex** *and* **Point** **re inaccessible.** Fig. 18.

oint p on the tangent run a line pq' to the oth tangent, and so determine pA as in § 127. Suppose the curve produced backward to p' on the perpendicular offset pp'.

Then

$$\sin p'OA = \frac{pA}{R} \text{ and } pp' = R \text{ vers } p'OA$$

Having located the point p', a parallel chord $p'q$ may be laid off, giving a point q on the curve, since $p'q = 2 \times pA$. At q deflect from qp' an angle equal to $p'OA$ for a tangent to the curve at q.

If any obstacle prevents using the chord $p'q$, any other chord as $p's$ may be used, by deflecting from $p'q$ the angle $qp's = \frac{1}{2}(qOs)$ and laying off its length,

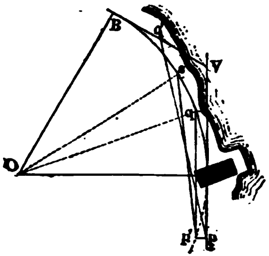

FIG. 18.

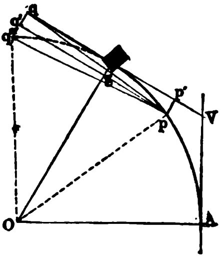

FIG. 19.

$$p's = 2R \sin (p'OA + qp's).$$

At s a deflection from the chord sp' of $(p'OA + qp's)$ will give the tangent at s.

If obstacles prevent the use of any chord, the methods described in § 131 may be resorted to.

130. *To pass from a curve to the forward tangent when the* **Point of Tangent is inaccessible.** Fig. 19.

From any transit point p on the curve, near the end of the curve, run a chord parallel to the tangent. The middle point g of the chord will be on the radius through the point of tangent B. At any convenient point beyond this an offset equal to $pp' = R$ vers pOB may be made to the tangent, and at *some other point an* equal offset will fix the direction of *tangent.*

Otherwise, if an unobstructed line pq can be found intersecting the tangent at a reasonable distance from B, measure the angle $q'pq = pqp'$, and lay off the distance

$$pq' = \frac{pp'}{\sin q'pq}$$

to fix the point q. Then

$$Bq = p'q - p'B = pp' \cot q'pq - R \sin pOB.$$

Otherwise; assume an arc of any number of stations from p to q'' on the curve produced, and take the length of chord from Tab. VII. Lay off pq'', and from q'' lay off $q''q = R$ vers $q''OB$, perpendicular to the tangent, to locate q. The angle $pq''q = 90° - q'pq''$, and the distance $qB = R \sin q''OB$.

131. *To pass an* **obstacle on a curve.** Fig. 20. From any transit point A' on the curve take the direction of a long chord which will miss the obstacle, as $A'B'$. The length of this chord is $2R \sin V'A'B'$, $V'A'$ being tangent to the curve at A' (see eq. 22), and by measuring this distance, the point B' on the curve is obtained. If the angle $V'A'B'$ is made equal to the deflection for an exact number of stations, the chord may be taken from Tab. VII.

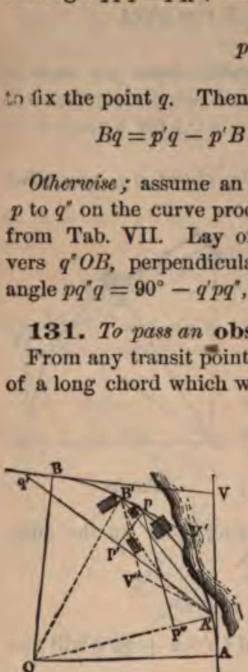

FIG. 20.

If the chord which will clear the obstacles would be too long for convenience, as $A'q'$, we may measure a part of it as $A'p'$, and then, by an ordinate to some station, regain the curve at p. The distance on the curve from A' to p being assumed, the distances $A'p'$ and $p'p$ are calculated by the methods given in § 121 to § 125. If $p'p$ can be made a middle ordinate the work will be much simplified. If more convenient the middle ordinate may first be laid off from A' to p'', and the half chord afterwards measured from p'' to locate p.

Again, we may calculate the auxiliary tangent $A'V'$ for any assumed length of curve $A'B'$, and lay off the distan $A'V'$ and $V'B'$, deflecting at V' an angle equal to tw

$V'A'B'$. But if the point V' should prove inaccessible, we may conceive the auxiliary tangents to be revolved about the chord $A'B'$ as an axis, so that V' will fall at V'', and the lines $A'V''$ and $V''B'$ may be laid out accordingly. If these in turn meet obstructions, we may run a curve from A' to B' of same radius as the given curve, but tangent to $A'V''$ and $V''B'$.

Again, the entire curve or any portion of it may be laid out by offsets from the tangents, or by ordinates from a long chord, as already explained, § 119 to § 126.

In case any distance on a curve must be measured by a triangulation, as in crossing a stream, a long chord may be chosen, either end of which is accessible, and the triangulation is then performed with respect to this chord or a part of it, as upon any other straight line.

SPECIAL PROBLEMS IN SIMPLE CURVES.

132. *Given: a curve joining two tangents, to find the change required in the radius* **R,** *and external distance* **E,** *for an assumed change in the value of the tangent distance* **T.** Fig. 21.

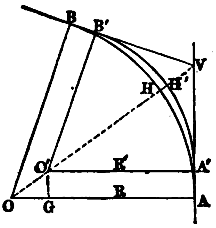

FIG. 21.

Let $T = AV = VB$ and $T' = A'V = VB'$
" $R = AO$ " $R' = A'O'$
" $E = VH$ " $E' = VH'$

Then $T - T' = AA' =$ the given change.

By eq. (25)
$$R = T \cot \tfrac{1}{2}\Delta$$
$$R' = T' \cot \tfrac{1}{2}\Delta$$

$$\therefore \quad OG = R - R' = (T - T') \cot \tfrac{1}{2}\Delta \qquad (48)$$

By eq. (26), similarly,

$$HH' = E - E' = (T - T')\tan\tfrac{1}{2}\Delta \tag{49}$$

Eqs. (48) (49) give the changes in R and E for any change in T. When T is increased R and E will be increased also, and *vice versa.*

Example.—A 4° curve joins two tangents, making an angle of $38° = \Delta$, and it is necessary to shorten the last tangent distance 80 feet. What will be the change in the radius and in the external distance?

Eq. (48)	$T - T' = 80$		log 1.903090
	$\frac{1}{2}\Delta$	19°	log cot 0.463028
Ans.	$R - R'$	232.34	log. 2.366118
	R	1432.69	
	$R' =$	1200.35	or about $4°\ 46' = D'$.

If the tangent distance had been increased 80 feet we should add the above to R.

$\therefore \quad R' = 1665.03$ or about $3°\ 26' = D'$

Eq. (49)	$T - T' = 80$		log 1.903090
	$\frac{1}{2}\Delta$	9° 30′	log tan 9.223607
Ans.	$E - E'$	13.387	log 1.126697

133. *Given: a curve joining two tangents, to find the change required in the radius* **R,** *and tangent distance* **T,** *for any assumed change in the value of the external distance* **E.** Fig. 21.

We suppose HH' given to find OG and AA'.

By eq. (24)

$$E = R \text{ ex sec } \tfrac{1}{2}\Delta$$
$$E' = R' \text{ ex sec } \tfrac{1}{2}\Delta$$

$$\therefore \quad OG = R - R' = \frac{E - E'}{\text{ex sec } \frac{1}{2}\Delta} \tag{50}$$

By eq. (49)

$$AA' = T - T' = (E - E')\cot\tfrac{1}{2}\Delta \tag{51}$$

Example.—A 4° curve joins two tangents, making an angle of 38° = Δ, and it is necessary to bring the middle point of the curve 25 feet nearer the vertex *V.* What changes are required in the radius and point of curve?

Eq. (50)	$E - E' =$	25	log	1.397940
	$\frac{1}{2}\Delta$	19°	log ex sec	8.760578
Ans.	$R - R'$	433.87	log	2.637362
	R	1432.69		
	R'	998.82 or about 5° 44′ = D'		

Eq. (51)	$E - E'$	25	log	1.397940
	$\frac{1}{2}\Delta$	9° 30	log cot	0.776393
	$T - T'$	149.39		2.174333

or the *P.C.* will be moved toward the vertex 149.39 feet.

But if the point *H*, Fig. 21, were to be moved 25 feet further from the vertex *V*, then

$$R' = 1866.56 \text{ or about } 3^\circ\ 04' = D'$$

and the *P.C.* will be moved 149.39 feet further from the vertex.

It is preferable to assume some radius from Table IV. near the value of R' found as above, and from this calculate the value of T' by eq. (21).

134. *Given: a curve joining two tangents, to find the change made in the tangent distance* **T**, *and external distance* **E**, *by any assumed change in the value of the radius* **R.** Fig. 21.

By eq. (48)

$$AA' = T - T' = (R - R') \tan \tfrac{1}{2}\Delta \qquad (52)$$

By eq. (50)

$$HH' = E - E' = (R - R') \text{ ex sec } \tfrac{1}{2}\Delta \qquad (53)$$

The changes calculated by eqs. (52) (53) will be added to or *subtracted from* T and E respectively, according as the radius *increased or diminished.*

135. *Since for* a constant value of the central angle the homologous parts of any two curves are proportional to each other, we may write at once

$$\left.\begin{aligned} R' &= R\frac{T'}{T} = R\frac{E'}{E} = R\frac{C'}{C} = R\frac{M'}{M} \\ T' &= T\frac{R'}{R} = T\frac{E'}{E} = T\frac{C'}{C} = T\frac{M'}{M} \end{aligned}\right\} \qquad (54)$$

etc. etc. etc.

136. *Given: a curve joining two tangents, to change the position of the* **Point of curve** *so that the curve may end in a* **parallel tangent.** Fig. 22.

Let AB be the given curve, AV, VB the tangents, and $V'B'$ the parallel tangent. Then VV' is the distance from one vertex to the other; and since there is no change in the form or dimensions of the curve, we may conceive it to be moved bodily, parallel to the line AV, until it touches the line $V'B'$, when every point of the curve will have moved a distance equal to VV'. Hence $AA' = OO' = BB' = VV'$. Therefore, run a line from B parallel to AV, intersecting the new tangent in B', measure BB', and lay off the distance from A to find A'. In the figure the new tangent is taken outside the curve, and so A' falls beyond A, but if the new tangent were taken inside the curve at $V''B''$, the new $P.C.$ would fall back of A at some point A''.

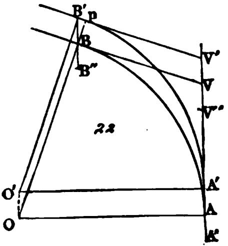

FIG. 22.

If the parallel tangent is defined by a perpendicular offset from B, as Bp; since the angle $BB'p = \Delta$

$$AA' = BB' = \frac{Bp}{\sin \Delta} \qquad (55)$$

137. *Given: a curve joining two tangents, to find the* **radius** *of a curve that, from the same Point of curve, will end in a* **parallel tangent.** Fig. 23.

Let AB be the given curve, AV, VB the tangents, and $V'B'$ the parallel tangent; and let $AO = R$ and $AO' = R'$.

Since the central angle Δ remains unchanged, the angle $\frac{1}{2}\Delta$ between the tangent and long chord remains unchanged; therefore $V'AB' = VAB$, and the new point of tangent is on the long chord AB produced. Find on the ground the intersection of $V'B'$ with AB produced and measure BB'. In the diagram draw Be parallel to AO, then $BeB' = \Delta$, and by eq. (22)

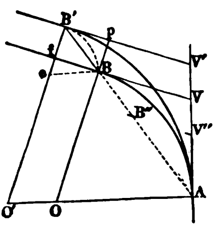

FIG. 23.

$$BB' = 2Be \sin \tfrac{1}{2}\Delta$$

but

$$Be = OO' = R' - R$$

$$\therefore \quad R' = R + \frac{BB'}{2 \sin \frac{1}{2}\Delta} \qquad (56)$$

The $+$ sign is used when B' is beyond B, as in the figure; but if the parallel tangent is within the given curve it will cut the chord in some point B'', and then the $-$ sign must be used, since R' will evidently be less than R.

If the parallel tangent is defined by a perpendicular offset, as $Bp = B'f$; since $BeB' = \Delta$

$$Bp = Be \text{ vers } \Delta = (R' - R) \text{ vers } \Delta$$

$$\therefore \qquad R' = R + \frac{Bp}{\text{vers } \Delta} \qquad (57)$$

Add or subtract as explained above.

If the long chord $C = AB$ is known, then the new long chord $C' = AB'$ or $AB'' = C \pm BB'$, and by eq. (54)

$$R' = R\,\frac{C \pm BB'}{C} \qquad (58)$$

138. *Given: a curve joining two tangents, to change the* **radius,** *and also the* **Point of curve,** *so that the new curve may end in a* **parallel tangent directly opposite** *the given Point of tangent.* Fig. 24.

Let AB be the given curve, AV, VB the tangents, $V'B'$ the *parallel tangent,* and B' the given tangent point on the radius *OB produced.*

In the diagram, produce the tangent AV and the radius OB to intersect at K. Then

$$BK = R \text{ exsec } \Delta$$

$$B'K = R' \text{ exsec } \Delta$$

Subtracting we have

$$BB' = (R - R') \text{ exsec } \Delta$$

$$\therefore \qquad R - R' = \frac{BB'}{\text{exsec } \Delta} \tag{59}$$

from which R' is easily determined, as in §§ 132 and 133.

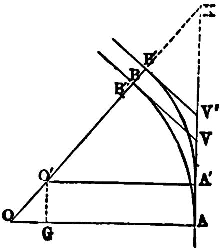

FIG. 24.

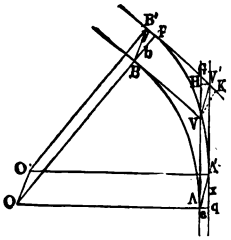

FIG. 25.

To find the change AA' of the $P.C.$, in the diagram draw $O'G$ parallel to $A'A$; then

$$O'G = OG \tan \Delta$$

or

$$AA' = (R - R') \tan \Delta \tag{60}$$

By substituting the value of $(R - R')$ from eq. (59) and observing Table II. 42 we have

$$AA' = BB' \times \cot \tfrac{1}{2}\Delta \tag{61}$$

Observe that eqs. (59), (60), and (61) may be derived directly from eqs. (50), (52), and (51) respectively by writing Δ for $\frac{1}{2}\Delta$.

139. *Given: a curve joining two tangents;* to find the new **tangent points** *after* **each tangent** has been move **parallel to itself** *any distance in either direction.* Fig.

and B be the given tangent points, and A' and B' tangent points required. Let the known perpendicu-nces $Aq = a$, and $Bp = b$. We then require the n parallel distances $qA' = x$ and $pB' = y$.

the form and dimensions of the curve remain un- we may conceive the curve to be moved bodily new position on lines parallel and equal to the joining the vertices. Then $AA' = OO' = BB' =$

diagram draw VK parallel and equal to $Bp = b$ and allel and equal to $Aq = a$. Then $VH = qA' = x$, and $B'p = y$. Since $VGV' = \Delta$, we have

$$VG = \frac{b}{\sin\Delta} \text{ and } GH = \frac{a}{\tan\Delta}$$

e

$$VH = VG - GH = x$$

y

$$\left.\begin{aligned} x &= \frac{b}{\sin\Delta} - \frac{a}{\tan\Delta} \\ y &= \frac{b}{\tan\Delta} - \frac{a}{\sin\Delta} \end{aligned}\right\} \qquad (62)$$

the new tangents are outside of the given curve, the and b are considered positive; if either new tangent side of the given curve its uld be considered negative. g eqs. (62) if x and y are be positive they are to be *forwards* from q and p, as 5; if either is found to be it is to be laid off in the direction.

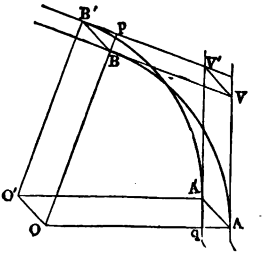

FIG. 26.

ple.—A certain curve has a angle of 50° = Δ, and it is l to move the first tangent *et and the second tangent t. Required,* the distances on the tangents from the *t points to the new.* Fig. 26.

Here $a = -20$ and $b = +12$

$+b$	12		1.079181	$-a$	20		1.301030
Δ	50°	log sin	9.884254	Δ	50°	log tan	0.076186
	15.665		1.194927	$-$	16.782		1.224844

$$\therefore \qquad x = 15.665 - (-16.782) = +32.450$$

$+b$	12		1.079181	$-a$	20		1.301030
Δ	50°	log tan	0.076186	Δ	50°	log sin	9.884254
	10.069		1.002995	$-$	26.108		1.416776

$$\therefore \qquad y = 10.069 - (-26.108) = +36.177$$

For $+a$ and $-b$ $\quad \begin{cases} x = -32.450 \\ y = -36.177 \end{cases}$

For $+a$ and $+b$ $\quad \begin{cases} x = -\ \ 1.120 \\ y = -15.939 \end{cases}$

For $-a$ and $-b$ $\quad \begin{cases} x = +\ \ 1.120 \\ y = +15.939 \end{cases}$

If we have a and x given to find b and y: Solving eqs. (62) for b and y we obtain

$$\left.\begin{aligned} b &= x \sin \Delta + a \cos \Delta \\ y &= x \cos \Delta - a \sin \Delta \end{aligned}\right\} \qquad (63)$$

In which the algebraic signs of the quantities must be observed as above.

140. *Given: a curve joining two tangents, to find a new* **Radius** *and new position of the* **Point of curve,** *such that the curve may end at the same point as before, but with a given change in the* **direction** *of the forward tangent. Fig. 27.*

Let AB be the given curve, AV, VB the given tangents, V'B the new tangent, and VBV' the given change in direction. Let $\Delta' = \Delta + VBV'$.

In the diagram draw BG perpendicular to AV produced; then

$$\begin{aligned} BG &= R \text{ vers } \Delta \\ &= R' \text{ vers } \Delta' \end{aligned}$$

Hence

$$R' = R \frac{\text{vers } \Delta}{\text{vers } \Delta'} \tag{64}$$

and

$$AA' = AG - A'G = R \sin \Delta - R' \sin \Delta' \tag{65}$$

In the figure the change in direction of tangent makes Δ' greater than Δ; therefore V' falls beyond V, and A' beyond A; but if the change made Δ' less than Δ, then V' and A' would fall behind V and A respectively, and R' would be greater than R.

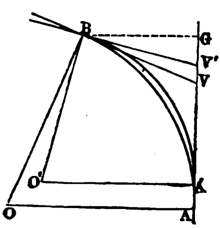

FIG. 27.

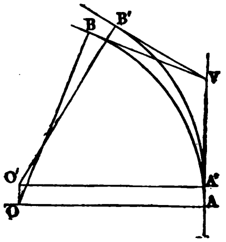

FIG. 28.

The same formulæ apply to the converse problem in which B is taken as the point of curve, and A and A' as points of tangent.

141. *Given a curve joining two tangents, to find the change in the* **Point of curve** *when the forward tangent takes a new* **direction** *from the* **vertex V.** Fig. 28.

By eq. (21)

$$VA = R \tan \tfrac{1}{2}\Delta, \quad VA' = R \tan \tfrac{1}{2}\Delta'$$

$$\therefore \quad AA' = R\,(\tan \tfrac{1}{2}\Delta - \tan \tfrac{1}{2}\Delta') \tag{66}$$

142. *Given: a curve joining two tangents, to find the new*

radius, R', ***when the forward tangent takes a new*** **direction** *from the* **vertex, V.** Fig. 29.

By eqs. (21) (25)

$$VA = R \tan \tfrac{1}{2}\Delta, \quad R' = VA \cot \tfrac{1}{2}\Delta'$$

$$\therefore \qquad R' = R \tan \tfrac{1}{2}\Delta \cot \tfrac{1}{2}\Delta' \qquad (67)$$

143. *Given: a curve joining two tangents, and a given change in the* **direction** *of the forward tangent from the vertex, to find the* **radius** *and* **point of curve** *of a curve that shall pass at the* **same distance, VH,** *from the vertex.* Fig. 30.

Let AB be the given curve, BVB' the given change in

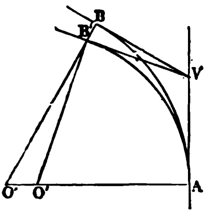

FIG. 29.

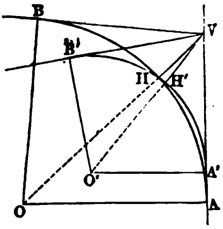

FIG. 30.

direction of tangent, and $VH' = VH$. Let $\Delta' = \Delta + BVB'$; then eq. (24)

$$VH = R \text{ ex sec } \tfrac{1}{2}\Delta = VH' = R' \text{ ex sec } \tfrac{1}{2}\Delta'$$

$$\therefore \qquad R' = R\,\frac{\text{exsec } \tfrac{1}{2}\Delta}{\text{exsec } \tfrac{1}{2}\Delta'} \qquad (68)$$

By eq. (28)

$$VA = VH \cot \tfrac{1}{2}\Delta, \quad VA' = VH' \cot \tfrac{1}{2}\Delta'$$

$$\therefore \qquad AA' = VH (\cot \tfrac{1}{2}\Delta - \cot \tfrac{1}{2}\Delta') \qquad (69)$$

But in case $\Delta' = \Delta - BVB'$, AA' becomes negative *nust be laid off* **backward from** A.

Example.—Given a 2° curve, $\Delta = 80°$ and $BVB' = -10°$ $\therefore\ \Delta' = 70°$

R			log 3.457114
$\frac{1}{2}\Delta$	40°		log exsec 9.484879
VH	874.97		2.941993
$\frac{1}{2}\Delta'$	35°		log exsec 9.343949
R'	1° 27′ nearly		3.598044
$\frac{1}{4}\Delta$	20°	cot 2.74748	
$\frac{1}{4}\Delta'$	17° 30′	cot 3.17159	
		− 0.42411	

$$\therefore \qquad AA' = 874.97 \times (-.42411) = -371.08$$

and must be laid off backward from A.

144. *Given: two indefinite tangents, a point situated between them, and the angle* Δ, *to find the radius* **R,** *and tangent distance* **T** *of a curve joining the tangents which shall pass through the* **given point.** Fig. 31.

If the given point is on the bisecting line VO, as H, measure $VH = E$, and find R and T as in §§ 97, 98.

When the given point, as P is not on the bisecting line VO; if a line GK is passed through P perpendicular to VO, it will be parallel to any long chord, as AB, and the angle $VGK = \frac{1}{2}\Delta$. The curve passing through P will intersect GK in some other point P'; the line GK is bisected by the line VO at I, and $PI = P'I$.

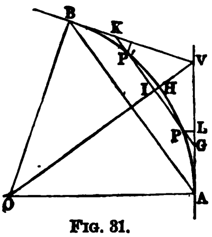

FIG. 31.

If the given point P is located by a perpendicular offset from the tangent, as PL; in the triangle PLG, $LG = PL \cot \frac{1}{2}\Delta$. Lay off LG, and at G deflect $VGK = \frac{1}{2}\Delta$, and measure GP and PK. Since by Geom. (Tab. I. 24) $\overline{GA}^2 = GP' \times GP$, and $GP' = PK$;

$$\therefore \qquad GA = \sqrt{GP \times PK} \qquad (70)$$

Lay off GA; and A is the Point of curve, $AV = T$, and $R = AV \cot \frac{1}{2}\Delta$.

If the given point were located by an offset from BV, find B first, and make $VA = BV$.

If the given point P is located by a perpendicular offset IP from the bisecting line VO; produce IP to intersect the tangent at G and measure PG. Since $P'G = GP + 2PI$

$$\therefore \qquad GA = \sqrt{GP(GP + 2PI)} \qquad (71)$$

whence we have the point of curve A, as before.

145. *Given: a curve, AP, and the* **radial offset PP′** *to find a curve which shall pass through the* **point P′**, *starting from the same point of curve* **A.** Fig. 32.

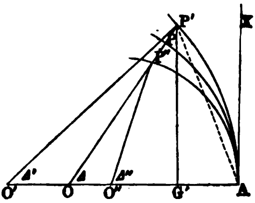

FIG. 32.

Let $b = PP'$, and in the diagram draw $P'G'$ parallel to the common tangent AX, and join AP'. Then

$$P'G' = (R \pm b) \sin \Delta$$
$$G'A = R - (R \pm b) \cos \Delta$$

$$\therefore \qquad \tan \tfrac{1}{2}\Delta' = \frac{G'A}{P'G'} = \frac{R}{(R \pm b) \sin \Delta} - \cot \Delta \qquad (72)$$

$$R' = \frac{P'G'}{\sin \Delta'} = \frac{(R \pm b) \sin \Delta}{\sin \Delta'} \qquad (73)$$

When the offset is *outward* use $R + b$, when it is *inward* use $R - b$.

Example.—Given: a 3° curve of 16 stations and a radial offset of 205 feet inward from the $P.T.$ to find the radius of the curve passing through the extremity of the offset.

Here $\Delta = 3° \times 16 = 48°$; and $b = 205$.

R 3° =	1910.08		
$R - b$	1705.08	log	3.231745
Δ 48°		log sin	9.871073
$P'G'$			3.102818
R 3°		log	3.281051
	1.50742		0.178233
Δ 48° cot	.90040		
$\frac{1}{2}\Delta'$ tan	.60702 = 31° 15½′		
	2		
Δ'	62° 31′	log sin	9.947995
$P'G'$		log	3.102818
R' (about 4° 01′). *Ans.*			3.154823

If the same offset were made *outside* of the curve we should find R' log 3.438350, or about a 2° 05′ curve.

This solution is inconveniently long for ordinary field practice. When the offset is small compared with the length of curve, we may use the following

Approximate Rule: Divide twice the offset b by the length of curve, look for the quotient in the table of nat. sines, and take out the corresponding angle, which multiply by 100, and divide by the length of curve. The *quotient* is the *correction* for the given degree of curve; to be *subtracted* when the offset is made *outward*, and *added* when the offset is made *inward*.

This rule is expressed by the formula

$$D' = D \mp \frac{100}{L} \sin^{-1} \frac{2b}{L} \qquad (74)$$

Taking the same example, we have

$$\frac{2b}{L} = \sin 14° \, 51'$$

$$\textit{and correction} = 14° \, 51' \times \frac{100}{1600} = \mp 0° \, 56'$$

Hence $D' = 3° \, 56'$ or $D' = 2° \, 04'$

THE VALVOID.

146. *Given: any number of circular curves of equal length L, all starting from a common point of curve A, in a common tangent AX, to find the* **equation of the curve** *joining their extremities.* Fig. 33.

Let AP be any one of the given curves,
" $R =$ its radius AO,
" $D =$ its degree of curve,
" $\Delta =$ its central angle AOP,
" $C =$ its long chord AP.

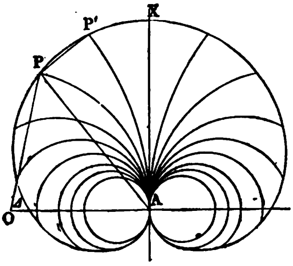

FIG. 33.

By substituting the value of R from eq. (16) in eq. (22) we have

$$C = 100 \frac{\sin \frac{1}{2}\Delta}{\sin \frac{1}{2}D} \qquad (75)$$

Substituting in this the value of D from eq. (20) and letting (theta) $\theta = \frac{1}{2}\Delta$, (rho) $\rho = \frac{C}{100}$ and $N = \frac{L}{100}$, we have for the polar equation of the required curve

$$\rho = \frac{\sin \theta}{\sin \frac{\theta}{N}} \qquad (76)$$

in which ρ *is* the radius-vector AP, θ the variable angle *XAP, the* unit of measure is one side of the inscribed polygon *by which the* circular curve AP is measured, and N the number *of these* sides in the length of the curve AP. By the conditions of the problem N is constant, but θ may have any value whatever. If we let θ vary from 0° to + 180° and from 0° to − 180° the point X will describe the curve $XP'PA$ shown in the figure, which is called the *Valvoid* from its resemblance to the shell of a bivalve. All circular curves tangent to AX at A and having a length $L = AX$ will terminate in the valvoid, and the line PP' joining the extremities of any two of them is a chord of the valvoid.

147. *To find a* **tangent to the valvoid** *at any point* **P.** Fig. 34. See Appendix.

Differentiating eq. (76)

$$\frac{d\rho}{d\theta} = \rho\left(\cot\theta - \frac{1}{N}\cot\frac{\theta}{N}\right) \tag{77}$$

which is essentially negative, since ρ is a decreasing function of θ.

Let (phi) $\varphi = APG$, the angle between the radius vector and the normal PG.

$$\therefore \qquad \tan\varphi = \frac{1}{N}\cot\frac{\theta}{N} - \cot\theta \tag{78}$$

The line PK perpendicular to PG is tangent to the valvoid at P, and PV perpendicular to PO is tangent to the curve AP.

Then $APV = \theta$ and $VPG = \theta - \varphi$, and letting $i = OPK = VPG$.

$$\therefore \qquad i = \theta - \varphi = \tfrac{1}{2}\Delta - \varphi \tag{79}$$

Therefore, to obtain the direction of a tangent to the valvoid at any point P, deflect from the radius PO an angle equal to $i = (\frac{1}{2}\Delta - \varphi)$, on the side of PO farthest from the point of curve A.

The value of i may be found by eqs. (78) (79), but we are saved this somewhat tedious calculation by the use of Table X. 1, which contains values of the ratio $\frac{i}{\Delta} = u$ for various values of Δ, and length of curve L. Multiplying Δ *by the proper tabulated* number gives the value of i = OPK *at once; or*

$$i = (\tfrac{1}{2}\Delta - \varphi) = u\Delta \tag{80}$$

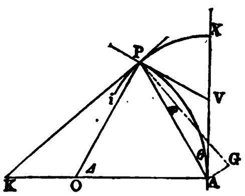

FIG. 34.

148. *To find the* **radius of curvature** *of the valvoid at any point* **P.** See Appendix.

Differentiating eq. (77) we have

$$\frac{d^2\rho}{d\theta^2} = \rho\left[-1 - \frac{2}{N}\cot\theta\cot\frac{\theta}{N} + \frac{1}{N^2}\left(2\cot^2\frac{\theta}{N} + 1\right)\right]$$

The general formula for the radius of curvature of polar curves is

$$r = \frac{\left(\rho^2 + \frac{d\rho^2}{d\theta^2}\right)^{\frac{3}{2}}}{\rho^2 + 2\frac{d\rho^2}{d\theta^2} - \rho\frac{d^2\rho}{d\theta^2}}.$$

Substituting in this the values of ρ, $\frac{d\rho}{d\theta}$, and $\frac{d^2\rho}{d\theta^2}$, and putting

$\left(\frac{1}{N}\cot\frac{\theta}{N} - \cot\theta\right) = a$ we have after reduction,

$$r = \frac{\rho}{2}\cdot\frac{(1 + a^2)^{\frac{3}{2}}}{1 - \frac{1}{2N^2} - a\cot\theta} \tag{81}$$

This formula being too complicated for convenient use in the field, its use is avoided by referring to Table X. 2, which contains values of the ratio $\frac{r}{L} = v$ for various values of $\triangle$ and L. Multiplying the given value of L by the proper tabular ratio, gives the value of the radius of curvature of the valvoid for a short distance either way from the given point P; or,

$$r = vL \tag{82}$$

149. *To find the* **length of arc** *of the valvoid corresponding to a change of* **one degree** *in the value of the angle* $\triangle$. Fig. 35.

From any chord AP suppose a deflection of ¼ degree to be made each way to Ap' and Ap''; then the angle $p'Ap'' = \frac{1}{2}°$ = the change in θ, and since $\triangle = 2\theta$, this makes a change of 1° in the value of $\triangle$. We then require to know the length of the arc $p'p''$, and we may, without sensible error, consider it to be described by the radius of curvature $r = Po$ for the point P, through an angle $p'op''$. Now

$$p'op'' = Xop' - Xop'' = \left(\frac{\Delta'}{2} + \varphi'\right) - \left(\frac{\Delta''}{2} - \varphi''\right) =$$
$$\frac{\Delta'}{2} - \frac{\Delta''}{2} + \varphi' - \varphi''$$

By eq. (80)

$$\varphi' = \frac{\Delta'}{2}(1 - 2u') \quad \text{and} \quad \varphi'' = \frac{\Delta''}{2}(1 - 2u'')$$

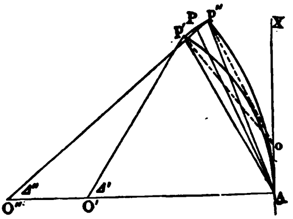

FIG. 35.

and since φ' is so nearly equal to φ'' we may assume $u' = u'' = u$; hence $\varphi' - \varphi'' = \frac{\Delta' - \Delta''}{2}(1 - 2u)$ and $p'op'' = (\Delta' - \Delta'')(1 - u)$.

But the condition of the problem requires $\Delta' - \Delta'' = 1°$, hence $p'op'' = (1 - u)°$.

Therefore the length of arc $p'p''$ for a change of 1° in the value of Δ is

$$l_{,} = r(1 - u) \times \text{arc } 1°$$

or (Tab. XVII.) $l_{,} = r(1 - u)\,.0174533$

and since $r = vL$ (Tab. X. 2),

$$l_{,} = v(1 - u)\,L\,.0174533 \qquad (83)$$

By this formula Table X. 3 has been prepared, for various values of Δ and L.

150. *Given:* **two curves** *of the same length* L *but of different radii, starting from the same point of curve in a* *common tangent, to determine the* **direction** *and* **length** *of a line* **joining their extremities. Fig. 36.**

Let AX be the common tangent, and AP', AP'' the two curves, to determine the direction and length of $P'P''$.

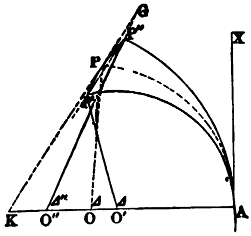

FIG. 36.

If we take the point P on the arc $P'P''$ determined by the angle $\Delta = \dfrac{\Delta' + \Delta}{2}$ and draw a tangent PK to the valvoid at P, we may assume without material error that the chord $P'P'$ will be parallel to PK for any value of $P'P''$ not exceeding $\frac{1}{4}L$, a limit not likely to be exceeded in practice.

Let O be the centre of the curve AP fixing the point P; then $AOP = \dfrac{\Delta' + \Delta''}{2}$, and

$$OPK = i = u\frac{\Delta' + \Delta''}{2}$$

$$PKO = K = \frac{\Delta' + \Delta''}{2} - i = \frac{\Delta' + \Delta''}{2}(1 - u)$$

Since $P'P''$ is assumed parallel to PK,

$$P'P''O'' = KGO'' = \Delta'' - K = \Delta'' - \frac{\Delta' + \Delta''}{2}(1 - u)$$

$$\therefore \quad P'P''O'' = i'' = \frac{\Delta''(1 + u'') - \Delta'(1 - u'')}{2} \tag{84}$$

Similarly producing $P''P'$ to any point H,

$$HP'O' = i' = \frac{\Delta'(1 + u'') - \Delta''(1 - u'')}{2} \tag{85}$$

whence also

$$i' = i'' + \Delta' - \Delta'' \tag{85$'$}$$

The slight error involved in the above assumption is corrected by taking out the value of u (Table X. 1) corresponding to Δ'', the *less* of the two given central angles; we have *therefore written* u with the double accent in equations (84) and (85).

When i' and i'' are positive, they will be deflected as in Fig. 36, on the side of the radius *farthest from* A; should i'' be negative it will of course be deflected from $P''O''$ *toward* A.

The arc $P'P''$ corresponds to a change of the central angle from Δ' to Δ''; hence

$$1^\circ : \Delta' - \Delta'' :: l_, : P'P''$$

or

$$P'P'' = (\Delta' - \Delta'')\, l_, \qquad (86)$$

in which $l_,$ is taken from Table X. 3 for $L = AP$, and $\Delta = \dfrac{\Delta' + \Delta''}{2}$.

As in practice, the distance $P'P''$ is usually small compared with L, the arc and chord will be almost identical and no further calculation is necessary. If $P'P''$ is large, it will be found that equation (86) gives the length of *arc* very correctly when $\dfrac{\Delta' - \Delta''}{2}$ does not exceed 20°, and the length of *chord* when $\dfrac{\Delta' + \Delta''}{2}$ exceeds 60°; for intermediate mean angles it gives a value to $P'P''$ between that of the arc and chord. The arc $P'P''$ may be considered to be described by the radius $r = vL$, v being taken for $\dfrac{\Delta' + \Delta''}{2}$ (Table X. 2), and its total curvature is found by multiplying its length by the degree of curve corresponding to r (Table IV).

Example. Given, a 2° 30′ curve, and a 1° curve of 12 stations each from the same PC, to determine the distance between their extremities.

$$\Delta' = 2\tfrac{1}{2}^\circ \times 12 = 30^\circ, \qquad \Delta'' = 12^\circ, \qquad \frac{\Delta' + \Delta''}{2} = 21^\circ,$$

$$\Delta' - \Delta'' = 18^\circ, \qquad u'' = .33446$$

Eq. (84). $i'' = 2^\circ.9737 = 2^\circ 58' 25''$

Eq. (85)′. $i' = i'' + \Delta' - \Delta'' = 20^\circ.9737 = 20^\circ 58' 25''$

Eq. (86). Arc $P'P'' = 18^\circ \times 10.425 = 187.65$ ft. *Ans.*

Eq. (82). $r = 1200 \times .7479 = 897.48$ ft. = (say) a 6°23′ curve.

Total curvature, $P'P'' = 6^\circ.383 \times 1.8765 = 11^\circ.9777$.

(*The distance* $P'P''$ may be found by solving the triangle *formed by itself* and the long chords of the curves AP′, P''.)

151. *Given: a curve* **AP**, *to find a curve starting from the same point* **A**, *that shall shift the station* **P** *any desired distance* **PP′** *to the right or left.* Fig. 86.

Before we can determine what distance PP' is desired, we must know (approximately) its direction. We have given, therefore, D, L, and Δ to find the angle OPP', and (after measuring PP') to find Δ' and D'.

The solution is necessarily somewhat approximate, yet close enough for all practical purposes. For if the required value of D' were obtained *precisely*, it would probably involve some seconds, and would therefore be discarded in favor of some value in even minutes.

When P′ is inside the given curve:

Eq. (80). $i = OPK = u\Delta$. Table X. 1.

Eq. (82). $r = Po = vL$. Table X. 2.

Let δ (delta) = degree of curve corresponding to r, by Table IV.

$$\therefore \qquad OPP' = i - \frac{PP'}{100} \cdot \tfrac{1}{2}\delta \text{ nearly.}$$

Eq. (86). $$\Delta' = \Delta + \frac{PP'}{l_{,}}. \quad \text{Table X. 3.}$$

Instead of taking $l_{,}$ from Table X. 3 for the exact value of Δ it is well to take it for the *estimated* value of $\frac{\Delta' + \Delta}{2}$.

Eq. (20). $$D' = \frac{100}{L}\Delta'$$

When P′ is outside of the given curve:

$$i = u\Delta, \qquad r = vL,$$

$$180° - OPP' = i + \frac{PP'}{100} \cdot \tfrac{1}{2}\delta \text{ nearly.}$$

$$\Delta' = \Delta - \frac{PP'}{l_{,}}, \qquad D' = \frac{100}{L}\Delta'$$

Example. Given, a 4° curve of 800 feet, or $\Delta = 32°$ to find a curve from the same *P.C.* which shall shift the last station, *in*, about 55 feet. (Fig. 36.)

$$i = 32° \times .3355 = 10°.736$$

$$r = 800 \times .7450 = 596, \quad \therefore \; \delta = 9° \, 36' = 9°.6$$

$$OPP' = 10°.736 - \frac{55}{100} \times 4°.8 = 8° \, 06'$$

$$\Delta' = 32° + \frac{55}{6.87} = 40°$$

$$D' = \frac{40°}{8} = 5°. \quad \textit{Ans.}$$

For a 5° curve, the true distance $PP' = 55.53$
" " 4°59' " " " " $PP' = 54.60$
which proves this method practically correct.

152. *Given: a tangent and curve, and a* **straight line intersecting** *them, making a given angle with the tangent at a given point; to determine the* **distance on the line** *from the tangent to the curve.* Fig. 37.

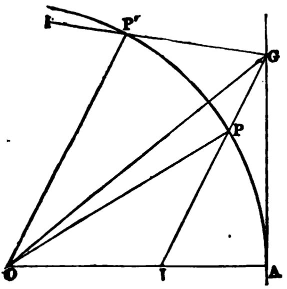

FIG. 37.

We have *OA*, *AG*, and the angle *AGP* to find *GP*.

$$\tan AGO = \frac{R}{AG} \qquad PGO = AGO - AGP$$

$$\sin OPI = \frac{OG}{R} \sin PGO = \frac{\sin PGO}{\sin AGO}$$

$$PG = R \frac{\sin(OPI - PGO)}{\sin PGO}$$

When $AGP = AGO$, eq. (24),

$$GP = R \text{ exsec } (90° - AGO)$$

When $AGP = 90°$, §§ (92), (119),

$$GP = R \text{ vers } POA, \qquad \sin POA = \frac{GA}{R}$$

When $AGP' > AGO$, we have

$$P'GO = AGP' - AGO$$

but the other formulæ remain unchanged.

Example.—Let $R = 955.37$, $AG = 350$, $AGP = 40°$

R	955.37		log 2.980170
AG	350.		log 2.544068
AGO		69° 52′ 47″	log tan 0.436102
AGP		40°	
PGO		29° 52′ 47″	log sin 9.697387
AGO	69° 52′ 47″		log sin 9.972653
OPI		32° 02′ 36″	log sin 9.724734
POG		2° 09′ 49″	log sin 8.576953
			8.879566
R			log 2.980170
PG	72.40 *Ans.*		log 1.859736

This problem may be used in passing from a tangent to a curve when the tangent point is obstructed. The distance AP on the curve is defined by the angle AOP, which is readily found.

If $AGP' > 2AGO$ the line will not cut the curve.

153. *Given: a curve and a* **distant point** *to find a* **tangent** *that shall pass* **through the point.** Fig. 38.

We have the curve *adg* and the point P visible, but distance *unknown, to* find the point of tangent B.

Any chord, as *bf*, parallel to the required tangent, if produced will pass the point *P* at a perpendicular distance equal to the middle ordinate of that chord. Ranging across every two consecutive stakes on the curve we at first find the range falling outside of the required tangent, as *bcG*, *cdH*, etc.; but finally the range falls inside, as *deK*. We then know that the required point is between *c* and *e*. If the range *ce* falls inside the point *P*, a perpendicular distance equal to the middle ordinate of *ce*, the tangent point is at *d*. If the perpendicular distance is greater than this, the point *B* is between *c* and *d*. If less, or if the range *ce* falls outside of *P*, the point *B* is between *d* and *e*. The middle ordinate for *ce* (200 feet) equals the tangent offset for 100 feet, given in Tab. IV., and it is generally so small that it can be estimated at *P* without going to lay it off.

To find the exact point *B*, when it falls between *d* and *e*, find by trial a point *x* on the arc *cd* in range with *e* and a point inside of *P* a perpendicular distance equal to the middle ordinate of *ex*. The point *B* is at the middle point of the arc *ex*. If the point *B* is between *c* and *d*, stand at *c* and find a point *x* on the arc *de* in the same way. *B* is at one half the arc *cx*.

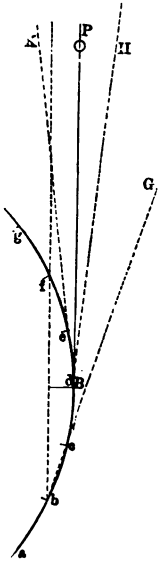

FIG. 38.

The middle ordinate of any chord *ex* is less than *M* for 200 feet, and greater than *m* for 100 feet. If necessary, its exact value *m'* can be found by

$$m' = \frac{m \times \overline{ex}^2}{10000} \qquad (87)$$

and this equation is nearly true when *ex* is as great at 300 or 400 feet. That is, middle ordinates on the same curve are to each other as the squares of their chords *very nearly*.

By this method the point *B* is found without the use of the transit, so that the *plug* can be driven at *B* before the transit is brought up from the rear. It is therefore preferable to the following solution. Fig. 39.

From any two points a and c of the curve measure the angles to the point P, so that with the chord ac as a base, and the measured angles, we may find cP by the formula

$$cP = ac\frac{\sin caP}{\sin cPa}$$

Knowing the angle c that cP makes with a tangent at c, we find the length of the chord cd by $cd = 2R \sin c$.

By Geom. Tab. I. 24,

$$PB = Pe = \sqrt{cP \times dP}$$

whence we know ce. Opposite e, or on the arc eB described with the radius Pe, we find B.

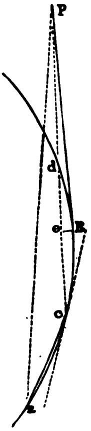

FIG. 39.

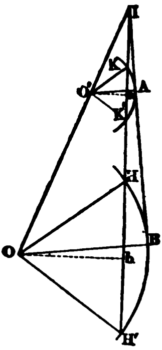

FIG. 40.

154. *Given:* **two curves exterior** *to each other, to find the tangent points of a line* **tangent to both** *and its* **length** *between tangent points.* Fig. 40.

Let B and A be the required tangent points. Let $OB = R$, and $O'A = R'$.

On the curve of greater radius R select a point H supposed *to be near the* unknown tangent point B, and knowing the direction of the radius OH, find on the other curve a point K having a radius $O'K$ parallel to OH, and measure HK. In the diagram draw Ob and $O'a$ perpendicular to HK. Then the angle $KO'a = 90° - HKO' = KO'A$ nearly, which is the angle required. We have therefore to find the correction $aO'A = x$, and apply it to $KO'a$.

$$Aa = R' \text{ vers } KO'a; \qquad Bb = R \text{ vers } KO'a \text{ nearly.}$$
$$Ka = R' \sin KO'a; \qquad Hb = R \sin KO'a$$

$$\therefore \quad Bb - Aa = (R - R') \text{ vers } KO'a$$
$$ab = HK + (R - R') \sin KO'a$$

$$\therefore \quad \sin x = \frac{(R - R') \text{ vers } KO'a}{HK + (R - R') \sin KO'a} \text{ nearly.} \tag{88}$$

$$KO'A = (KO'a - x) = HOB$$

Observe that $KO'a$ = the angle between the tangent at K or H and the line HK; and $KO'A$ = the angle between the tangent at K or H and the required tangent BA.

If, instead of H and K, the points H' and K' had been selected, then

$$\sin x = \frac{(R - R') \text{ vers } H'Ob}{H'K' - (R - R') \sin H'Ob} \text{ nearly,} \tag{88'}$$

and

$$H'OB = K'O'A = H'Ob + x.$$

The length of BA should be obtained by measurement, but it may be calculated by

$$AB = ab - (R - R') \sin x \tag{89}$$

When $R = R'$, $x = 0$, and HK is parallel to BA.

In case the **curves** *are* **reverse** *to each other, as in Fig.* 41,

$$\sin x = \frac{(R + R') \text{ vers } KO'a}{HK + (R + R') \sin KO'a} \text{ nearly.} \tag{90}$$

$$KO'A = HOB = KO'a - x$$

If the points H' and K' are selected, Fig. 41,

$$\sin x = \frac{(R + R') \text{ vers } H'Ob}{H'K' - (R + R') \sin H'Ob} \text{ nearly.} \tag{91}$$

$$H'OB = K'O'A = H'Ob + x.$$

The lines HK, AB, and OO' all intersect in a common point I, Fig. 41.

$$HI = \frac{HK \times R}{R + R'} \tag{92}$$

$$IB = \sqrt{HI(HI + 2R \sin HOb)} \tag{93}$$

$$AB = IB\frac{R + R'}{R} \tag{94}$$

These last three equations furnish **another method** of solving the same problem. They may be applied to Fig. 40 by changing the sign of R'.

In Fig. 41, if $R = R'$, then $HI = \frac{1}{2}HK$ and $AB = 2IB$.

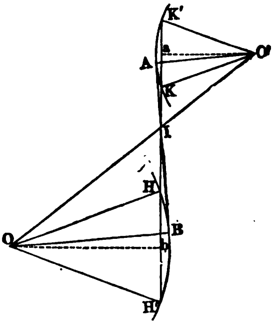

FIG. 41.

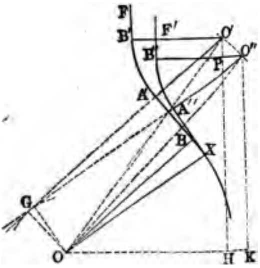

FIG. 42.

155. *Given:* **two curves, O and O′, reverse** *to each other, joined by a tangent* **BA′**, *and terminating in another tangent,* **B′F** ; **to change** the position of the **Point of Tangent B** *of the first curve, so that the second curve may terminate in a given* **parallel tangent, B′F′.** Fig. 42.

Let X be the required new position of B.

" O'' be the corresponding position of O'.

" $\triangle' = A'O'B'$ and $\triangle'' = A''O''B''$.

Since the radii and the connecting tangent are unchanged *in length,* and all rotate together about O as a centre, O″ *will be on a circle* passing through O′, described with a radi *OO′,* and the required angle $BOX = O'OO''$.

In the diagram, produce $O'A'$ and draw the perpendicular G, and let $\alpha =$ the angle $OO'G$. Also, draw OK parallel nd $O''K$ and $O'H$ perpendicular to $B'O'$. In the triangle $O'G$ we have

$$\cot OO'G = \frac{GO'}{GO}, \quad \text{or} \quad \cot\alpha = \frac{R+R'}{BA'} \qquad (95)$$

nd

$$OO' = \frac{R+R'}{\cos\alpha} \qquad (96)$$

The angle $KOO' = OO'B' = \alpha + \Delta'$.
The angle $KOO'' = OO''B'' = \alpha + \Delta''$.

$$KO = OO''.\cos(\alpha + \Delta''), \quad HO = OO'.\cos(\alpha + \Delta').$$

$$\therefore \ HK = OO'[\cos(\alpha + \Delta'') - \cos(\alpha + \Delta')] = B'F'$$

$$\therefore \quad \cos(\alpha + \Delta'') = \cos(\alpha + \Delta') + \frac{B'F'}{OO'} \qquad (97)$$

$$BOX = O'OO'' = (\alpha + \Delta') - (\alpha + \Delta'') \qquad (98)$$

If we conceive a line to be drawn through O bisecting the rc $O'O''$, the angle it makes with $B''O''$ is a mean between $'O'O$ and $B''O''O$; hence the chord $O'O''$, perpendicular to is line, makes an angle with $O'P$ perpendicular to $B'O'$ of

$$PO'O'' = \tfrac{1}{2}[(\alpha + \Delta') + (\alpha + \Delta'')]$$

d since

$$O'P = PO''\cot PO'O''$$

$$F'B'' = B'F'\cot\tfrac{1}{2}[(\alpha + \Delta') + (\alpha + \Delta'')] \qquad (99)$$

hich gives the distance, measured on the parallel tangent, tween the old tangent point and the new.

This problem occurs in practice when both the connecting ngent and the radius of the last curve are at their *minimum nit*, and the parallel tangent is *inside* of the old one, as in e figure. Should the new tangent be *outside*, the same formulæ apply, only changing the sign of $B'F'$ in eq. (97). But this last case it is usually preferable to employ problem *136 or § 137.*

xample.—A 1° 40′ curve is followed by a tangent of 200 ft. *that by a 4°* curve of 10 stations ending in a ta and the offset to the given parallel tangent is 80 ft. on the inside. Required, the position of the new tangent points X and B''.

Here $R = 3437.87$, $R' = 1432.69$, $BA' = 200$, $B'F' = 80$.

Eq. (95)	$R + R'$	4870.56	log 3.687579
	BA'	200.	log 2.301030
∴	α	2° 21′	log cot 1.386549
Eq. (96)	α	2° 21′	log cos 9.999635
∴	OO'		3.687944
Eq. (97)	$B'F'$	80	1.903090
		.01641	8.115146
	$\alpha + \Delta'$	42° 21′ cos .73904	
	$\alpha + \Delta''$	40° 56′ cos .75545	
Eq. (98)	BOX	1° 25′ ∴ $BX = 85$ ft. *Ans.*	
Eq. (99)	$PO'O''$	41° 38′ 30″ cot 1.12468 × 80 = 89.97 = $F'B'$	

156. When the tangents of a proposed road are to be in general much longer than the curves, it is desirable to establish the tangents first in making the location, and afterwards determine suitable curves. On the other hand, if the curves necessarily predominate, they should be first selected and adjusted to the ground with reference to grade and easy alignment, and afterwards joined by tangents. In the latter case the field work cannot be successfully accomplished unless the location has been previously worked out upon a correct map constructed from the preliminary surveys. The map should show contours of the surface, and also the grade contour, or intersection of the surface and plane of the grade. In side-hill work the grade contour indicates approximately the degree and position of the necessary curves. In the work of selecting proper curves upon the map, **templets** or pattern curves are almost indispensable. The templets are cut to form a series of curves, the radii being taken from *Table IV.* to a scale corresponding to the scale of the map, which ranges from 400 to 100 feet per inch, according to the difficulty of the location. The templets should represe *convenient* curves, or those in which the number of minu per station bear a simple ratio to 100. Curves of 50′ and multiples of 50′ are most convenient; 40′ curves and multiples standing next in order, and 30′ curves and multiples next.

TABLE OF CONVENIENT CURVES.

D.	Ratio of Min. to Feet.	D.	Ratio of Min. to Feet.	D.	Ratio of Min. to Feet.
50′	1 : 2	40′	2 : 5	30′	3 : 10
1° 40′	1 : 1	1° 20′	4 : 5	1° 00′	3 : 5
2° 30′	3 : 2	2° 00′	6 : 5	1° 30′	9 : 10
3° 20′	2 : 1	2° 40′	8 : 5	2° 00′	6 : 5
4° 10′	5 : 2	3° 20′	2 : 1	2° 30′	3 : 2
5° 00′	3 : 1	4° 00′	12 : 5	3° 00′	9 : 5
5° 50′	7 : 2	4° 40′	14 : 5	3° 30′	21 : 10
6° 40′	4 : 1	5° 20′	16 : 5	4° 00′	12 : 5
7° 30′	9 : 2	6° 00′	18 : 5	4° 30′	27 : 10
8° 20′	5 : 1	6° 40′	4 : 1	5° 00′	3 : 1
9° 10′	11 : 2	7° 20′	22 : 5	5° 30′	33 : 10
10° 00′	6 : 1	8° 00′	24 : 5	6° 00′	18 : 5

After drawing the curves and tangents upon the map, the tangent points and central angles are carefully determined, the latter being compared with the lengths of the curves obtained by a pair of stepping dividers set *precisely* by scale to the length of one station. Field notes are then prepared from the map, and if the work has been well done these notes may be followed in the field with scarcely any alterations.

No ordinary protractor will measure the angles closely enough for this purpose; it is better to use a radius as large as convenient, of 50 parts. The chord of any arc drawn with this radius equals 100 times the sine of one half the angle subtended.

The importance of having absolutely straight-edged rulers in such work is obvious. In case a very long line is to be projected upon the map, it is well to use a piece of fine sewing silk for the purpose. See §§ 53, 54.

CHAPTER VI.

COMPOUND CURVES.

A. Theory.

157. A compound curve consists of two or more consecutive circular arcs of different radii, having their centres on the same side of the curve; but any two consecutive arcs must have a common tangent at their meeting point, or their radii at this point must coincide in direction. The meeting point is called the point of compound curve, or *P.C.C.* Compound curves are employed to bring the line of the road upon more favorable ground than could be done by the use of any simple curve.

When a compound curve of two arcs connects two tangent lines, the tangent points are at unequal distances from the intersection or vertex, the shorter distance being on the line which is tangent to the arc of shorter radius.

158. Let VA, VB (Fig. 43) be any two right lines intersecting at V, and let Δ be the deflection angle between them. Let A and B be the tangent points of a compound curve (VA less than VB), and let AP, PB be the two arcs of the curve. The centre O_1 of the arc AP will be found on AS, drawn perpendicular to VA; the centre O_2 of the arc PB will be found on BS produced perpendicular to VB; and the angle ASB will evidently equal Δ. Join VS, and on VS as a diameter describe a circle; it will pass through the points A and B, since the angles VAS, VBS are right angles in a semicircle. Draw the chord VQ, bisecting the angle AVB, and join AQ, BQ. Then AQ, BQ are equal, since they are chords subtending the equal angles AVQ, BVQ. From Q as a centre, and with radius QA, describe a circle; it will cut the tangent lines at A and B, and also at two other points G and Y, such that $VG = VA$, and $VY = VB$. Hence $BG = AY$, and the parallel chords AG, BY are perpendicular to VQ. Join AB; then $AQB = ASB = \Delta$, since both angles are subtended by *the same chord* AB.

In the triangle VAB, the sum of the angles at A and B i *equal to* the exterior angle Δ between the tangents; wh *their difference* $(A - B)$ is equal to the angle at the centr nded by the chord *BG*, which is the difference of the ($VB - VA$). For the angle $VAB = VAG + GAB$, and ıngle $VBA = VBY - ABY$. But $VAG = VBY$ and ' $= ABY$, and by subtraction $VAB - VBA = 2GAB =$, since *A* is on the circumference and *Q* at the centre.

ı9. THEOREM.—*The circle YAGB, whose centre is Q, is* ›cus *of the point of compound curve P, whatever be the* ve *lengths of the arcs AP, PB composing the curve.*

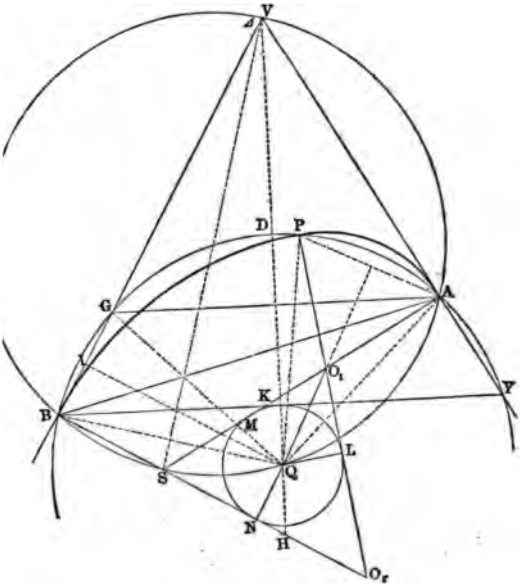

FIG. 43.

the circle *YAGB*, and between *A* and *G*, take any point d on *AS* find a centre O_1, from which a circular arc may ıwn cutting the circle at *A* and *P*; also on *BS* produced ı centre O_2, from which a circular arc may be drawn g the circle at *B* and *P*. Join PQ, PO_1 and PO_2. *when two circles* intersect, the angles are equal beadii *drawn to* the points of intersection, $QPO_1 = QAO_1$

and $QPO_2 = QBO_2$. Draw the chord QS and it subtends the equal angles $QAO_1 = QBO_2$. Hence $QPO_1 = QPO_2$ and the radius PO_1 coincides in direction with the radius PO_2, which is the condition essential to a compound curve.

Now, if we imagine another point P' to be taken on QP or on QP produced, and the arcs AP' BP', drawn from centres found on AS and BS, it is evident that the equality of angles found in respect to P could not exist in respect to P'. Hence the arcs would intersect in P' at some angle O_1PO_2 and would not form a compound curve. Therefore, Q. E. D.

160. THEOREM.—*In any compound curve the radial lines passing through the three tangent points A, P, and B are all tangent to a circle having the point Q for its centre, and for its diameter the difference of the sides VB and VA.*

Draw the three lines QM, QN, QL perpendicular to the radial lines BO_2, AS, and PO_2 respectively. Then the three right-angled triangles BQN, PQL, and AQM are equal, since $BQ = PQ = AQ =$ radius of the circle AGB, and the angles at B, P, and A are equal by the last theorem. Hence $QM = QL = QN$, and if a circle be described with this radius about Q, the three lines BO_2, PO_2, and AO_1 produced will be tangent to it. Draw QI perpendicular to VB; it will bisect the chord GB in I; and $QN = BI = \frac{1}{2}BG$. Hence the diameter $2QN = BG = VB - VA$; which was to be proved.

Corollary 1. The compound curve intersects the circle AGB in the point P, at an angle equal to half the difference of the angles VAB, VBA. For $QPL = QBN = BQI = \frac{1}{2}BQG$. The arc AP is exterior, and the arc PB interior to the circle AGB.

Cor. 2. Since both centres are on the line PL, the position of the point P fixes the lengths of the radii of a compound curve. As P is moved toward G both radii are increased, until when P reaches G, AO_1 becomes AK, a *maximum*, while BO_2 becomes infinite. As P moves toward A both radii are diminished, but the least value of the arc AP depends upon the least radius allowed on the road. If in the diagram we make AO_1 equal to the least radius allowed, a right line drawn through the point O_1 tangent to the circle LMN fixes the *corresponding minimum* value of the arc AP, and also of the radius BO_2 for given values of VA, VB, and Δ. Between these limits any desired values of the radii may be employed.

Cor. 3. In the triangle SO_1O_2, the sum of the two central angles AO_1P and PO_2B is equal to the exterior angle $ASB = \Delta$; consequently, as the central angle of one arc is increased by any change in the position of the point P, the central angle of the other will be diminished an *equal amount.*

Cor. 4. Only one value of the angle AO_1P is consistent with a given value of the radius AO_1, since both depend on the variable position of the line PL; and for the same reason only one value of the angle BO_2P is consistent with a given value of the radius BO_2. Hence only one radius or one central angle can be assumed at pleasure, the remaining parts being deducible therefrom in terms of the sides VA, VB, and the angle Δ.

B. General Equations.

161. Let $S_1 =$ the side VA, $S_2 =$ the side VB
Let $R_1 =$ the radius AO_1 $R_2 =$ the radius BO_2
" $\gamma =$ diff. $VAB - VBA$, $\Delta =$ the sum $VAB + VBA$
" $\Delta_1 =$ central angle AO_1P, $\Delta_2 =$ central angle BO_2P.

In the triangle BQI, $\cot BQI = \dfrac{IQ}{BI}$. But $IQ = VI \times \cot IQV = \frac{1}{2}(S_2 + S_1) \cot \frac{1}{2}\Delta$, and $BI = \frac{1}{2}(S_2 - S_1)$.

$$\therefore \qquad \cot \tfrac{1}{2}\gamma = \frac{S_2 + S_1}{S_2 - S_1} \cot \tfrac{1}{2}\Delta \qquad (100)$$

By Cor. 3,

$$\Delta_1 + \Delta_2 = \Delta \qquad (101)$$

In the triangle AQM, $AO_1 = AM - MO_1$. But $AM = MQ \cot \frac{1}{2}\gamma$, and $MO_1 = MQ \cot \frac{1}{2}\Delta_1$.

$$\left.\begin{aligned} \therefore \qquad R_1 &= \tfrac{1}{2}(S_2 - S_1)(\cot \tfrac{1}{2}\gamma - \cot \tfrac{1}{2}\Delta_1) \\ \text{Similarly,} \qquad R_2 &= \tfrac{1}{2}(S_2 - S_1)(\cot \tfrac{1}{2}\gamma + \cot \tfrac{1}{2}\Delta_2) \end{aligned}\right\} \qquad (102)$$

Subtracting,

$$R_2 - R_1 = \tfrac{1}{2}(S_2 - S_1)(\cot \tfrac{1}{2}\Delta_2 + \cot \tfrac{1}{2}\Delta_1) \qquad (103)$$

From (102),
$$\left.\begin{aligned}\cot\tfrac{1}{2}\Delta_1 &= \cot\tfrac{1}{2}\gamma - \frac{R_1}{\tfrac{1}{2}(S_2 - S_1)}\\ \cot\tfrac{1}{2}\Delta_2 &= \frac{R_2}{\tfrac{1}{2}(S_2 - S_1)} - \cot\tfrac{1}{2}\gamma\end{aligned}\right\} \qquad (104)$$

In the triangle ABG,
$$BG = \frac{AB \sin BAG}{\sin AGV}$$
or
$$\tfrac{1}{2}(S_2 - S_1) = \frac{\tfrac{1}{2}AB \sin\tfrac{1}{2}\gamma}{\sin\tfrac{1}{2}\Delta} \qquad (105)$$

by which we find $\tfrac{1}{2}(S_2 - S_1)$, when, instead of the sides and Δ, we have given AB, and the angles VAB and VBA.

From (103),
$$\tfrac{1}{2}(S_2 - S_1) = \frac{R_2 - R_1}{\cot\tfrac{1}{2}\Delta_2 + \cot\tfrac{1}{2}\Delta_1} \qquad (106)$$

From (102),
$$\left.\begin{aligned}\cot\tfrac{1}{2}\gamma &= \frac{R_1}{\tfrac{1}{2}(S_2 - S_1)} + \cot\tfrac{1}{2}\Delta_1\\ \cot\tfrac{1}{2}\gamma &= \frac{R_2}{\tfrac{1}{2}(S_2 - S_1)} - \cot\tfrac{1}{2}\Delta_2\end{aligned}\right\} \qquad (107)$$

From (100),
$$\tfrac{1}{2}(S_2 + S_1) = \frac{\tfrac{1}{2}(S_2 - S_1)\cot\tfrac{1}{2}\gamma}{\cot\tfrac{1}{2}\Delta} \qquad (108)$$

S_2 and S_1 are found by adding and subtracting the values found by eqs. (106), (108).

From (105),
$$\tfrac{1}{2}AB = \frac{\tfrac{1}{2}(S_2 - S_1)\sin\tfrac{1}{2}\Delta}{\sin\tfrac{1}{2}\gamma} \qquad (109)$$

which may be used instead of (108) when the sides are not required. $VAB = \tfrac{1}{2}(\Delta + \gamma)$ and $VBA = \tfrac{1}{2}(\Delta - \gamma)$.

162. Given: *the sides* $VA = S_1$ *and* $VB = S_2$ *and the angle* Δ; *assuming the shorter radius* R_1, **to find** Δ_1, Δ_2, *and* R_2.

Use equations (100), (104), (101), (102), and (18).

Example.—Let $VA = 1899.90$, $VB = 1091.12$, $\Delta = 74°$, an *assume* $R_1 = 955.37$.

(100) $\frac{1}{2}(S_2 + S_1)$ 1495.51			log	3.174789
$\frac{1}{2}(S_2 - S_1)$ 404.39			"	2.606800
			"	0.567989
$\frac{1}{2}\Delta$	37°		cot "	0.122886
∴ $\frac{1}{2}\gamma$	11° 31′ 01″.5	cot 4.90769	" "	0.690875
(104) R_1 $(D = 6°)$			"	2.980170
$\frac{1}{2}(S_2 - S_1)$			"	2.606800
		2.36249	"	0.373370
∴ $\frac{1}{2}\Delta_1$	21° 27′	cot 2.54520		
(101) $\frac{1}{2}\Delta$	37°			
∴ $\frac{1}{2}\Delta_2$	15° 33′	" 3.59370		
(102) $\frac{1}{2}\gamma$		" 4.90769		
		8.50139	"	0.929490
$\frac{1}{2}(S_2 - S_1)$			"	2.606800
∴ R_2 $(D = 1° 40')$				3.536290

18) ∴ $\Delta_1 = 42° 54'$, $L_1 = 715$; $\Delta_2 = 31° 06'$, $L_2 = 1866$.

163. Given: *the line AB, and the angles VAB, VBA; ssuming the longer radius* R_2, **to find** Δ_2, Δ_1, *and* R_1.

Example.—Let $AB = 2437.82$, $VAB = 48° 31'$, $VBA = 25° 29'$, nd assume $R_2 = 3437.87$.

05) $\frac{1}{2}AB$ 1218.91			log	3.085972
$\frac{1}{2}\gamma$	11° 31′		sin "	9.300276
			"	2.386248
$\frac{1}{2}\Delta$	37°		" "	9.779463
$\frac{1}{2}(S_2 - S_1)$ 404.38			"	2.606785
)4) R_2				3.536289
		8.50166	"	0.929504
$\frac{1}{2}\gamma$	11° 31′	cot 4.90785		
$\frac{1}{2}\Delta_2$	15° 33′	cot 3.59381		
)1) $\frac{1}{2}\Delta$	37°			
$\frac{1}{2}\Delta_1$	21° 27′	cot 2.54516		
)2) $\frac{1}{2}\gamma$		" 4.90785		
		2.36269	log	0.373407
$\frac{1}{2}(S_2 - S_1)$				2.606785
R_1 $(D = 6°)$				2.9801

164. Usually a compound curve is fitted by trial to the shape of the ground, after which it may be desirable to **calculate** the sides *VA*, *VB*, or the line *AB*, and the angles *VAB*, *VBA*.

Example.—From the point of curve *A*, a 6° curve is run 715 feet to the *P.C.C.*; thence a 1° 40′ curve is run 1866 feet to the *P.T.* *Required,* the sides *VA*, *VB*, and the line *AB*, and angles *VAB*, *VBA*. Here $R_1 = 955.37$, $\Delta_1 = 42° 54'$, $R_2 = 3437.87$, $\Delta_2 = 31° 06'$.

(106) $R_2 - R_1$	2482.50			log	3.394889
$\frac{1}{2}\Delta_1$		21° 27′	cot 2.54516		
$\frac{1}{2}\Delta_2$		15° 33′	" 3.59370		
			6.13886	"	0.788088
∴ $\frac{1}{2}(S_2 - S_1)$	404.39			"	2.606801
(107) R_1				"	2.980170
			2.36248	"	0.373369
$\frac{1}{2}\Delta_1$		21° 27′	cot 2.54516		
∴ $\frac{1}{2}\gamma$		11° 31′ 01″.7	" 4.90764	"	0.690873
(108) $\frac{1}{2}(S_2 + S_1)$					2.606801
				"	3.297674
$\frac{1}{2}\Delta$		37°		cot "	0.122886
∴ $\frac{1}{2}(S_2 + S_1)$	1495.51			"	3.174788
S_2	1899.90				
S_1	1091.12				
VAB		48° 31′			
VBA		25° 29′			
(109) $\frac{1}{2}(S_2 - S_1)$				"	2.606801
$\frac{1}{2}\Delta$		37°		sin "	9.779463
					2.386264
$\frac{1}{2}\gamma$		11° 31′ 01″.7		sin "	9.300294
∴ $\frac{1}{2}AB$	1218.91			"	3.085970
AB	2437.82				

165. Given: *the radii R_1, R_2, the angle Δ, and one side, VA, or VB,* **to find** *the other side and the central angles Δ_1, Δ_2. Fig. 43.*

In the triangle AMQ, $AO_1 = AM - MO_1 = IQ - MQ \cot MO_1Q$; or

$$R_1 = \tfrac{1}{2}(S_2 + S_1)\cot\tfrac{1}{2}\Delta - \tfrac{1}{2}(S_2 - S_1)\cot\tfrac{1}{2}\Delta_1$$

whence

$$\tfrac{1}{2}(S_2 + S_1) = \tfrac{1}{2}(S_2 - S_1)\cot\tfrac{1}{2}\Delta_1 \tan\tfrac{1}{2}\Delta + R_1\tan\tfrac{1}{2}\Delta$$

By eq. (106)

$$\tfrac{1}{2}(S_2 - S_1) = (R_2 - R_1)\frac{\sin\frac{1}{2}\Delta_2 \sin\frac{1}{2}\Delta_1}{\sin\frac{1}{2}\Delta}$$

Substituting this above, subtracting and reducing

$$S_1 = (R_2 - R_1)\sin\tfrac{1}{2}\Delta_2\,\frac{\sin\frac{1}{2}(\Delta - \Delta_1)}{\frac{1}{2}\sin\Delta} + R_1\tan\tfrac{1}{2}\Delta$$

ut $\frac{1}{2}(\Delta - \Delta_1) = \frac{1}{2}\Delta_2$ and $2\sin^2\frac{1}{2}\Delta_2 = \text{vers}\ \Delta_2$, whence

$$S_1 = \frac{(R_2 - R_1)\,\text{vers}\ \Delta_2 + R_1\,\text{vers}\ \Delta}{\sin\Delta} \qquad (110)$$

ansposing,

$$\text{vers}\ \Delta_2 = \frac{S_1\sin\Delta - R_1\,\text{vers}\ \Delta}{R_2 - R_1} \qquad (111)$$

nilarly, from the triangle BQO_2

$$R_2 = \tfrac{1}{2}(S_2 + S_1)\cot\tfrac{1}{2}\Delta + \tfrac{1}{2}(S_2 - S_1)\cot\tfrac{1}{2}\Delta_2$$

m which and eq. (106) we derive

$$S_2 = \frac{R_2\,\text{vers}\ \Delta - (R_2 - R_1)\,\text{vers}\ \Delta_1}{\sin\Delta} \qquad (112)$$

d

$$\text{vers}\ \Delta_1 = \frac{R_2\,\text{vers}\ \Delta - S_2\sin\Delta}{R_2 - R_1} \qquad (113)$$

Example.—Given : $VA = S_1 = 1091.12$, $\Delta = 74°$, and the radii $R_1 = 955.37$, $R_2 = 3437.87$, to find Δ_1, Δ_2, and S_2.

(111) S_1	1091.12			log	3.037873
Δ		74°		sin "	9.982842
			1048.85	"	3.020715
R_1				"	2.980170
Δ		74°		vers "	9.859956
			692.03	"	2.840126
			356.82	"	2.552449
$R_2 - R_1$	2482.50			"	3.394889
∴ Δ_2		31° 06′		vers "	9.157560
∴ Δ_1		42° 54′		" "	9.427254
(112) $R_2 - R_1$				"	3.394889
			663.96	"	2.822143
R_2				"	3.536289
Δ				vers "	9.859956
			2490.26	"	3.396245
			1826.30	"	3.261572
Δ				sin "	9.982842
∴ S_2			1899.90	"	3.278730

166. Given : *one side, and the radius and central angle of the adjacent arc,* **to find** *the other radius and side.*

From eqs. (111), (113) we have

$$\left.\begin{aligned} R_2 - R_1 &= \frac{S_1 \sin \Delta - R_1 \operatorname{vers} \Delta}{\operatorname{vers} \Delta_2} \\ R_2 - R_1 &= \frac{R_2 \operatorname{vers} \Delta - S_2 \sin \Delta}{\operatorname{vers} \Delta_1} \end{aligned}\right\} \qquad (114)$$

by one of which the required radius may be found; the required side is then found by eq. (110) or (112), as in the last problem.

Example.—Given : $VA = S_1 = 1091.12$ $\Delta = 74°$, $R_1 = 955.37$ *and* $\Delta_1 = 42°54'$; *to find* R_2 $\Delta_2 = 74° - 42°\ 54' = 31°\ 06'$.

) S_1	1091.12			log	3.037873
$\triangle$		74°		"	sin 9.982842
			1048.85	"	3.020715
R_1	955.37			"	2.980170
$\triangle$		74°		"	vers 9.859956
			692.03	"	2.840126
			356.82	"	2.552449
$\triangle_2$		31°06		"	vers 9.157556
$\therefore R_2 - R_1$	2482.52			"	3.394893
$\therefore R_2$	3437.89				

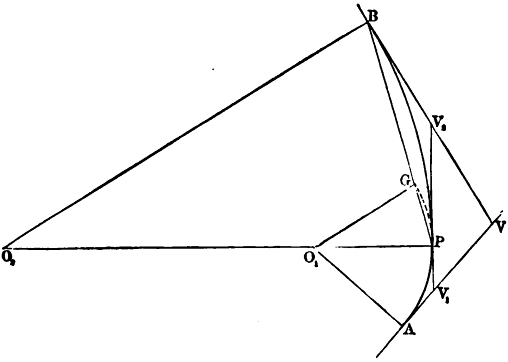

FIG. 44.

Otherwise: Fig. 44. If convenient in the field, a tan-ent PV_2 may be run from the point P to intersect the arther tangent. The distance PV_2 multiplied by cot $\frac{1}{2}\triangle_2$ vill equal the radius R_2 by eq. (25).

167. *Remarks.*—If the first arc AP be produced to G, ig. 44, so that $AO_1G = \triangle$, then G is the tangent point of a ngent parallel to VB, and by §137, the tangent point B must e *on the line PG produced.* Conversely, if the point B is *sumed, and the arc AG* given, the point P must be on *line BG* produced. The radius R_2 may be found b

$R_2 = \frac{BP}{2 \sin \frac{1}{2} \Delta_2}$, BP being measured on the ground; or by similar triangles $R_2 : R_1 :: BP : GP$.

The distance VD, Fig. 43, from the vertex to the circle AGB is expressed by the formula

$$VD = \tfrac{1}{2}(S_2 - S_1) \frac{\text{ex sec} \frac{1}{2} \Delta}{\sin \frac{1}{2} \gamma}. \tag{115}$$

If the point P falls at D, then VD is also the distance of the curve from the vertex measured on the line VQ. But when P falls at D, the radius PO_2 is perpendicular to the line AB, and $\Delta_1 = VAB$, and $\Delta_2 = VBA$. When Δ_1 is greater than VAB, the arc AP, being exterior to the circle, cuts the line VD; but when Δ_1 is less than VAB, the arc PB cuts the line DQ.

If the line O_2P produced passes through V, we have

$$\sin QVL = \frac{S_2 - S_1}{S_2 + S_1} \sin \tfrac{1}{2} \Delta \tag{116}$$

giving $\Delta_1 = \frac{1}{2}\Delta + QVL$ and $\Delta_2 = \frac{1}{2}\Delta - QVL$.

When Δ_1 is *greater* than this, we have for the external distance of the vertex

$$E_1 = R_1 \text{ ex sec } AO_1V$$

in which the angle AO_1V is found by the formula $\tan AO_1V = \frac{R_1}{S_1}$, and E_1 is measured on a line VO_1, making the angle $AVO_1 = 90° - AO_1V$.

When Δ_1 is less than $(\frac{1}{2}\Delta + QVL)$, we have similar expressions with respect to the arc BP and centre O_2.

168. *To locate a compound curve when the point of compound curve is inaccessible.* Fig. 45.

Each arc being in itself a simple curve is located as such. When the *P.C.C.* is accessible, the transit is placed over it, and the direction of the common tangent found, from which the second arc is then located.

When the *P.C.C.* is not accessible, the common tangent V_1V_2 *may be found* by locating the points V_1 and V_2, which *may be easily* done, since $V_1A = V_1P = R_1 \tan \frac{1}{2}\Delta_1$, and

$V_2B = V_2P = R_2 \tan \frac{1}{2} \Delta_2$, from which each arc may then be located by offsets or otherwise, as in the case of simple curves.

Should the points $V_1 V_2$ be obstructed, the common tangent may be found by an offset $HG = LP$ from any convenient point H; for knowing the angle HO_1P, we have $HG = R_1$ vers HO_1P, and $GP = R_1 \sin HO_1P$.

If the entire tangent $V_1 V_2$ is too much obstructed for use, the parallel line HK may be employed, observing that the angle PO_2K is found by vers $PO_2K = \frac{LP}{R_2}$, and the distance LK by $LK = R_2 \sin PO_2K$, by which a point K on the second arc is found having a tangent offset $KI = HG$.

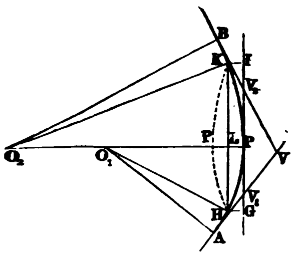

FIG. 45.

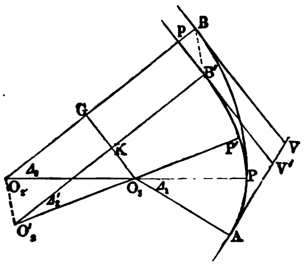

FIG. 46.

Should the line HK be also obstructed, we may run the inverted curve $HP' = HP$ and $P'K = PK$ to find the point K from which so much of the second arc as is accessible may be located.

C. Special Problems in Compound Curves.

169. *Given: a compound curve ending in a tangent;* **to change** *the* **P.C.C.** *so that the curve may end in a given* **parallel tangent.** Fig. 46.

Let APB be the given curve ending in VB,
" $V'B'$ be the given parallel tangent,
" p = perpendicular distance between tangents.

It is required to change the point P, and with it the values *of* Δ_1 *and* Δ_2, *so that* with the same radii R_1 and R_2 the new *curve* $AP'B'$ *may* end in the parallel tangent $V'B'$.

a. *When the tangent $V'B'$ is* **inside** *of VB:*

Let $\Delta_1 = AO_1P$, $\Delta_1' = AO_1P'$, $\Delta_2 = PO_2B$, $\Delta_2' = P'O_2'B'$, and in the diagram draw O_1G perpendicular to BO_2; then $GO_2 = O_1O_2 \cos \Delta_2$, $KO_2' = O_1O_2' \cos \Delta_2'$. Subtracting since $O_1O_2 = O_1O_2' = (R_2 - R_1)$, and $KO_2' - GO_2 = GB - KB' = p$,

$$p = (R_2 - R_1)(\cos \Delta_2' - \cos \Delta_2)$$

whence

$$\cos \Delta_2' = \frac{p}{R_2 - R_1} + \cos \Delta_2 \qquad (117)$$

$PO_1P' = (\Delta_2 - \Delta_2')$ and the point P is *advanced.*

b. *When the tangent $V'B'$ is* **outside** *of VB:*

$$p = (R_2 - R_1)(\cos \Delta_2 - \cos \Delta_2')$$

whence

$$\cos \Delta_2' = \cos \Delta_2 - \frac{p}{R_2 - R_1} \qquad (118)$$

$PO_1P' = (\Delta_2' - \Delta_2)$ and the point P is *moved back* and the arc AP diminished.

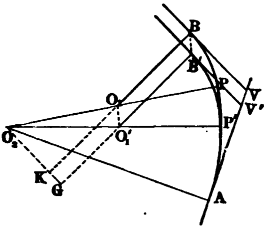

FIG. 47.

In case the curve **terminates with** *the arc of* **shorter radius,** *or R_1 follows R_2.* Fig. 47.

c. *When $V'B'$ is* **inside** *of VB:*

$$p = (R_2 - R_1)(\cos \Delta_1 - \cos \Delta_1')$$

whence

$$\cos \Delta_1' = \cos \Delta_1 - \frac{p}{R_2 - R_1} \qquad (119)$$

$PO_2P' = (\Delta_1' - \Delta_1)$ and the point P is *moved back.*

d. *When* $V'B'$ *is* **outside** *of* VB:

$$p = (R_2 - R_1)(\cos \Delta_1' - \cos \Delta_1)$$

whence

$$\cos \Delta_1' = \cos \Delta_1 + \frac{p}{R_2 - R_1} \qquad (120)$$

$PO_2P' = (\Delta_1 - \Delta_1')$ and the point P is *advanced.*

Example.—Let $R = 2292.01$, $R_1 = 1432.69$, $\Delta_2 = 28°$, and $p = 20.07$ inside of VB; case **a.**

p	20.07		log 1.302547
(117) $R_2 - R_1$	859.32		" 2.934155
		.023356	" 8.368392
Δ_2	28°	cos .88295	
∴ Δ'_2	25°	" .906306	
∴ PO_1P'	3°		

170. *Given: a compound curve terminating in a tangent,* **to change** *the* **P.C.C.** *and also the* **last radius,** *so that the curve shall end in a* **parallel tangent** *at a point on the* **same radial line** *as before.* Fig. 48.

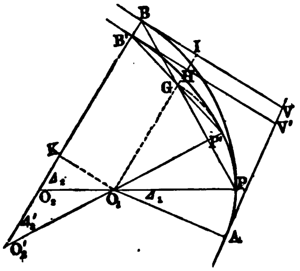

FIG. 48.

Let APB be the given curve ending in the tangent VB; let $V'B'$ be the given parallel tangent; and let $p = BB' = HI =$ the perpendicular distance between tangents.

It is required to change the point P to P', and also the value of R_2 to R_2', so that the new curve may end in $V'B'$ at B' **inside** of VB on the same radial line BO_2.

In the diagram produce the arc AP to G to meet O_1G *drawn parallel to* O_2B; then $PO_1G = \Delta_2$. Draw the chord PB, *and it will pass* through G. Lay off the distance p from

B on BO_2 to find B'; draw $B'G$ and produce it to intersec the arc APG in P'. Then P' is the *P.C.C.* required. Joi $P'\,O_1$ and produce it to meet BO_2 produced in O_2'. The $P'O_2' = B'O_2' = R_2'$ the new radius, with which describe th arc $P'B'$.

By Geom. Tab. I. 18:

$PBV = \frac{1}{2}\,PO_2B = \frac{1}{2}\Delta_2$, and $GB'V' = \frac{1}{2}P'O_2'B' = \frac{1}{2}\Delta_2'$.

$$\therefore \qquad PGP' = BGB' = \tfrac{1}{2}(\Delta_2 - \Delta_2')$$

Draw O_1K perpendicular to BO_2.

Then $O_1K = B'H = BI = O_1O_2 \sin \Delta_2 = (R_2 - R_1) \sin \Delta_2$

$$\tan \tfrac{1}{2}\Delta_2 = \frac{GI}{BI} \qquad \tan \tfrac{1}{2}\Delta_2' = \frac{GH}{B'H} = \frac{GI - p}{B'H}$$

$$\therefore \qquad \tan \tfrac{1}{2}\Delta_2' = \tan \tfrac{1}{2}\Delta_2 - \frac{p}{(R_2 - R_1)\sin \Delta_2} \qquad (121)$$

In the triangle $O_1O_2O_2'$

$$\sin \Delta_2' : \sin \Delta_2 :: O_1O_2 : O_1O_2' :: (R_2 - R_1) : (R_2' - R_1)$$

$$R_2' - R_1 = \frac{\sin \Delta_2}{\sin \Delta_2'}\,(R_2 - R_1)$$

and

$$R_2' = (R_2 - R_1)\frac{\sin \Delta_2}{\sin \Delta_2'} + R_1 \qquad (122)$$

If $B'V'$ were **outside** *of VB;*

$$\tan \tfrac{1}{2}\Delta_2' = \tan \tfrac{1}{2}\Delta_2 + \frac{p}{(R_2 - R_1)\sin \Delta_2} \qquad (123)$$

$$R_2' = (R_2 - R_1)\frac{\sin \Delta_2}{\sin \Delta_2'} + R_1 \qquad (122)$$

When the **smaller radius** *R_1 follows R_2: If the given tangent $B'V'$ is* **inside** *of BV.* Fig. 49.

$$\tan \tfrac{1}{2}\Delta_1' = \tan \tfrac{1}{2}\Delta_1 + \frac{p}{(R_2 - R_1)\sin \Delta_1} \qquad (124)$$

$$R_1' = R_2 - (R_2 - R_1)\,\frac{\sin \Delta_1}{\sin \Delta_1'} \qquad (125)$$

If $B'V'$ is **outside** *of BV:*

$$\tan \tfrac{1}{2}\Delta_1' = \tan \tfrac{1}{2}\Delta_1 - \frac{p}{(R_2 - R_1)\sin \Delta_1} \qquad (126)$$

$$R_1' = R_2 - (R_2 - R_1)\frac{\sin \Delta_1}{\sin \Delta_1'} \qquad (125)$$

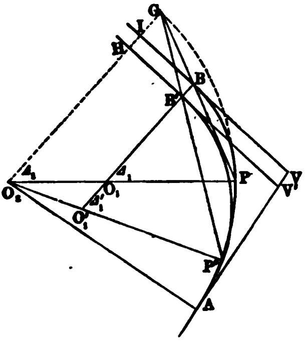

FIG. 49.

Example 1.—*Fig.* 48.

Let $R_2 = 2292.01$ $p = 20.07$ *inside.*
" $R_1 = 1432.69$ $\Delta_2 = 28°$

(121) $R_2 - R_1 =$	859.32		log 2.934155
Δ_2		28°	log sin 9.671609
			2.605764
p	20.07		1.302547
	.04975		8.696783
$\tan \frac{1}{2}\Delta_2$	.24933		
$\therefore \tan \frac{1}{2}\Delta_2'$	.19958	11° 17′	
(122) Δ_2'		22° 34′	sin 9.584058
$(R_2 - R_1)$			2.934155
			3.350097
Δ_2		28°	sin 9.671609
$(R_2' - R_1)$	1051.25		3.021706
R_1	1432.69		
Ans. R_2'	2483.94	$\therefore D = 2° 18' 25''$	

$PO_1P = 28° - 22° 34' = 5° 26'$ $\therefore$ PP′ $= 135.83$ ft.

Example 2.—*Fig.* 49.

Let $R_2 = 2292.01$ $p = 20.07$ inside.
$R_1 = 1432.69$ $\Delta_1 = 46°$

(124) $R_2 - R_1$	859.32		log 2.934155
Δ_1		46°	log sin 9.856934
			2.791089
p	20.07		1.302547
	.03247		8.511458
tan $\frac{1}{2}\Delta_1$	42447	23°	
∴ tan $\frac{1}{2}\Delta_1'$	.45694	24° 33½'	
∴ Δ_1'		49° 07'	log sin 9.878547
$R_2 - R_1$			2.934155
			3.055608
Δ_1		46°	log sin 9.856934
	817.60		2.912542
R_2	2292.01		

Ans. $R_1' = 1474.41$ ∴ $D = 3° \, 53' \, 12''$

$$PO_2P' = \Delta_1' - \Delta_1 = 3° \, 07' \; \therefore \text{arc } PP' = \frac{3°.1166}{2.5} = 124.67 \text{ ft.}$$

Observe that in either figure both tangents must be on the same side of the point G, in order to a solution.

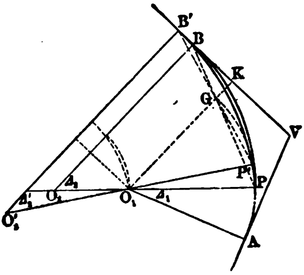

FIG. 50.

171. *Given: a compound curve ending in a tangent, to change the* **last radius** *and also the position of the* P.C.C., *so that the curve may end in the* **same tangent.** Fig. 50.

I. *When the curve ends with the* **greater** *radius* R_2.

Let APB be the compound curve in which R_1 R_2 Δ_1 and Δ_2 are known.

In the diagram draw the chord PB and produce the first arc AP to meet it in G; draw O_1G, and produce it to meet the tangent in K. Then by § 137 O_1K is parallel to O_2B, and by eq. (57)

$$GK = (R_2 - R_1) \text{ vers } \Delta_2 \qquad (127)$$

If we **assume P**′ *as the new P.C.C.*, we have $\Delta_2' = P'O_2'B'$, and the chord $P'G$ produced will intersect the tangent at the new point of tangent B', and $BO_2' = R_2'$. Similar to eq. (127) we have

$$GK = (R_2' - R_1) \text{ vers } \Delta_2'$$

and equating the two expressions, we obtain

$$R_2' = R_1 + \frac{(R_2 - R_1) \text{ vers } \Delta_2}{\text{vers } \Delta_2'} = R_1 + \frac{GK}{\text{vers } \Delta_2'} \qquad (128)$$

If we **assume** $\mathbf{R_2'}$, *we have*

$$\text{vers } \Delta_2' = \frac{R_2 - R_1}{R_2' - R_1} \text{ vers } \Delta_2 = \frac{GK}{R_2' - R_1} \qquad (129)$$

In the two right-angled triangles BKG and $B'KG$, we have

$$BK = GK \cot \tfrac{1}{2}\Delta_2$$
$$B'K = GK \cot \tfrac{1}{2}\Delta_2'$$

and by subtraction,

$$BB' = GK (\cot \tfrac{1}{2}\Delta_2' - \cot \tfrac{1}{2}\Delta_2) \qquad (130)$$

in which GK is obtained from eq. (127).

When BB' as given by eq. (130) is *negative*, the point B' falls between B and V.

If we **assume** *the distance* **BB**′ *on the tangent*, we have from the last equation,

$$\cot \tfrac{1}{2}\Delta_2' = \cot \tfrac{1}{2}\Delta_2 \pm \frac{BB'}{GK} \qquad (131)$$

GK being obtained from eq. (127) and R_2' from eq. (128). In eq. (131) use the + sign when B' is beyond B as in the Fig. 50.

II. *When the given curve ends with the* **smaller radius** R_1. Fig. 51.

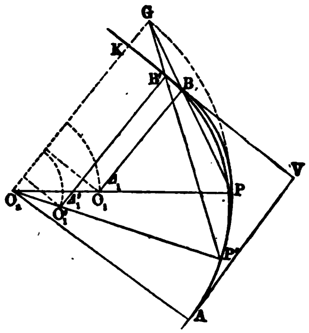

FIG. 51.

We have by a similar reasoning

$$GK = (R_2 - R_1) \text{ vers } \Delta_1 \tag{132}$$

$$R_1' = R_2 - \frac{(R_2 - R_1) \text{ vers } \Delta_1}{\text{vers } \Delta_1'} = R_2 - \frac{GK}{\text{vers } \Delta_1'} \tag{133}$$

$$\text{vers } \Delta_1' = \frac{R_2 - R_1}{R_2 - R_1'} \text{ vers } \Delta_1 = \frac{GK}{R_2 - R_1'} \tag{134}$$

$$BB' = GK (\cot \tfrac{1}{2} \Delta_1 - \cot \tfrac{1}{2} \Delta_1') \tag{135}$$

$$\cot \tfrac{1}{2} \Delta_1' = \cot \tfrac{1}{2} \Delta_1 \pm \frac{BB'}{GK} \tag{136}$$

using the − sign when B' is beyond B.

Example.—Fig. 51.

Let $R_2 = 2292.01$, $R_1 = 1432.69$, $\Delta_1 = 46°$, and let the *P.C.C. be* moved back 200 feet from P to P'; hence $PO_1'P' = 5°$ *and* $\Delta_1' = 51°$; to find the new radius R_1' and the distanc *BB'*.

Eq. (132) $R_2 - R_1$	859.32		log	2.934155
Δ_1		46°	" vers	9.484786
$\therefore GK$			log	2.418941
eq. (133) Δ_1'		51°	" vers	9.568999
$R_2 - R_1'$	707.85			2.849942
R_2	2292.01			
$\therefore R_1'$	1584.16	and $D = 3° 37'$		
eq. (135) GK			log	2.418941
$\cot \frac{1}{2}\Delta_1$	2.35585	23°		
$\cot \frac{1}{2}\Delta_1'$	2.09654	25° 30′		
	0.25931		log	9.413819
$\therefore BB'$	68.04			1.832760

172. *Given: a compound curve ending in a tangent, the last radius being the* **greater, to change** *the* **last radius** *and also the position of the* **P.C.C.** *so that the curve may end at the* **same tangent point,** *but with a given difference in the* **direction** *of the tangent.* Fig. 52.

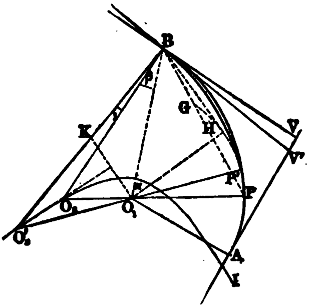

FIG. 52.

Let APB be the given compound curve, $PO_1 = R_1$ and $PO_2 = R_2 > R_1$.

Let $V'B$ be the new tangent, and the angle $VBV' = i$, the given difference in direction: to find $BO_2' = R_2'$, $BO_2'P' = \Delta_2'$ *and the angle* PO_1P'.

We have

$$BO_2 - O_1O_2 = R_2 - (R_2 - R_1) = R_1$$

$$BO_2' - O_1O_2' = R_2' - (R_2' - R_1) = R_1$$

From which we see that whatever may be the value of the new radius, the *difference* of the distances from B and O_1 to the new centre is *constant*, and equal to R_1. We therefore conclude that the centres O_2 and O_2' are on an *hyperbola* of which B and O_1 are the foci, and R_1 the major axis.

This suggests an easy **graphical method** of solving the problem.

Through B draw a line perpendicular to the new tangent $V'B$ which will give the direction of the required centre O_2'. On this line lay off BK equal to R_1, and since $(R_2' - R_1) = O_1O_2' = KO_2'$, if we join KO_1, the triangle $KO_2'O_1$ is isosceles; therefore bisect KO_1 and erect a perpendicular from the middle point to intersect the line BK produced in O_2'. Draw $O_2'O_1$ and produce it to intersect the arc AP (produced if necessary) in P'. Then P' is the new $P.C.C.$ required, and $BO_2' = P'O_2' = R_2'$, the new radius.

The analytical solution is as follows:
Adopting the usual notation of the hyperbola

Let $2a = R_1$ = the major axis,
" $2c = BO_1$ = the distance between foci.

Produce the arc AP and through B draw the tangent BH, and join $HO_1 = R_1$. Then in the right-angled triangle BHO_1

$$BH^2 = BO_1^2 - R_1^2 = 4c^2 - 4a^2$$

Now by Anal. Geom., $c^2 - a^2 = b^2$.
Therefore $2b = BH$ = the minor axis.
Draw the chord PB and produce the arc AP to cut it in G. Then by Geom. (Table I. 24)

$$BH^2 = PB \times GB = 2R_2 \sin \tfrac{1}{2}\Delta_2 \times 2(R_2 - R_1) \sin \tfrac{1}{2}\Delta_2$$

$$\therefore BH = 2 \sin \tfrac{1}{2}\Delta_2 \sqrt{R_2 (R_2 - R_1)} \qquad (137)$$

Let α = the angle HO_1B, then

$$\tan \alpha = \frac{BH}{R_1} \text{ and } BO_1 = \frac{R_1}{\cos \alpha} \tag{138}$$

In the triangle BO_1O_2 let $O_1BO_2 = \beta$; then

$$\sin \beta = \frac{R_2 - R_1}{BO_1} \sin \Delta_2 \tag{139}$$

The polar equation of the hyperbola for the branch IO_2O_2', taking the pole at B and estimating the variable angle v from the line BO_1, is

$$r = \frac{b^2}{c \cdot \cos v - a}$$

When $v = \beta \pm i$, $r = R_2'$, and substituting the values of a, b, and c found above, we have

$$R_2' = \frac{BH^2}{2\,(BO_1 \cos (\beta \pm i) - R_1)} \tag{140}$$

using $(\beta + i)$ when V' falls between V and A, as in the figure, and $(\beta - i)$ when V' falls beyond V.

In the triangle BO_1O_2', the angle $BO_2'O_1 = \Delta_2'$ and

$$\sin \Delta_2' = \frac{BO_1}{R_2' - R_1} \sin (\beta \pm i) \tag{141}$$

Finally

$$PO_1P' = \Delta_2 - (\Delta_2' \pm i) \tag{142}$$

Remark.—When V' falls between V and A, as in Fig. 52, if the angle i be greater than the angle VBH, the curve ceases to be a compound, and becomes reversed. Therefore $VBH = \alpha - \beta$ is the maximum value of i possible in this case. When V' *falls beyond* V, *the* point P' will fall between P and A; *and the largest* possible value of i will then be that which *renders* $PO_1P' = \Delta_1$, and makes the point P' coincide with A.

Example.—Fig. 52. Let $R_1 = 1432.69$ $\Delta_1 = 31°$
$i = 6°$ $R_2 = 2292.01$ $\Delta_2 = 56°$

(137)	$R_2 - R_1$	859.32		log	2.
	R_2	2292.01			3.
				2)	6.
					3.1
	$\frac{1}{2}\Delta_2$		28°	log sin	9.
	2				0.
∴	BH				3.1
(138)	R_1	1432.69			3.1
∴	α		42° 36′ 23″.7	log tan	9.
	α		42° 36′ 23″.7	log cos	9.
∴	BO_1				3.2
(139)	$R_2 - R_1$				2.
					9.
	Δ_2		56°	log sin	9.
∴	β		21° 28′ 06″.3	log sin	9.5
(140)	$\beta + i$		27° 28′ 06″.3	log cos	9.
	BO_1				3.
		1727.09			3.
	R_1	1432.69			
		294.40 × 2 = 588.80			2.7
	BH^2				6.
∴	R_2'	2949.05			3.4

(141) ∴ $\Delta_2' = 36° 18' 26''$

(142) ∴ $PO_1P' = 13° 41' 34'' = 342.3$ feet.

Remark—This problem may also be solved by first fi the new sides $V'A$, $V'B$, from which and the new c angle ($\Delta \pm i$), and the radius R_1, may be found Δ_1', Δ_2 R_2', as in § 162. The new sides are readily found fro old ones by solving the triangle VBV'. If the original are not given, they must be calculated as in § 164.

173. *Given: a compound curve ending in a tange last radius being the* **less,** *to change the* **last radius** *a position of the* **P.C.C.** *so that the curve may end at the* **tangent point,** *but with a* **given difference direction** *of tangent.* Fig. 53.

Let APB be the given curve, and $PO_2 = R_2$, and $PO_1 = R_1 < R_2$. Let $V'B$ be the new tangent, and $VBV' = i$, the given angle; to find $BO_1' = R_1'$, $BO_1'P' = \Delta_1'$, and PO_2P'.

We have

$$BO_1 + O_1O_2 = R_1 + (R_2 - R_1) = R_2$$

$$BO_1' + O_1'O_2 = R_1' + (R_2 - R_1') = R_2$$

from which we infer that the locus of the centre O_1' is an *ellipse*, of which B and O_2 are the foci, and R_2 the major axis,

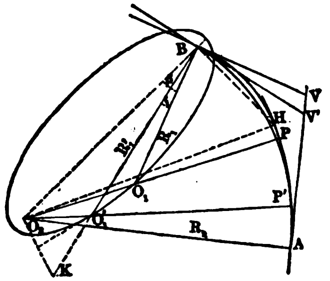

FIG. 53.

since the *sum* of the distances BO_1' and O_2O_1' is always equal to R_2.

This suggests an easy **graphical solution** of the problem, as follows:

Perpendicular to $V'B$ draw the indefinite line BK, which will contain the required centre O_1', and lay off $BK = R_2$. Join KO_2, bisect it, and from the middle point erect a perpendicular to intersect BK in O_1'. Join O_2O_1', and produce the line to intersect the arc AP (produced if necessary) in P', which is the new $P.C.C.$ required. $P'O_1' = BO_1' = R_1'$, the required radius, and $P'O_1'B = \Delta_1'$.

The analytical solution is as follows: Adopting the usual notation of the ellipse,

let $2a = R_2$ = the major axis,

" $2c = BO_2$ = the distance between foci.

At B erect BH perpendicular to BO_2 to intersect the arc A (produced if necessary) in H, and join $HO_2 = R_2$. Then

$$BH^2 = R_2^2 - BO_2^2 = 4a^2 - 4c^2$$

But by Anal. Geom., $a^2 - c^2 = b^2$.

Hence $2b = BH =$ the minor axis.

In the triangle BO_1O_2 we know $BO_1 = R_1$, and $O_1O_2 = R_2 - R_1$, and the included angle $BO_1O_2 = 180^\circ - \Delta_1$; hence by Trig. (Tab. II. 25)

$$\tan \tfrac{1}{2}(O_1O_2B - O_1BO_2) = \frac{2R_1 - R_2}{R_2} \tan \tfrac{1}{2}\Delta_1 \qquad (143)$$

The angles at B and O_2 are then found by (Tab. II. 26). Let $\beta =$ the angle O_1BO_2; then

$$BO_2 = (R_2 - R_1)\frac{\sin \Delta_1}{\sin \beta} \qquad (144)$$

The value of BH^2 above may be written

$$BH^2 = (R_2 + BO_2)(R_2 - BO_2) \qquad (145)$$

The polar equation of the ellipse, taking the pole at B, and estimating the variable angle v from the axis BO_2, is

$$r = \frac{b^2}{a - c \, . \cos v}$$

When $v = \beta \mp i$, then $r = R_1'$, and substituting the values of a, b, and c, given above, we have

$$R_1' = \frac{BH^2}{2(R_2 - BO_2 \cos(\beta \mp i))} \qquad (146)$$

using $(\beta - i)$ when V' falls between V and A, as in Fig. 53, and $(\beta + i)$ when V' falls beyond V.

In the triangle $BO_1'O_2$, the angle $O_1'BO_2 = (\beta \mp i)$, and the exterior angle $BO_1'P' = \Delta_1'$; hence

$$\sin \Delta_1' = \frac{BO_2}{R_2 - R_1'} \sin(\beta \mp i) \qquad (147)$$

Finally $$PO_2P' = (\Delta_1 \mp i) - \Delta_1' \qquad (148)$$

When V' is on AV, then PO_2P' is negative, showing that it must be laid off from P toward A; but when V' is beyon

V, then PO_2P' is positive, and P' will be on AP produced. The only limits imposed on the angle i are that the resulting value of PP' shall not exceed PA, and that R_1' shall not be less than a practical minimum.

Example.—Fig. 53.

Let $D_2 = 3° 20'$ $R_2 = 1719.12$ $\Delta_2 = 23° 20'$
$D_1 = 6°$ $R_1 = 955.37$ $\Delta_1 = 48°$ $i = 7° 45'$

The resulting values are as follows:

β		21° 09′ 32″.6	
BO_2	1572.42		3.196567
BH^2			5.683829
R_1'	1273.65		3.105052
Δ_1'		54° 56′	
PO_2P'		14° 41′	
PP'	440.5		

(See also remark at end of § 172.)

174. *Given a* **simple curve** *joining two tangents, to replace it by a* **three-centred compound** *curve between the* **same tangent points.** Fig. 54.

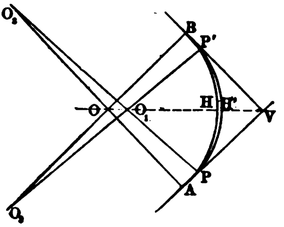

FIG. 54.

Let $R = AO$ = radius of simple curve.

$$R_1 = PO_1 = P'O_1 < R \quad \Delta_1 = PO_1P'$$

$$R_2 = AO_2 = BO_3 > R \quad \Delta_2 = AO_2P = BO_3P'$$

$$\Delta = AOB$$

nce AO_2 is made equal to BO_3 and $VA = VB$, AO_2P *must* ual BO_3P', and the compound curve will be symmetrical out the bisecting line VO; and the centre O_1 will be on the ne VO.

We have at once from the figure,

$$2\Delta_2 + \Delta_1 = \Delta \tag{149}$$

In the triangle OO_1O_2 we have

$$O_1O_2 : OO_2 :: \sin AOV : \sin PO_1V$$

whence

$$R_2 - R_1 = \frac{(R_2 - R)\sin \tfrac{1}{2}\Delta}{\sin \tfrac{1}{2}\Delta_1} \qquad (150)$$

which expresses the general relation between the quantities, R and Δ being given.

We may now assume values for R_1 *and* R_2 subject to the above conditions, viz., $R_1 < R$ and $R_2 > R$; whence

$$\sin \tfrac{1}{2}\Delta_1 = \frac{(R_2 - R)\sin \tfrac{1}{2}\Delta}{R_2 - R_1} \qquad (151)$$

In selecting values for R_1 and R_2, the degree of curve D_1 should be but little greater than D of the simple curve, say from 30 to 60 minutes, while D_2 may be taken at $\frac{1}{2}D$ to $\frac{1}{4}D$.

Example.—Given: $R = 1719.12$ $D = 3° 20'$ $\Delta = 40°$

Let $R_1 = 1432.69$ $D_1 = 4°$
" $R_2 = 5729.65$ $D_2 = 1°$

∴	$R_2 - R$	4010.53		log 3.603202
	$R_2 - R_1$	4296.96		" 3.633161
				" 9.970041
	$\frac{1}{2}\Delta$		20°	log sin 9.534052
∴	$\frac{1}{2}\Delta_1$		18° 36′ 57″	" " 9.504093
	Δ_1		37° 13′ 54″	
∴	Δ_2		1° 23′ 03″	
	$AP = P'B$	138.4 ft.		

Again we may assume Δ_2 *and* R_1, whence

$$\Delta_1 = \Delta - 2\Delta_2$$

and

$$R_2 = \frac{R\sin \tfrac{1}{2}\Delta - R_1 \sin \tfrac{1}{2}\Delta_1}{\sin \tfrac{1}{2}\Delta - \sin \tfrac{1}{2}\Delta_1} \qquad (152)$$

Example.—Given: $R = 1719.12$ $\Delta = 40°$
Let $R_1 = 1432.69$ $\Delta_2 = 1°$ ∴ $\Delta_1 = 38°$
Ans. $R_2 = 7387.24$ ∴ $D_2 = 0° 46\tfrac{1}{2}'$ $AP = 129.$

Finally we may assume Δ_2 *and* R_2, and deduce Δ_1 and R_1 *from eqs. (149) (150)*; but this is the least desirable because the value of R_1 so found will not usually give a convenient value to the degree of curve D_1.

175. *To determine the* **distance HH'** *between the middle points of a simple curve and a three-centred compound curve joining the* **same tangent points** *AB.* Fig. 54.

In the triangle OO_1O_2, we have

$$OO_1 = (R_2 - R_1)\frac{\sin \Delta_2}{\sin \frac{1}{2}\Delta}$$

$$HH' = OO_1 + O_1H' - OH$$

$$\therefore HH' = (R_2 - R_1)\frac{\sin \Delta_2}{\sin \frac{1}{2}\Delta} - (R - R_1) \qquad (153)$$

In the first example given above $HH' = 14.55$, and in the second $HH' = 17.05$ ft.

In many instances the distance HH' is so great as to render this problem practically useless, unless the distance HH_1 is discounted beforehand by putting the simple curve AHB a sufficient distance inside of the proper location through the point H'. But the problem given below is usually preferable.

176. *Given, a* **simple curve** *joining two tangents to replace it by a* **three-centred compound** *curve which shall pass through the* **same middle point H.**

I. *The curve* **flattened at the tangents.** Fig. 55.

Let $R = AO$, the radius, and Δ = the central angle of the simple curve AHB, and let H be the middle point.

Let $R_1 = PO_1 = HO_1$ $\qquad \Delta_1 = PO_1P'$

" $R_2 = PO_2 = A'O_2 = B'O_2$ $\qquad \Delta_2 = PO_2A' = P'O_2B'$

" A' and B' be the new tangent points required.

We have at once, as in the last problem,

$$2\Delta_2 + \Delta_1 = \Delta. \qquad (154)$$

Since the curve is to be symmetrical about VO, $HP = HP$ $PA = P'B$, and $AA' = BB'$.

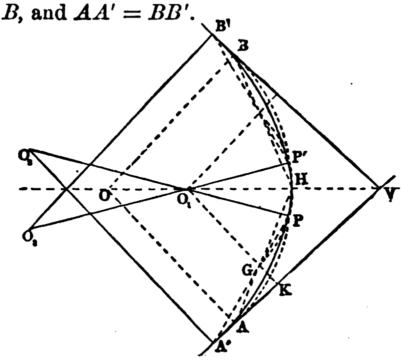

FIG. 55.

In the diagram produce the arc HP to G, and draw O_1G parallel to OA, and produce it to K. Then a tangent line at G will be parallel to VA; and by § 137 the point G will be on the long chord HA, and on the long chord PA'. GK is the perpendicular distance between parallel tangents, and the problem is similar to that given in § 171; whence by eq. (57) we have, in this case,

$$GK = (R_2 - R_1) \text{ vers } \Delta_2 = (R - R_1) \text{ vers } \tfrac{1}{2}\Delta. \qquad (155)$$

for the general equation in which R and Δ are given.

Analagous to eq. (130) we have

$$AA' = KA' - KA = GK \cot GA'K - GK \cot GAK.$$

$$\therefore AA' = GK (\cot \tfrac{1}{2}\Delta_2 - \cot \tfrac{1}{2}\Delta) \qquad (156)$$

in which GK is obtained from (155).

We may now assume values for R_1 and R_2, making $R_1 < R$ and $R_2 > R$, and deduce the values of Δ_2, Δ_1, and AA'.

Solving eq. (155)

$$\text{vers } \Delta_2 = \frac{(R - R_1) \text{ vers } \frac{1}{2}\Delta}{R_2 - R_1} = \frac{GK}{R_2 - R_1} \qquad (157)$$

Eq. (154) gives Δ_1, and eq. (156) gives AA'.

Example.—Fig. 55.

Given: $R =$	764.489	$D = 7° 30'$	$\Delta = 40°$
Let $R_1 =$	716.779	$D_1 = 8°$	
" $R_2 =$	3437.870	$D_2 = 1° 40'$	
(155) $R - R_1$	47.71		log 1.678609
$\frac{1}{2}\Delta$		20°	log vers 8.780370
∴ GK			log 0.458979
$R_2 - R_1$	2721.091		" 3.434743
∴ Δ_2		(say) 2° 38'	log vers 7.024236
∴ $A'P$	158.00	$\Delta_1 = 34° 44'$	
(156) $\frac{1}{2}\Delta_2$	43.5081	= cot 1° 19'	
$\frac{1}{2}\Delta$	5.6713	cot 10°	
	37.8368		log 1.577914
GK			" 0.458979
∴ AA'	108.87		" 2.036893

Again, we may assume Δ_2 and $R_1 < R$; whence

$$\Delta_1 = \Delta - 2\Delta_2$$

and

eq. (155) $GK = (R - R_1) \text{ vers } \frac{1}{2}\Delta$

and

$$R_2 = R_1 + \frac{GK}{\text{vers } \Delta_2} \qquad (158)$$

Eq. (156) gives AA'.

Again, we may assume Δ_2 *and the distance* AA'; whence, from eq. (156)

$$GK = \frac{AA'}{\cot \frac{1}{2}\Delta_2 - \cot \frac{1}{2}\Delta} \qquad (159)$$

eq. (155) $R_1 = R - \dfrac{GK}{\text{vers } \frac{1}{2}\Delta}$

eq. (158) gives R_2.

Again, we may assume $R_1 < R$ *and* AA'; then, eq. (155)

$$GK = (R - R_1) \text{ vers } \frac{1}{2}\Delta$$

and eq. (156)

$$\cot \tfrac{1}{2}\Delta_2 = \cot \tfrac{1}{2}\Delta + \frac{AA'}{GK} \qquad (160)$$

d eq. (158) gives R_2.

Example.—Fig 55.

Given: $R =$ 764.489 $D = 7° 30'$ $\Delta = 40°$
Let $R_1 =$ 716.779 $D_1 = 8°$
" $AA' =$ 110.

Hence by last example,

	GK				log 0.458979
eq. (160)	AA'	110.			2.041393
		38.2309			1.582414
	cot $\frac{1}{4}\Delta$	5.6713		10°	
∴	cot $\frac{1}{2}\Delta_2$	43.9022		1° 18′ 18″	log 1.642486
(158)	Δ_2		(say)	2° 37′	log vers 7.018147
	GK				0.458979
	$R_2 - R_1$	2759.5			3.440832
∴	R_2	3476.3	$D_2 =$	1° 39′	
∴	AP'	157.	$\Delta_1 =$	34° 46′	

II. *The curve* **sharpened at the tangents.** Fig. 56.

This case will only occur when, with a given external distance VH, a simple curve would absorb too much of the tangents.

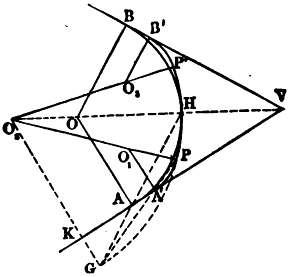

Fig. 56.

Let AHB be the simple curve, and
" $A'PHP'B'$ the required compound curve.
" $R_2 = PO_2 = HO_2$; $\Delta_2 = PO_2P'$
" $R_1 = PO_1 = A'O_1 = B'O_3$; $\Delta_1 = A'O_1P = P'O_3B'$.

We have from the figure,

$$2\Delta_1 + \Delta_2 = \Delta. \qquad (161)$$

In the diagram draw O_2G parallel to OA cutting the tangent at K, and produce the arc HP to G. Draw the chords GH and GP, passing through A and A' respectively. We have then a discussion similar to the preceding case, and to the problem § 171, Fig. 51, whence we derive the general formulæ:

$$GK = (R_2 - R_1) \text{ vers } \Delta_1 = (R_2 - R) \text{ vers } \tfrac{1}{2}\Delta \qquad (162)$$

and

$$AA' = GK (\cot \tfrac{1}{2}\Delta_1 - \cot \tfrac{1}{2}\Delta) \qquad (163)$$

1. Assuming $R_1 < R$ and $R_2 > R$

$$\text{vers } \Delta_1 = \frac{GK}{R_2 - R_1} = \frac{R_2 - R}{R_2 - R_1} \text{ vers } \tfrac{1}{2}\Delta \qquad (164)$$

2. Assuming $\Delta_1 < \frac{1}{2}\Delta$ and $R_1 < R$

$$R_2 = \frac{R \text{ vers } \frac{1}{2}\Delta - R_1 \text{ vers } \Delta_1}{\text{vers } \frac{1}{2}\Delta - \text{vers } \Delta_1} \qquad (165)$$

3. Assuming $\Delta_1 < \frac{1}{2}\Delta$ and AA'

$$GK = \frac{AA'}{\cot \frac{1}{2}\Delta_1 - \cot \frac{1}{2}\Delta} \qquad (166)$$

$$R_2 = R_1 + \frac{GK}{\text{vers } \frac{1}{2}\Delta} \qquad (167)$$

$$R_1 = R_2 - \frac{GK}{\text{vers } \Delta_1} \qquad (168)$$

4. Assuming $R_2 > R$ and AA'

$$GK = (R_2 - R) \text{ vers } \tfrac{1}{2}\Delta$$

$$\cot \tfrac{1}{2}\Delta_1 = \cot \tfrac{1}{2}\Delta + \frac{AA'}{GK} \qquad (169)$$

The third assumption will usually secure most readily the desired curve. AA' should be assumed as small as the nature of the case will allow, and Δ_1 should not be much smaller than $\frac{1}{2}\Delta$.

It is evidently not *necessary* that the new curve should be symmetrical; for having laid out the curve $A'PH$, the simple *curve HB may then be* used, or, if desirable, some compound *urve $HP'B'$ determined* by an assumed value of BB' not *'ual to AA'.*

These formulæ (154) to (169) are readily adapted to the case of substituting a compound for a simple curve when it is necessary to keep one tangent point fixed, but to move the other a certain distance in either direction on the tangent. For if in Figs. 55, 56, we draw a tangent at H, and make H the fixed point of tangent, it is evident that the central angle of the curve will then be AOH. The only change necessary, therefore, to adopt the formulæ to this case is to write Δ in place of $\frac{1}{2}\Delta$, and to observe, instead of eqs. (154) (161), that

$$\Delta_1 + \Delta_2 = \Delta.$$

Example.—Fig. 55.

Let	$R =$	1910.08	$\Delta = 84°$		
Assume	$AA' =$	260.	$\Delta_1 = 38°$	$\therefore \Delta_2 = 8°$	
Eq. (166)	$AA' =$	260.		log	2.414973
	$\cot \frac{1}{2}\Delta_1$	2.90421	19°		
	$\cot \frac{1}{4}\Delta$	2.60509	21°		
		.29912		log	9.475846
$\therefore$	GK			"	2.939127
Eq. (167)	$\frac{1}{2}\Delta$		42°	" vers	9.409688
		3384.07			3.529439
	R	1910.08			
$\therefore$	R_2	5294.15		$D =$ say 1° 05′	
Eq. (168)	GK			log	2.939127
	Δ_1		38°	" vers	9.326314
		4100.27			3.612813
$\therefore$	R_1	1193.88		$D = 4° 48'$	
	$A'P$	791.67		$PH = 369.23$	

177. *Given,* **two curves** *joined by a common tangent* **to replace the tangent by a curve** *compounded with the given curves.* Fig. 57.

Let $R_1 = BO_1$ the radius of one curve,
" $R_3 = AO_3$ the radius of the other curve, $> R_1$,
" $l = BA$ the common tangent,
" $R_2 = PO_2 = P'O_2$ the radius of connecting curve.
" $\Delta_2 = PO_2P'$ the central angle of " "
" $\alpha = AO_3P'$ and $\beta = BO_1P$.
" $i = AO_3O_1$.

In the diagram join O_1O_3 and draw O_1G parallel to BA. Then in the right-angled triangle O_1GO_3 we have,

$$\cot i = \frac{GO_3}{GO_1} = \frac{R_3 - R_1}{l} \qquad (170)$$

$$O_1O_3 = \frac{R_3 - R_1}{\cos i} = \frac{l}{\sin i} \qquad (171)$$

which gives the distance between the centres of the given curves.

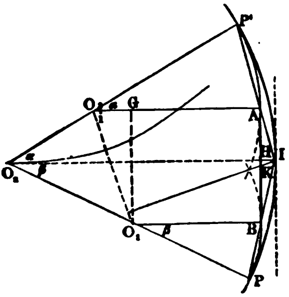

FIG. 57.

We shall now assume the following geometrical truths, which may be easily demonstrated.

If two circles intersect in one point, they intersect in two points; and the line joining the two points is the common chord.

The common chord is perpendicular to the line joining the centres, and when produced it bisects the common tangents.

If a third circle is drawn touching the two circles, a tangent to the third circle, parallel to the common tangent, will have its tangent point on the common chord produced.

Conversely, therefore, if the tangent BA be bisected at K, and a line, KI, drawn perpendicular to O_1O_3, KI will coincide with the common chord produced, and the angle $IKA = AO_3O_1 = i$. If on KI we assume a point I through which it is desirable that the connecting curve should pass, then I is the tangent point of a tangent parallel to BA; consequently a line through I perpendicular to BA contains the required centre O_2.

I. Let $p = HI =$ the perpendicular distance between the tangents.

If in the diagram we join IA and IB, and produce the chords to intersect the given curves in P and P', then P and P' are the points of compound curvature; and the lines PO_1 and $P'O_3$ produced will intersect IO_2 in the same point O_2; and the angles $P'O_2I = \alpha$ and $PO_2I = \beta$.

In the triangle AIB the line KI bisects the base AB, and we have by Geom. Tab. I. 25.

$$AI^2 + BI^2 = 2AK^2 + 2KI^2$$

By eq. (56)
$$AI = 2(R_2 - R_3) \sin \tfrac{1}{2}\alpha$$

$$BI = 2(R_2 - R_1) \sin \tfrac{1}{2}\beta$$

$$AK = \tfrac{1}{2}l \quad \text{and} \quad KI = \frac{p}{\sin i}$$

$$\therefore\ 4(R_2 - R_3)^2 \sin^2 \tfrac{1}{2}\alpha + 4(R_2 - R_1)^2 \sin^2 \tfrac{1}{2}\beta = \tfrac{1}{2}l^2 + \frac{2p^2}{\sin^2 i}$$

Dividing by 2 and putting vers $\alpha = 2 \sin^2 \frac{1}{2}\alpha$ and vers $\beta = 2 \sin^2 \frac{1}{2}\beta$ (Tab. II. 46)

$$(R_2 - R_3)^2 \operatorname{vers} \alpha + (R_2 - R_1)^2 \operatorname{vers} \beta = \tfrac{1}{4}l^2 + \frac{p^2}{\sin^2 i}$$

But by eq (57)

$$(R_2 - R_3) \operatorname{vers} \alpha = (R_2 - R_1) \operatorname{vers} \beta = p \qquad (172)$$

$$\therefore\ p\,(2R_2 - (R_3 + R_1)) = \tfrac{1}{4}l^2 + \frac{p^2}{\sin^2 i}$$

$$2R_2 = (R_3 + R_1) + \frac{l^2}{4p} + \frac{p}{\sin^2 i} \qquad (173)$$

From (172)

$$\operatorname{vers} \alpha = \frac{p}{R_2 - R_3}; \ \operatorname{vers} \beta = \frac{p}{R_2 - R_1} \qquad (174)$$

and from the figure

$$\Delta_3 = \alpha + \beta \qquad (175)$$

These formulæ solve the problem when p is assumed. If *desirable* we may find α and β independently of R_2, for he triangle AIB, $IAB = \frac{1}{2}\alpha$ and $IBA = \frac{1}{2}\beta$; and since $HK = p \cot i$,

$$\cot \tfrac{1}{2}\alpha = \frac{AH}{HI} = \frac{\frac{1}{2}l - HK}{p} = \frac{l}{2p} - \cot i \qquad (176)$$

$$\cot \tfrac{1}{2}\beta = \frac{BH}{HI} = \frac{\frac{1}{2}l + HK}{p} = \frac{l}{2p} + \cot i \qquad (177)$$

II. *In case α or β is assumed*, we have from the last equation

$$p = \frac{l}{2(\cot \frac{1}{2}\alpha + \cot i)} = \frac{l}{2(\cot \frac{1}{2}\beta - \cot i)} \qquad (178)$$

III. *In case the radius R_2 is assumed*, then in the triangle $O_1O_2O_3$ we know all three sides; for $O_1O_2 = (R_2 - R_1)$, $O_2O_3 = (R_2 - R_1)$, and $O_1O_3 = \dfrac{R_3 - R_1}{\cos i}$

By Trig. (Table II. 31.)

$$\text{vers } \Delta_2 = \frac{2(s - O_1O_2)\,(s - O_2O_3)}{O_1O_2 \times O_2O_3}$$

in which $s = \frac{1}{2}$ sum of the three sides.

Substituting values, and reducing, observing that,

$$\left(\frac{1}{\cos i} - 1\right)\left(\frac{1}{\cos i} + 1\right) = \sec^2 i - 1 = \tan^2 i$$

nd that $(R_3 - R_1) \tan i = l$, we have

$$\text{vers } \Delta_2 = \frac{l^2}{2(R_2 - R_1)\,(R_2 - R_3)} \qquad (179)$$

n the same triangle.

$$\sin O_3O_1O_2 = \sin \Delta_2 \frac{O_3O_2}{O_1O_3}$$

t *from the figure* $O_3O_1O_2 = i - \beta$, and taking the value $_1O_3$ *from eq. (171).*

$$\sin(i-\beta)=\frac{(R_2-R_3)\sin\Delta_2\sin i}{l} \qquad (180)$$

We then find α from eq. (175) and p from (172).

The angles α and β may be found otherwise, for by Trig (Tab. II. 27) we have in the triangle $O_1O_2O_3$

$$\sin\tfrac{1}{2}(O_1O_3O_2-O_3O_1O_2)=\frac{O_1O_2-O_2O_3}{O_1O_3}\cos\tfrac{1}{2}\Delta_2$$

or

$$\sin\left(90^\circ-\left(i+\frac{\alpha-\beta}{2}\right)\right)=\frac{(R_3-R_1)\cos i\ \cos\frac{1}{2}\Delta_2}{R_3-R_1}$$

$$\therefore\ \cos\left(i+\frac{\alpha-\beta}{2}\right)=\cos i\,.\,\cos\tfrac{1}{2}\Delta_2 \qquad (181)$$

which is a convenient formula when i and Δ_2 are not too small. Having obtained $\frac{\alpha-\beta}{2}$, we have

$$\alpha=\tfrac{1}{2}\Delta_2+\frac{\alpha-\beta}{2} \qquad \beta=\tfrac{1}{2}\Delta_2-\frac{\alpha-\beta}{2} \qquad (182)$$

For a constant value of l the *less* the difference of $R_3 - R_1$ the *greater* will be the value of the angle i. When $R_3 = R_1$, $\cot i = 0$ and $i = 90^\circ$ and the tangent point I will be on a perpendicular to BA drawn through the middle point K; and $\alpha = \beta$. On the contrary, as $(R_3 - R_1)$ increases, i becomes less, and the foot, H, of the perpendicular HI moves toward B, the tangent point of the curve of smaller radius R_1. The distance $HK = p \cot i$. The connecting curve is farthest from the tangent BA at I. To find the ordinate from BA to the curve at any other point, subtract from p the tangent offset for the length of curve from I to the ordinate in question. §115, eq. (39) may be used on flat curves with tolerable accuracy, even when the distance equals several hundred feet.

IV. It is evident that in this problem R_2 must be greater *than either R_1* or R_3. As the centre O_2 is taken nearer the line O_1O_3, R_2 grows less, and is a minimum when O_2 falls on the line O_1O_3. In this case we have $\Delta_2 = 180°$, and

$$R_2 = \tfrac{1}{2}(R_3 + R_1 + O_1O_3);\ \text{a minimum.} \qquad (183)$$

This limit must be regarded in assuming the value of R_2. Since

$$O_1O_2 - O_2O_3 = (R_2 - R_1) - (R_2 - R_3) = (R_3 - R_1)$$

a constant value, independent of R_2, we infer that the centre O_2 is always on a hyperbola of which O_1 and O_3 are the foci; $(R_3 - R_1)$ equals the diameter on the axis joining the foci; and l equals the diameter at right angles to it, for in the triangle O_1GO_3,

$$l^2 = \overline{O_1O_3}^2 - (R_3 - R_1)^2 \qquad (184)$$

Example.—Fig. 57.

Given: $R_1 = 1432.69$ $R_3 = 1910.08$ and $l = 400$.
Assume $p = 11.4$ to find R_2, α and β.

Eq. (170)	$R_3 - R_1$	477.39		log	2.678873
	l	400.		"	2.602060
∴	i		39° 57′ 34″	log cot	0.076813
Eq. (173)	i		39° 57′ 34″	" sin	9.807701
	i		39° 57′ 34″	" $\sin^2$	9.615402
	p	11.4		log	1.056905
	*	27.64		"	1.441503
	$\frac{1}{4}l^2$			"	4.602060
	p			"	1.056905
	*	3508.77		"	3.545155
	$R_3 + R_1$	3342.77			
	2)	6879.18			
∴	R_2	3439.59	(say) 3437.87		
Eq. (174)	p	11.4		"	1.056905
	$R_2 - R_3$	1527.79		"	3.184064
∴	α		7° 00′	log vers	7.872841
	p	11.4		log	1.056905
	$R_2 - R_1$	2005.18		"	3.302153
	β	(nearly)	6° 07′	log vers	7.754752
	Δ_2		13° 07′		

Example.—Fig 57.

Given: $R_1 = 1432.69$, $R_3 = 1910.08$, and $l = 400$.
Assume $R_2 = 3437.87$, to find Δ_2, β, α and p.

Eq. (179)		2.		log 0.301030
	$R_2 - R_1$	2005.18		" 3.302153
	$R_2 - R_3$	1527.79		" 3.184064
				" 6.787247
	l^2			5.204120
∴	Δ_2		13° 07′ 22″	log vers 8.416873
Eq. (170)	$R_3 - R_1$	477.39		log 2.678873
	l	400.		" 2.602060
∴	i		39° 57′ 34″	log cot 0.076813
Eq. (180)	i		39° 57′ 34″	log sin 9.807701
	Δ_2		13° 07′ 22″	" " 9.356099
	$R_2 - R_3$	1527.79		log 3.184064
				log sin 2.347864
	l	400.		log 2.602060
∴	$i - \beta$		33° 50′ 39″	log sin 9.745804
∴	β		6° 06′ 55″	
Eq. (175)	α		7° 00′ 27″	
Eq. (172)	$R_2 - R_3$			log 3.184064
	α		7° 00′ 27″	log vers 7.873309
∴	p	11.41		1.057373

178. *Given: a three-centred compound curve* **to replace the middle arc** *by an arc of different radius.*

I. *When the radius of the middle arc is the* **greatest.** Fig. 57.

First find the length and direction of the common tangent *AB.* Let Δ_2 = central angle of the middle arc, R_2 = its radius, and R_1 and R_3 the radii of the other arcs. From eq. (179):

$$l = \sqrt{2(R_2 - R_1)(R_2 - R_3) \text{ vers } \Delta_2} \qquad (185)$$

Then find i by eq. (170), α and β by eqs. (181) (182), and p by eq. (172).

For the new arc we may now assume a new value for p, or *for R_2, or for α.* Indicating the new values by an accent, **if we assume p′** we proceed as in the last problem, using eqs. *(173), etc.* If we assume R_2, we use eq. (179), etc. If we assume α', we use eq. (178).

II. *When the radius of the middle arc is the* **least** *of the three.* Fig. 58.

In this case the middle arc is within the other two produced; and for the same values of $R_1 R_3$ and $O_1 O_3$, the *locus*

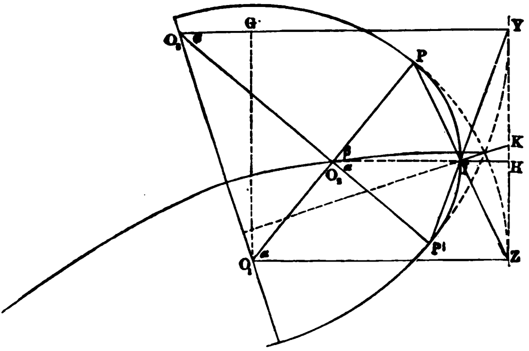

FIG. 58.

of the centre O_2 is the opposite branch of the hyperbola found in §177. When the centre O_2 falls on the line $O_1 O_3$, $\triangle_2 = 180°$, and

$$R_2 = \tfrac{1}{2}(R_3 + R_1 - O_1 O_3), \text{ a maximum.} \tag{186}$$

Analogous to eq. (185), we have

$$l = \sqrt{2(R_1 - R_2)(R_3 - R_2) \text{ vers } \triangle_2} \tag{187}$$

which gives the length of the common tangent YZ.

We then have the values of i and of $O_1 O_3$ by eqs. (170) (171), and of α and β by eqs. (181) (182), and analogous to eq. (172),

$$p = (R_1 - R_2) \text{ vers } \alpha = (R_3 - R_2) \text{ vers } \beta \tag{188}$$

in which p is the perpendicular distance HI between parallel tangents.

For the new arc we may now assume a new value for p, for R_2, or for α. *Indicating* the new values by an accent, ***if we assume*** p', ***we have***, analogous to eq. (173)

$$2R_2' = R_3 + R_1 - \left(\frac{l^2}{4p'} + \frac{p'}{\sin^2 i}\right) \tag{189}$$

and from eq. (188)

$$\text{vers}\ \alpha' = \frac{p'}{R_1 - R_2}\ ;\ \text{vers}\ \beta' = \frac{p'}{R_3 - R_2} \tag{190}$$

If we assume R_2', we have, analogous to eq. (179),

$$\text{vers}\ \Delta_2' = \frac{l^2}{2(R_1 - R_2')(R_3 - R_2')} \tag{191}$$

and we find α and β by eqs. (181) (182), and p' by eq. (188).

III. *When the radius of the middle arc has an* **intermediate value,** *compared with the other radii.* **Fig. 59.**

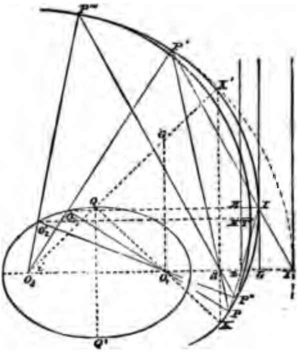

FIG. 59.

In this case, whatever be the value of R_2, we have

$$O_3O_2 + O_2O_1 = (R_3 - R_2) + (R_2 - R_1) = (R_3 - R_1)$$

a constant value independent of R_2; hence we infer that the *locus* of O_2 is an ellipse, of which O_1 and O_3 are the foci, and $(R_3 - R_1)$ equal to the transverse axis.

Let $l = QQ' =$ the conjugate axis, and let $i = QO_3O_1 = QO_1O_3$.

Produce O_3Q *to* G, making $QG = O_3Q$, and join GO_1.

Then by similar triangles GO_1 is perpendicular to O_1O_3, and $GO_1 = l$; and in the right-angled triangle GO_3O_1

$$\sin i = \frac{GO_1}{GO_3} = \frac{l}{R_3 - R_1} \qquad (192)$$

$$O_1O_3 = (R_3 - R_1)\cos i = l \cot i \qquad (193)$$

Analogous to eqs. (185) and (187), we have

$$l = \sqrt{2(R_3 - R_2)(R_2 - R_1)\ \text{vers}\ \Delta_2} \qquad (194)$$

which may also be derived from the triangles $O_1O_2O_3$ and O_1O_3Q.

Let $\alpha = O_2O_3O_1$, and $\beta = O_2O_1O_3$

Then

$$\sin \alpha = \frac{O_1O_2}{O_1O_3} \sin \Delta_2 = \frac{R_2 - R_1}{l} \tan i \,.\, \sin \Delta_2 \qquad (195)$$

$$\text{From the figure } \beta = \Delta_2 - \alpha \qquad (196)$$

In the diagram produce the line O_3O_1 and it will intersect all the arcs. At the points Z and Y, where it cuts the inner and outer arcs, draw tangent lines perpendicular to O_3O_1. Draw the radius O_2I parallel to O_3O_1, and the tangent line IL at I.

Let $q = ZY$ and $p = ZL = HI$

Then by the theory of parallel tangents, §137, the point I is on the chord PZ produced, and it is also on the chord $P'Y$; and we have

$$p = ZL = (R_2 - R_1)\ \text{vers}\ \beta\,. \qquad (197)$$

$$q - p = LY = (R_3 - R_2)\ \text{vers}\ \alpha \qquad (198)$$

and q equals the *sum* of these. But $q = ZY$ is the shortest distance between the inner and outer arcs, and has a constant value independent of R_2. If we assume $R_2 = \frac{1}{2}(R_3 + R_1)$ the centre O_2 will be at Q, and $\alpha = \beta = i$, and $p = \frac{1}{2}q$. Making these substitutions above,

$$q = (R_3 - R_1)\ \text{vers}\ i. \qquad (199)$$

lso, from the figure,

$$ZY = O_3Y - O_1Z - O_1O_3,$$

or,

$$q = R_3 - R_1 - O_1O_3. \tag{200}$$

In the triangle ZIY we have by Geom. Tab. I. 26,

$$ZI^2 = IY^2 + ZY^2 - 2ZY(ZY - ZL)$$

or

$$ZY^2 - 2ZY.ZL = IY^2 - ZI^2$$

Now,

$$ZI^2 = 4(R_2 - R_1)^2 \sin^2 \tfrac{1}{2}\beta = 2(R_2 - R_1)^2 \text{ vers } \beta$$

$$IY^2 = 4(R_3 - R_2)^2 \sin^2 \tfrac{1}{2}\alpha = 2(R_3 - R_2) \text{ vers } \alpha$$

Hence

$$ZI^2 = 2(R_2 - R_1)\,p \quad\text{and}\quad IY^2 = 2(R_3 - R_2)\,(q - p)$$

Substituting these values, and solving for p, we have

$$p = \frac{q(R_3 - R_2 - \frac{1}{2}q)}{R_3 - R_1 - q} = \frac{q(R_3 - R_2 - \frac{1}{2}q)}{O_1O_3} \tag{201}$$

Also

$$R_2 = (R_3 - \tfrac{1}{2}q) - p\,.\,\frac{O_1O_3}{q} \tag{202}$$

For any other value of R_2, we have

$$R_2' = (R_3 - \tfrac{1}{2}q) - p'\frac{O_1O_3}{q}$$

Hence

$$R_2' - R_2 = \frac{O_1O_3}{q}\,(p - p') \tag{203}$$

which gives the change in R_2 for a given change in the value of p

Observe that as p diminishes R_2 increases and *vice versa.*

Having determined the value of R_2', we find p' by substituting R_2' for R_2 in eq. (201); and from eqs. (197) (198) we have

$$\text{vers } \beta' = \frac{p'}{R_2' - R_1} \tag{204}$$

$$\text{vers } \alpha' = \frac{q - p'}{R_3 - R_2'} \tag{205}$$

nd the change in the points of compound curvature is found y $(\beta - \beta')$ and $(\alpha' - \alpha)$.

Remark.—When $R_2 = \frac{1}{2}(R_3 + R_1)$, $\Delta_2 = 2i$, a minimum, nd the long chord PP' is perpendicular to O_1O_3. When R_2 s greater than this, α is greater than β, and *vice versa.* Whatver be the value of R_2, the long chord PP' always cuts the ine O_1O_3 produced in the same point S, at a distance from Z of

$$ZS = R_1 \text{ vers } i;$$

r from O_1 of $$O_1S = R_1 \cos i.$$

This item will be found useful in solving the problem raphically.

Example.

Let $R_1 = 781.84$ $D_1 = 7° 20'$
" $R_2 = 1375.40$ $D_2 = 4° 10'$ $\Delta_2 = 48°$
" $R_3 = 1910.08$ $D_3 = 3° 00'$
Let $p - p' = 11.30$

Eq. (194)	2			log 0.301030
	$R_3 - R_2$	534.68		" 2.728094
	$R_2 - R_1$	593.56		" 2.773465
	Δ_2		48°	log vers 9.519657
				2) 5.322246
∴	l	458.27		log 2.661123
192)	$R_3 - R_1$	1128.24		" 3.052402
∴	i		23° 57′ 55″	log sin 9.608721
193)	i		23° 57′ 55″	log cos 9.960847
	$R_3 - R_1$			log 3.052402
∴	O_1O_3	1030.98		log * 3.013249
95)	$R_2 - R_1$			log 2.773465
	Δ_2		48°	log sin 9.871073
				log * 2.644538
∴	α		25° 19′ 52″	log sin 9.631289
96)	β		22° 40′ 08″	
03)	O_1O_3			log 3.013249
00)	q	97.26		1.987934
	$\frac{O_1O_3}{q}$			1.025315
	$p - p'$	11.30		log 1.053078
∴	$R_2' - R_2$	119.78		" 2.078393
∴	R_2'	1495.18 (say) 1494.95 for 3° 50′ curve.		

(201)	$R_3 - R_2' - \frac{1}{2}q$	366.50		log 2.564074
	$\frac{O_1 O_3}{q}$			" 1.025315
$\therefore$	p'	34.57		" 1.538759
(197)	$R_2' - R_1$	713.11		" 2.853157
	β'		17° 55′	log vers 8.685602
(198)	$q - p'$	62.69		log 1.797198
	$R_3 - R_2'$	415.18		" 2.618184
	α'		31° 54′	log vers 9.179014
	Δ_2'		49° 49′	

$$\alpha' - \alpha = 6^\circ\ 34' \ \therefore\ P'P''' = 218.89$$
$$\beta - \beta' = 4^\circ\ 45' \ \therefore\ PP'' = 64.77$$

The practical difficulty in changing the middle arc of three centred curves lies in the difference of measurement that ensues. Thus, in the last problem, although the total central angle is the same, the new curve is 6.56 feet shorter than the original, making a fractional station at P''. If the change is made during the location, it is well to re-run the last arc from P''' to the tangent following, so as to eliminate the fractional station from the curve.

CHAPTER VII.

TURNOUTS.

179. A turnout is a curved track by which a car may leave the main track for another. At the point where the outer rail of the turnout crosses the rail of the main track a *frog* is introduced which allows the flanges of the wheels to pass the rails. A frog consists essentially of a solid block of iron or steel having two straight channels crossing each other on the upper surface, in which the flanges of the wheels pass. The triangular portion of the upper surface formed by the channels is called the *tongue* of the frog, and the angle which the channels make with each other is called the *frog-angle.* Every railroad is provided with a set of frogs of different angles, from which may be selected one best adapted to any particular case.

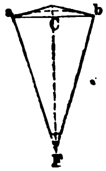

FIG. 60.

The frogs may be designated by their angles, but it is customary to designate them by numbers expressing the ratio of the bisecting line FC of the tongue to the base line ab, Fig. 60. Observe that F is at the intersection of the edges produced, and not at the blunt point of the tongue.

In the triangle aFC,

$$\frac{FC}{aC} = \cot \tfrac{1}{2} aFb$$

and if we let n = the number of the frog, and F = the frog angle, then

$$n = \frac{FC}{ab} = \frac{FC}{2aC} = \tfrac{1}{2}\cot \tfrac{1}{2}F \qquad (206)$$

On some roads, however, the frogs are numbered arbitrarily, or according to their length in feet, while on others they are designated by letters of the alphabet. In any case the true number (n) *of a frog may be* determined by the above formula.

The first rail of the turnout is common to both tracks, and is called the *switch-rail*. It has one end free, so as to be shifted from one track to the other as required; the free end, D (Fig. 61), is called the *point of switch*. The tangent point of the turnout, at A, is called the *heel of switch*, and the distance, AD, is the *length of switch*. The switch-rail should be several feet longer than AD, and the excess be spiked down in the line of the main track back of the point A. Then if the point D is thrown over to meet the rail of the turnout at K, the switch rail is sprung into an arc, which coincides with the arc of the turnout, provided that the length of switch AD has been properly taken. The distance DK through which the point moves is called the *throw* of the switch. It varies on different roads from 4½ to 6 inches, but is usually made about 5 inches, or 0.42 feet. A turnout should be a simple curve from the heel of the switch to the point of the frog.

180. *Given: a main track, straight, and a frog angle F, to determine the distance* **BF,** *on the main track from the heel of switch to point of frog, the radius,* **r,** *of the centre line of the turnout, the length of chord* **af,** *and the proper length of switch* **AD.** **Fig. 61.**

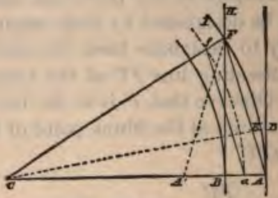

FIG. 61.

Let C be the centre of the turnout.

" $F =$ the frog angle, $HFI = FCB$.

" $g =$ the gauge of track AB.

" $r =$ radius, $aC = fC$.

" $DK =$ the throw of switch.

Then the radius of the gauge side of the outer rail is $(r + \frac{1}{2}g)$, and we have

$$AB = FC \,.\, \text{vers}\, FCB$$

or,

$$g = (r + \tfrac{1}{2}g)\, \text{vers}\, F$$

whence

$$(r + \tfrac{1}{2}g) = \frac{g}{\text{vers}\, F} \tag{207}$$

The angle $AFB = \tfrac{1}{2}F$

and $BF = AB \cot AFB = g \,.\, \cot \tfrac{1}{2}F$ (208)

Again, in the triangle FCB

$$BF = FC \,.\, \sin FCB = (r + \tfrac{1}{2}g) \sin F \tag{209}$$

The chord af is evidently

$$af = 2r \sin \tfrac{1}{2}F \tag{210}$$

Similar to eq. (207), we have

$$\text{vers}\, ACD = \frac{DK}{KC} = \frac{DK}{r + \tfrac{1}{2}g}$$

But since the inside rail has the same throw, while its radius is $(r - \tfrac{1}{2}g)$, we may, if convenient, drop the $\tfrac{1}{2}g$, and hence the length of switch is

$$AD = r \,.\, \sin ACD \tag{211}$$

The degree of curve corresponding to r is found from Table IV., or by eq. (17), and the centre line of the turnout may be located by transit deflections from the tangent point a, using chords of 20 or 25 feet + the correction found in §§ 106, 107; or the deflection for a 20-foot chord may be calculated at once by

$$\sin (\tfrac{1}{2}d_{20}) = \frac{10}{r} \tag{212}$$

181. Simple as these formulæ are, they may be rendered still more convenient by introducing the **number of the frog,** n. By eq. (206) we have $\cot \tfrac{1}{2}F = 2n$, which substituted in eq. (208) gives

$$BF = 2gn \tag{213}$$

Drawing the chord AF to the outer rail,

$$AF = \sqrt{AB^2 + BF^2} = g\sqrt{1 + 4n^2} \tag{214}$$

Make $BA' = AB$ and join FA'; then by similar triangles, $AA'F$ and AFC,

$$AA' : AF :: AF : FC$$

whence

$$FC = \frac{AF^2}{AA'}$$

or

$$(r + \tfrac{1}{2}g) = \tfrac{1}{2}g\,(1 + 4n^2) \qquad (215)$$

whence

$$r = 2gn^2 = BF \,.\, n \qquad (216)$$

The chord af to the arc of the centre line is to AF as r is to $(r + \frac{1}{2}g)$; hence $af = \dfrac{AF \,.\, r}{r + \frac{1}{2}g}$, and substituting values from eqs. (214) (215) we have

$$af = \frac{2r}{\sqrt{1 + 4n^2}} \qquad (217)$$

Assuming that, for small angles, the tangent offsets vary as the squares of their distances from the tangent point, which will lead to no material error in this case;

$$AB : DK :: BF^2 : AD^2$$

whence

$$\left.\begin{aligned} AD &= BF\sqrt{\frac{DK}{AB}} \\ \text{or} \quad AD &= \sqrt{4n^2\, g \,.\, DK} = \sqrt{2r \,.\, DK} \end{aligned}\right\} \qquad (218)$$

It is not necessary to determine the degree of curve in order to locate the turnout, for having fixed the position of BF, the position of af is found by laying off Ba, and Ff, each equal to $\frac{1}{2}g$. Whatever be the length of the chord af, found by eq. (217) or (210), its middle ordinate is always $\frac{1}{4}g$, and the ordinates at the quarter points, $\frac{3}{4} \,.\, \frac{1}{4}g = \frac{3}{16}g$. Thus for the standard gauge of 4.708 the middle ordinate is 1.177, and the side ordinates 0.883.

By the preceding formulæ Table XI. has been calculated, which gives the required parts of a turnout for various frogs when the gauge is 4 feet 8½ inches and the throw 5 inches; *also for* a gauge of 3 feet and throw of 4 inches. For any *other throw,* only AD must be calculated. For a different *gauge* the engineer will do well to construct a similar table, *adapted to the* frogs used on the road.

In the table the frog angle is given to seconds, in order that the results may agree, whether found by equations in § 180 or § 181; but in practice the nearest minute is sufficiently exact. The frogs most used for single turnouts are those from No. 7 to No. 9, inclusive.

182. *In case of a* **double turnout** *from the same switch,* three frogs are required, as at F, F' and F'', Fig. 62., and the switch is called a *three-throw switch*, because its point takes three positions. The frogs F and F' are usually alike, and placed exactly opposite each other in the main track. The other frog F'' is placed on the centre line of the main track. Its angle F'' and its distance from a are now to be determined in terms of F.

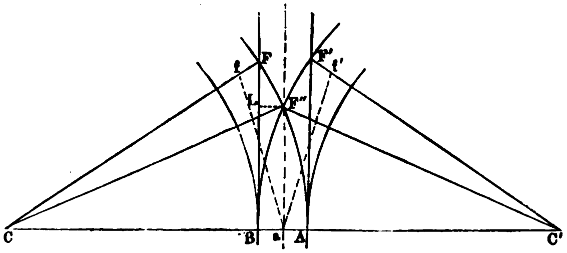

FIG. 62.

In the figure we have vers $F''Ca = \frac{Aa}{F''C}$ or

$$\text{vers}\ \tfrac{1}{2}F'' = \frac{g}{2(r + \frac{1}{2}g)} \qquad (219)$$

The distance $$aF'' = (r + \tfrac{1}{2}g) \sin \tfrac{1}{2}F'' \qquad (220)$$

also $$aF'' = r \,.\, \tan \tfrac{1}{2}F'' \qquad (221)$$

All the parts of the turnout required to locate the frogs F and F'' are calculated by the formulæ in the preceding sections, or are taken from Table XI.

If we let n'' = the number of the frog F'', then by eq. (206) $\tan \frac{1}{2}F'' = \frac{1}{2n''}$, which substituted in eq. (221) gives

$$aF'' = \frac{r}{2n''} \qquad (222)$$

Also, in the triangle $aF''C$,

$$aF'' = \sqrt{(r+\tfrac{1}{2}g)^2 - r^2} = \sqrt{g(r+\tfrac{1}{4}g)} \qquad (223)$$

Equating these and replacing r by $2gn^2$, we obtain

$$n'' = \sqrt{\frac{n^4}{2n^2 + \frac{1}{4}}} \qquad (224)$$

If we neglect the $\frac{1}{4}$, we have

$$\text{(approx.)} \qquad n'' = \frac{n}{\sqrt{2}} = .7071n \qquad (225)$$

Example.—If $F = F' = 6° 44'$, or $n = n' = 8.5$, then $n' = 6.0 +$ or $F'' = 9° 32'$.

183. *In case no frog is at hand of the angle or number given* by eq. (219) or (225), we may select one as nearly like it as possible, and locate the turnout as a **compound curve,** provided that F'' is less than $2F$. Fig. 63.

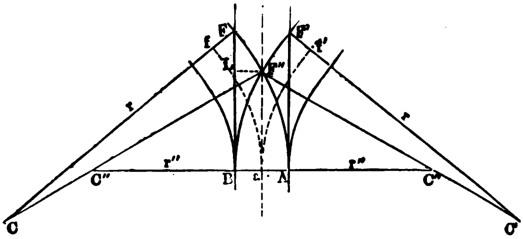

FIG. 63.

Let $r'' = C''a$, and $r = r' = Cf = C'f'$

Then analogous to the equations of § 180,

$$(r'' + \tfrac{1}{2}g) = \frac{\frac{1}{2}g}{\text{vers } \frac{1}{2}F''} \qquad (226)$$

$$\therefore r'' = \frac{\frac{1}{2}g}{\text{exsec } \frac{1}{2}F''} \qquad (227)$$

$$aF'' = (r'' + \tfrac{1}{2}g) \sin \tfrac{1}{2}F'' = r'' \tan \tfrac{1}{2}F'' \qquad (228)$$

The length of the switch, by eq. (218), is

$$AD = \sqrt{2r''\,\overline{DK}}$$

The curvature of the rail between the frogs F'' and F is $F''CF = (F - \frac{1}{2}F'')$.

Draw the chord $F''F$ and the perpendicular $F''L$; then the angle $LFF'' = F - \frac{1}{2}(F - \frac{1}{2}F'') = \frac{1}{2}(F + \frac{1}{2}F'')$; and since $LF'' = \frac{1}{2}g$,

$$\therefore\ F''F = \frac{\frac{1}{2}g}{\sin\frac{1}{2}(F + \frac{1}{2}F'')} \tag{229}$$

$$LF = \tfrac{1}{2}g \,.\, \cot\tfrac{1}{2}\,(F + \tfrac{1}{2}F'') \tag{230}$$

$$(r + \tfrac{1}{2}g) = \frac{\frac{1}{2}F''F}{\sin\frac{1}{2}\,(F - \frac{1}{2}F'')} \tag{231}$$

Example.—Let $F = 6° 44'$ $F'' = 10° 24'$

Eq. (226)	$\frac{1}{2}g$	2.354		log 0.371806
	$\frac{1}{2}F''$		5° 12′	log exs 7.616224
$\therefore$	r''	569.616		2.755582
Eq. (228)	$\frac{1}{2}F''$		5° 12′	log tan 8.959075
$\therefore$	aF''	51.839		1.714657
Eq. (229)	$\frac{1}{2}g$	2.354		log 0.371806
	$\frac{1}{2}(F + \frac{1}{2}F'')$		5° 58′	log sin 9.016824
$\therefore$	$F''F$	22.645		1.354982
Eq. (231)	$\frac{1}{2}(F - \frac{1}{2}F'')$		0° 46′	log sin 8.126471
	$2(r + \frac{1}{2}g)$	1692.432		3.228511
$\therefore$	r	843.862		

When $n'' > .707n$, r will be less than r''. Should F' not equal F, (F'' being given), then r' and $L'F'$ must be calculated also, by substituting F' for F in eqs. (230) and (231).

184. *From the same switch in a straight track it is required o lay* **two turnouts on the same side.** Fig. 64.

If we assume $F' = F$, and that these two frogs shall be *opposite each other,* we calculate all the distances of the first *urnout for the angle* F (or number n) by § 180, 181, whence *have the radius* $r = Ca$.

Let $r' = C'a$, the radius of the centre line of the secon turnout. The angle $ACF = F$, and since $F' = F$, the ang $CF'C' = F$, and the triangle $CF'C'$ is isosceles, and $C'F'$ = $C'C$. But $C'F' = C'A = \frac{1}{2}CA$.

or
$$(r' + \tfrac{1}{2}g) = \tfrac{1}{2}(r + \tfrac{1}{2}g) \tag{232}$$

∴
$$r' = \tfrac{1}{2}(r - \tfrac{1}{2}g) \tag{233}$$

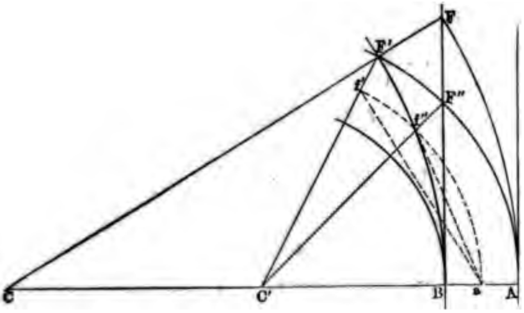

FIG. 64.

To calculate the remaining frog at F'', we have from co (207)

$$\text{vers } F'' = \frac{g}{r' + \frac{1}{2}g} \tag{234}$$

or from eq. (216)

$$n'' = \sqrt{\frac{r'}{2g}} \tag{235}$$

$$BF'' = (r' + \tfrac{1}{2}g)\sin F = 2gn'' \tag{236}$$

$$af'' = 2r' \sin \tfrac{1}{2}F'' = \frac{2r'}{\sqrt{1 + 4n''^2}} \tag{237}$$

and since $AC'F' = 2F$,

$$af' = 2r' \sin F \tag{238}$$

The length of switch may be calculated by either r or r *since for* r', which is about $\frac{1}{2}r$, the throw of switch is *doubl* *hus giving* practically identical results.

If we compare the values of F'' as obtained by eqs. (d *(219)*, we shall find them almost identical for given v of F and g; and since this may also be proved analytically by assuming that vers $\frac{1}{2}F''$ = $\frac{1}{4}$ vers F'', which is very nearly true for ordinary values of F'', we conclude that a set of frogs ($F = F'$, and F'') which is adapted to a double turnout in opposite directions from a straight line (as in Fig. 62) is also adapted to a double turnout on one side (as in Fig. 64), the curves being simple curves in every case. But this being true, the set is also adapted to a double turnout in opposite directions *from any curved track* the radius of which is not less than r as given for F, since any such case is intermediate between the two cases named. When, therefore, a certain frog, F, is adopted for general use on any road, another frog should also be adopted, whose angle, F″, is determined by eq. (219), or whose number n is determined by eq. (225). Thus, if $F = 6° 44'$, or $n = 8\frac{1}{2}$, then F'' should be 9° 32′, or $n' = 6$.

185. *In case no frog is at hand of the angle or number given by eqs.* (234) (235), we may select one as near the same angle as possible, and, calling this F'', calculate the distance BF'' and the radius $C''F''$ (Fig. 65) as for a single turnout; § 180.

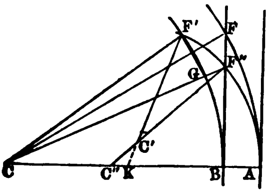

FIG. 65.

Then assuming any other frog F', whether equal to F or not, it is required to find the chord $F''F'$, and the radius $C'F'$ of the arc $F''F'$. The point F' may fall either side of the radius CF, according to the values given to F'' and F'.

a. *In case F' falls beyond the radius CF*, we will assume *first*, that the entire rail from B to F' is laid with *the same radius BC*, and centre C. (This investigation also applies to *the case* when F' falls between B and the line CF.)

In the diagram (Fig. 65) draw CF''. We then have

$$\tan BCF'' = \frac{BF''}{BC} = \frac{BF''}{r - \frac{1}{2}g} \qquad (239)$$

and

$$GF'' = (r - \tfrac{1}{2}g) \text{ exsec } BCF'' \qquad (240)$$

In the triangle $F''CF'$.

$$F''C - F'C : F''C + F'C :: \tan \tfrac{1}{2}(F''F'C - F'F''C)$$
$$: \cot F''CF'$$

Now, since $C'F'C = F'$, and $BC''F'' = F''$,

$$\therefore \; F''F'C = F''F'C' + F'$$

and

$$F'F''C = F'F''C'' - C''F''C = F''F'C' - (F'' - BCF'')$$

Letting $U = C''F''C = (F'' - BCF'')$

and subtracting, we have

$$F''F'C - F'F''C = F' + U$$

Hence the above proportion may be written

$$GF'' : 2BC + GF'' :: \tan \tfrac{1}{2}(F' + U) : \cot \tfrac{1}{2}F''CF'$$

whence

$$\cot \tfrac{1}{2}F''CF' = \frac{2BC + GF''}{GF''} \tan \tfrac{1}{2}(F' + U) \qquad (241)$$

(Since $BCF'' + F''CF' = BCF'$, and we know the radius BC, the chord or arc BF' is easily obtained, which fixes the position of the frog F'; and the problem may end here, frequently, in practice.)

Now in the same triangle $F''CF'$, *the half sum* of $F''F'C$ and $F'F''C$ is $90° - \frac{1}{2}F''CF'$; while, as we have just seen, the *half difference* is $\frac{1}{2}(F' + U)$; and by subtracting we have the less, or

$$F'F''C = 90° - \tfrac{1}{2}(F' + U + F''CF') \qquad (242)$$

Now

$$F''F' = \frac{F'C \sin F''CF'}{\sin F'F''C}$$

or

$$F''F' = \frac{BC \,.\, \sin F''CF'}{\cos \frac{1}{2}(F' + U + F''CF')} \qquad (243)$$

To find the angle $F''C'F'$; produce the line $F'C'$ in the diagram to intersect the line BC at K. Then the two triangles $KC''C'$ and KCF' have the angle K common, and the sum of the other angles will be equal; that is,

$$KC''C' + C''C'K = KCF' + CF'K$$

or

$$F'' + F''C'F' = BCF' + F'$$

and since

$$BCF' = BCF'' + F''CF'$$

$$\therefore\ F''C'F' = F''CF' + F' - U \qquad (244)$$

If we denote the radius $F'C'$ by $r' + \frac{1}{2}g$

$$r' + \tfrac{1}{2}g = \frac{\frac{1}{2}F''F'}{\sin \frac{1}{2}F''C'F'} \qquad (245)$$

Example.—Given: the three frogs $F = 6°\ 43'\ 59''$, $F' = 6°\ 01'\ 32''$, and $F'' = 8°\ 47'\ 51''$ to lay a double turnout on one side of a straight track. Fig. 65.

By Tab. XI. $BF = 80.036$ $r = 680.306$ $AD = 23.82$
$BF'' = 61.204$ $r'' = 397.826$

Eq. (239)	BF''	61.204		log	1.786779
	$(r - \frac{1}{2}g)$	677.952		"	2.831199
$\therefore$	BCF''		5° 09′ 38″	log tan	8.955580
Eq. (240)	BCF''		5° 09′ 38″	log exsec	7.609587
	$(r - \frac{1}{2}g)$	677.952		log	2.831199
	GF''	2.760		"	0.440786
Eq. (241)	$(2BC + GF'')$	1358.664		"	3.133112
			$(U = 3°\ 38'\ 13'')$		2.692326
	$\frac{1}{2}(F' + U)$		4° 49′ 52″.5	log tan	8.926968
$\therefore$	$\frac{1}{2}(F''CF')$		1° 22′ 35″	" cot	1.619294
Eq. (243)	$F''CF'$		2° 45′ 10″	" sin	8.681481
	$r - \frac{1}{2}g$	677.952			2.831199
					1.512680
	$\frac{1}{2}(F' + U + F''CF')$		6° 12′ 27″.5	" cos	9.997446
$\therefore$	$F''F'$	32.752			1.515234
Eq. (245)	$\frac{1}{2}F''C'F'$		2° 34′ 14″.5	" sin	8.651781
$\therefore$	$2(r' + \frac{1}{2}g)$	730.219			2.863453
	r'	362.755			

b. *We assume, secondly, that the middle track is straight beyond F, and tangent to the curve at F.* Fig. 66.

Then whenever the value of F'' is *less* than that given by eq. (234), the arc AF'', produced with the same radius AC'', will intersect the straight rail HF' at some point F', and the frog angles F and F' will be equal.

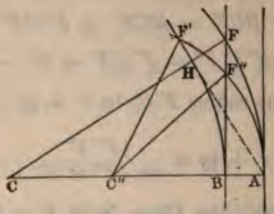

FIG. 66.

For the straight rail HF' produced backwards, passes through the point A, making an angle F with the main track, since the triangles CBF and CHA are equal, and $AH = BF$. Now any circle, tangent to the main rail at A, will intersect the line AH in some point F', and since AF' is the chord of the arc, the angle at F' equals the angle at A, which is F. Hence $F = F'$; and the angle $AC''F' = 2F$.

The length of the chord AF' is

$$AF' = 2AC'' \sin F \tag{246}$$

$$\text{The chord } F''F' = 2F''C'' \sin \tfrac{1}{2}(F''C''F')$$

$$= 2AC'' \sin \tfrac{1}{2}(2F - F'')$$

Hence, $$F''F' = 2(r'' + \tfrac{1}{2}g) \sin(F - \tfrac{1}{2}F'') \tag{247}$$

Example.—Let $F' = F = 6° 43' 59''$ and $F'' = 8° 47' 51''$

By Table XI. $r'' = 397.826$
Eq. (247) $2(r'' + \frac{1}{2}g) = 800.360$ log 2.903285
$F - \frac{1}{2}F''$ 2° 20′ 03″.5 log sin 8.609915
$\therefore$ $F''F'$ 32.60 1.513200

If the frog F' is required to be different from F, then the *inside curve* must be compounded at F'', giving other values *to the length* and radius of the arc F'' F'.

c. We assume, *thirdly*, that the curve of the *middle track is reversed at F.* Fig. 67.

In the diagram, let Q be the centre of the reversed portion, and F' the proper position of the frog F', and C' the centre of the required arc $F''F'$. Then Q is on the radial line CF, produced, and C' is on the radial line $F''C''$ produced. Join FQ and $F'Q$, and produce $C''F''$ to intersect these lines in L and M respectively. Also join $F''Q$, and denote the angle $LF''Q$ by U and the angle $F'QF''$ by Q.

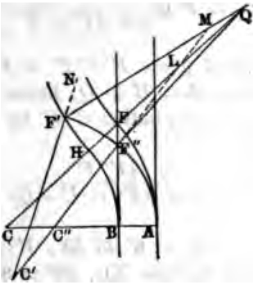

FIG. 67.

In the triangle $FF''Q$ we know $F''F = BF - BF''$, and the side FQ is given; and the included angle $F''FQ = 90° + F$. Hence we may calculate (Tab. II. 25) the angle $F''QF$ and the side $F''Q$.

The triangle $CC''L$ gives the angle at $L = F'' - F$; and the triangle $F''LQ$ gives $LF''Q = L - F''QF$

$$\therefore\ U = F'' - F - F''QF \qquad (248)$$

In the triangle $F'QF''$ we have

$$F'Q - F''Q : F'Q + F''Q :: \tan \tfrac{1}{2}(F'F''Q - F''F'Q)$$
$$: \cot \tfrac{1}{2}(F'QF'')$$

But $F'F''Q = F'F''L + U$ and $F''F'Q = F''F'N - F'$, and since $F''F'N = F'F''L$, we have by subtraction,

$$F'F''Q - F''F'Q = U + F'$$

Hence $$\cot \tfrac{1}{2}Q = \frac{F'Q + F''Q}{F'Q - F''Q} \tan \tfrac{1}{2}(U + F') \qquad (249)$$

(Now the angle $FQF' = Q - F''QF$, and is subtended by the chord HF', which is therefore easily found, and serves to locate the frog F', and frequently this is all that will be required.)

In the triangle $F''QF'$, the half sum of $QF''F'$ and $QF'F''$ is $90° - \frac{1}{2}Q$, while, as we have just seen, the *half difference* is $\frac{1}{2}(U + F')$; hence by adding, we have the greater, or

$$QF''F' = 90° + \tfrac{1}{2}(U + F' - Q)$$

$$\therefore \; F''F' = F'Q \frac{\sin Q}{\cos \frac{1}{2}(U + F' - Q)} \qquad (250)$$

The triangle $C'F'M$ gives $F''C'F' = F' - M$, while the triangle $F''MQ$ gives $M = U + Q$; hence $F''C'F' = F' - (U + Q)$; and denoting the radius $C'F'$ by $r' + \frac{1}{2}g$,

$$r' + \tfrac{1}{2}g = \frac{\frac{1}{2}F''F'}{\sin \frac{1}{2}(F' - U - Q)} \qquad (251)$$

Example.—Let $F = F' = 6° \, 43' \, 59''$, $F'' = 8° \, 47' \, 51''$, and $FQ = 953.012$. Then by Tab. XI., $BF = 80.036$ and $BF'' = 61.204$; hence $F''F = 18.832$; and the included angle is $96° \, 43' \, 59''$.

Solving the triangle $FF''Q$ we find $F''QF = 1° \, 07' \, 18''$, $FF''Q = 82° \, 08' \, 43''$, and $F''Q = 955.402$. Now $F'Q = FQ + g = 957.720$.

(249)	$F'Q + F''Q$	1913.122		log	3.281743
	$F'Q - F''Q$	2.318		"	0.365113
			(U 0° 56′ 34″)	"	2.916630
	$\frac{1}{2}(U + F')$		3° 50′ 16″.5	log tan	8.826231
$\therefore$	$\frac{1}{2}Q$		1° 02′ 08″.4	" cot	1.742861
(250)	Q		2° 04′ 16″.8	" sin	8.558033
	$\frac{1}{2}(U + F' - Q)$		2° 48′ 08″.1	" cos	9.999480
					8.558553
	$F'Q$	957.720		log	2.981239
$\therefore$	$F''F'$	34.633			1.539892
(251)	$\frac{1}{2}(F' - U - Q)$		1° 51′ 34″.1	log sin	8.511191
					3.028701
	$2(r' + \frac{1}{2}g)$	1068.32			
$\therefore$	r'	531.81			

186. *Given: a main track, curved, and a frog-angle F, to locate a turnout on the* **inside** *of the curve.* Fig. 68.

Let $R = Oa$ = radius of main track.

" $r = Ca$ = radius of turnout.

" $F = CFO$ = the frog angle.

In the diagram draw the chord AF and produce it to intersect the outer rail at G; and draw FO and GO. Since the chords AF and AG coincide, and the radii AC and AO coincide, the chords subtend equal angles at C and O respectively, and GO is parallel to FC. (See § 137.) Hence, $FOG = CFO = F$. Let θ = the angle FOA.

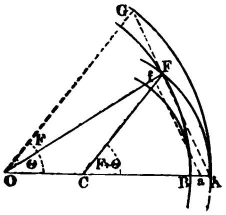

FIG. 68.

In the triangle FOA, $\theta = GFO - FAO = GFO - FGO$; and in the triangle GFO, $GO + FO : GO - FO :: \tan \frac{1}{2}(GFO + FGO) : \tan \frac{1}{2}(GFO - FGO)$, or $2R : g :: \cot \frac{1}{2}F : \tan \frac{1}{2}\theta$

$$\therefore \tan \tfrac{1}{2}\theta = \frac{g}{2R} \cot \tfrac{1}{2}F = \frac{gn}{R} \qquad (252)$$

In the triangle CFO,

$$(r + \tfrac{1}{2}g) = (R - \tfrac{1}{2}g) \frac{\sin \theta}{\sin (F + \theta)} \qquad (253)$$

In the triangle BOF,

$$BF = 2(R - \tfrac{1}{2}g) \sin \tfrac{1}{2}\theta \qquad (254)$$

In the triangle aCf,

$$af = 2r \sin \tfrac{1}{2}(F + \theta) \qquad (255)$$

The length of switch AD, for a given throw DK, may be found thus: from Table IV. take the tangent offsets, t and t', corresponding to R and r respectively, and assuming that the offsets may vary as the squares of their distances from the tangent point, we have

$$t - t' : DK :: (100)^2 : AD^2$$

$$\therefore\ AD = \sqrt{\frac{10000\ DK}{t - t'}} \qquad (256)$$

This result is practically the same as that found for length of switch in a turnout from a straight line with the same frog, when R is large.

Example.—Let $R = 1432.69$ and $F = 6° 43' 59''$.

Eq. (252)	$\frac{1}{2}g$	2.354		log 0.371806
	$\frac{1}{2}F$		3° 21' 59".5	log cot 1.230440
				log 1.602246
	R (Tab. IV.)			" 3.156151
$\therefore$	$\frac{1}{2}\theta$		1° 35' 59".8	log tan 8.446095
Eq. (254)	θ		3° 11' 59".6	" sin 8.746786
	$F + \theta$		9° 55' 58".6	" " 9.236778
				9.510008
	$R - \frac{1}{2}g$	1430.336		3.155438
$\therefore$	$r + \frac{1}{2}g$	462.856		2.665446
	r	460.502		
(254)	2			log 0.301030
	$(R - \frac{1}{2}g)$	1430.336		" 3.155438
	$\frac{1}{2}\theta$		1° 35' 59".8	log sin 8.445924
$\therefore$	BF	79.872		log 1.902392
(255)	$2r$	921.004		" 2.964262
	$\frac{1}{2}(F + \theta)$		4° 57' 59".3	log sin 8.937381
$\therefore$	af	79.734		log 1.901643

The values of BF and af are found to be so nearly identical in this case with those determined in case of a turnout from a straight line, that the values given in Table XI may be used at once for ordinary values of R; and the *degree of curve* of the turnout in this problem is *approximately* the *sum* of the degree *of curve of* the main track and the degree of curve given in *Table XI. opposite F.* Thus, in the example $4° + 8° 26' = 12° 26'\ \therefore\ r = 461.7$ nearly.

187. *Given: a main track, curved, and a frog-angle F, to locate a turnout on the* **outside** *of the curve.* Fig. 69.

In the diagram draw the chord AF, and produce it to meet the inner rail at G; and draw FO and GO. The triangles CAF and OAG are both isosceles, and have the angles at A equal; hence they are similar, and $FCA = AOG$. Hence $FOG = HFO = F$. Let $R = Oa$, $r = Ca$, and $\theta = FOA$.

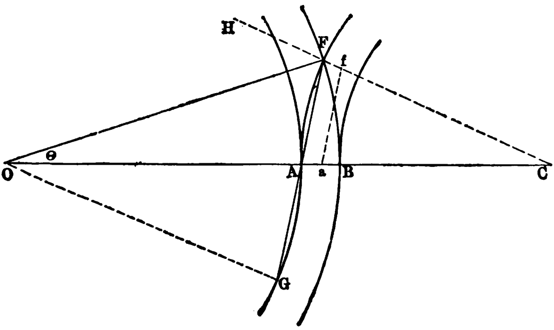

FIG. 69.

In the triangle FOA, $\theta = OAG - AFO = FGO - GFO$; and in the triangle FOG; $FO + GO : FO - GO :: \tan \frac{1}{2}(FGO + GFO) : \tan \frac{1}{2}(FGO - GFO)$, or $2R : g :: \cot \frac{1}{2}F : \tan \frac{1}{2}\theta$

$$\therefore\ \tan \tfrac{1}{2}\theta = \frac{g}{2R} \cot \tfrac{1}{2}F = \frac{gn}{R} \qquad (257)$$

which is identical with (252).

In the triangle CFO

$$(r + \tfrac{1}{2}g) = (R + \tfrac{1}{2}g)\,\frac{\sin\theta}{\sin\,(F - \theta)} \qquad (258)$$

In the triangle BOF,

$$BF = 2(R + \tfrac{1}{2}g) \sin \tfrac{1}{2}\theta \qquad (259)$$

In the triangle aCf,

$$af = 2r\,.\,\sin \tfrac{1}{2}(F - \theta) \qquad (260)$$

For a given throw, the length of switch will be

$$AD = \sqrt{\frac{10000\,DK}{t + t'}} \qquad (261)$$

in which t and t' are the tangent offsets (Tab. IV.) corresponding to R and r.

In this problem, as in the preceding, we may, for ordinary values of R, assume the values for BF and af given in Tab. XI. The *degree of curve* of this turnout is, *approximately*, $d - D$, taking d from Tab. XI. and D from Tab. IV. corresponding to R. Should $D = d$, this turnout becomes a straight line; and when $D > d$, or when R is less than r given in Tab. XI., the centre falls on the same side as O. Fig. 70. In this case, using the same notation, $\frac{1}{2}\theta$ is given by eq. (257).

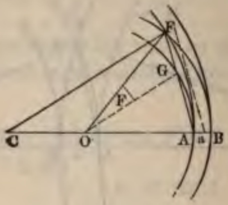

FIG. 70.

$$(r - \tfrac{1}{2}g) = (R + \tfrac{1}{2}g)\,\frac{\sin\theta}{\sin(\theta - F)} \qquad (262)$$

Eq. (259) $$BF = 2(R + \tfrac{1}{2}g)\sin\tfrac{1}{2}\theta$$

$$af = 2r\sin\tfrac{1}{2}(\theta - F) \qquad (263)$$

188. A tongue-switch is a short, stiff switch which, when moved, revolves at the heel as on a pivot. When it is thrown over to the turnout track, it makes an abrupt angle with the main track, called the *switch angle;* but in this position it should be tangent to the turnout curve. The use of this switch is generally confined to yards and warehouses, where but little space can be afforded, and where the motion of the cars is always slow.

189. *Given: a straight track, a frog-angle* **F,** *and the length and throw of a* **tongue-switch,** *to locate the turnout. Fig. 71.*

Let AD be the length, and DK the throw of switch, and let S denote the switch-angle DAK.

$$\text{Then} \qquad \sin S = \frac{DK}{AD} \text{ or } S^\circ = 57^\circ.3\,\frac{DK}{AD} \qquad (264)$$

(Compare § 86.)

Let C be the centre of the required turnout, and in the diagram draw CK and CF; also draw DG perpendicular to the straight track. Then $DGF = F$; and in the triangle KGC, $KCF = KGF - GKC$, and since CKA is a right-angle, $GKC = S$ $\therefore$ $KCF = F - S$.

Draw the chord KF, and since the triangle KCF is isosceles, the angle $CFK = 90^\circ - \frac{1}{2}(F - S)$. Now, $CFI = 90^\circ - F$; hence by subtraction, $KFI = \frac{1}{2}(F + S)$.

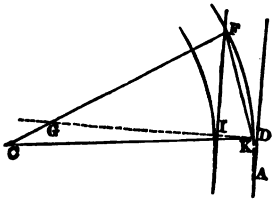

FIG. 71.

If g denote the gauge, we know $KI = g - DK$; and in the right-angled triangle KIF, we have

$$IF = KI \,.\, \cot \tfrac{1}{2}(F + S) \qquad (265)$$

$$KF = \frac{KI}{\sin \frac{1}{2}(F + S)} \qquad (266)$$

$$r + \tfrac{1}{2}g = \frac{\frac{1}{2}KF}{\sin \frac{1}{2}(F - S)} \qquad (267)$$

These equations are analogous to eqs. (229) (230) (231).

190. *Given:* *a* **double turnout** *with* **tongue-switch,** *from a straight track; to find the angle, F'', of the middle frog.*

Assuming $F' = F$ calculate $(r + \frac{1}{2}g)$ by the last equations. *Since the rails of the* turnouts intersect on the centre line of the straight track, as in Fig 63; if we substitute the value of $F''F'$, eq. (229) in eq. (231), we have

$$(r + \tfrac{1}{2}g) = \frac{\tfrac{1}{2}g}{2 \sin \tfrac{1}{2}(F + \tfrac{1}{2}F'') \sin \tfrac{1}{2}(F - \tfrac{1}{2}F'')}$$

and by Trig. Table II.

$$r + \tfrac{1}{2}g = \frac{\tfrac{1}{2}g}{\cos \tfrac{1}{2}F'' - \cos F}$$

whence $$\cos \tfrac{1}{2}F'' = \cos F + \frac{\tfrac{1}{2}g}{(r + \tfrac{1}{2}g)} \qquad (268)$$

If the angle of the middle frog to be used does not agree with F'' found by the last equation, the turnout will be compounded at F''.

191. *Given: a straight track, the frog-angles F, F' and F'',* and the **switch angle S,** *to locate a* **double turnout.** Fig. 72.

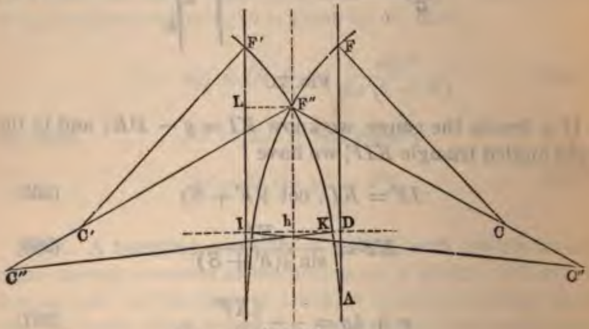

FIG. 72.

Assuming that F'' shall be placed on the centre line of the straight track, let h be a point on the centre line at the point of switch. Then $hK = \tfrac{1}{2}g - DK$; and since the angle F'' *is bisected* by the centre line the necessary formulæ in this *case are obtained* from § 189 by simply replacing F by ½F'' and KI by hK; and in the first members IF by hF'' and r by r''. *This is obvious* by the similarity of the figures.

Hence
$$hF'' = hK \,.\, \cot \tfrac{1}{2}(\tfrac{1}{2}F'' + S) \tag{269}$$

$$KF'' = \frac{hK}{\sin \tfrac{1}{2}(\tfrac{1}{2}F'' + S)} \tag{270}$$

$$r' + \tfrac{1}{2}g = \frac{\tfrac{1}{2}KF''}{\sin \tfrac{1}{2}(\tfrac{1}{2}F'' - S)} \tag{271}$$

The location of the remaining frogs is a problem already discussed, § 183, eq. (229), etc.

192. *Given: a straight track, the frog angles F, F', F'', and the* **switch angle S,** *to locate a* **double turnout on one side.** Fig. 73.

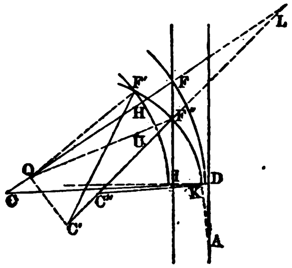

Fig. 73.

The frog F is located by § 189; but for the frog F'' we have evidently a double throw; hence eqs. (265) (266) (267) become

$$IF'' = (g - 2DK) \cot \tfrac{1}{2}(F'' + 2S) \tag{272}$$

$$KF'' = \frac{g - 2DK}{\sin \tfrac{1}{2}(F'' + 2S)} \tag{273}$$

$$r' + \tfrac{1}{2}g = \frac{\tfrac{1}{2}KF''}{\sin \tfrac{1}{2}(F'' - 2S)} \tag{274}$$

To locate the remaining frog F': when F' falls beyond the line CF, there are three cases.

a. The middle track reversed beyond F.

We find the distance $F''F$ by subtracting IF'', eq. (272) from *IF, eq. (265): after* which the solution is identical with that *given § 185, c., Fig. 67.*

b. The middle track compounded at F.

Let Q be the centre of the curve beyond F, and also let $Q =$ the angle $F'QF''$; and let $U =$ the angle $C''F''Q$.

Then by a course of reasoning analogous to that of case a, we derive

$$U = F'' - F + F''QF \tag{275}$$

$$\cot \tfrac{1}{2}Q = \frac{F''Q + F'Q}{F''Q - F'Q} \tan \tfrac{1}{2}(U + F') \tag{276}$$

Now since the radius $F'Q$ is given, and the angle $FQF' = Q - FQF''$, we readily determine the distance HF', and so locate the frog F'.

In the triangle $F''QF'$, the *half sum* of $QF''F'$ and $QF'F''$ is $90° - \tfrac{1}{2}Q$, while the *half difference* is $\tfrac{1}{2}(U + F')$; hence by subtraction we have the less, or

$$F'F''Q = 90° - \tfrac{1}{2}(U + F' + Q)$$

Hence

$$F'F'' = F'Q \frac{\sin Q}{\cos \tfrac{1}{2}(U + F' + Q)} \tag{277}$$

Join $C'Q$, and the quadrilateral $C'QF'F''$ gives

$$F' + Q = U + F''C'F'$$

hence $F''C'F' = F' - U + Q$; and denoting the radius $C'F$ by $r' + \tfrac{1}{2}g$, we have

$$r' + \tfrac{1}{2}g = \frac{\tfrac{1}{2}F''F'}{\sin \tfrac{1}{2}(F' - U + Q)} \tag{278}$$

Cor. Since the centre Q is assumed at pleasure, it may be made to coincide with the centre C, and then the compound curve becomes a simple curve. Then also, the above formulæ will apply when F' is such that the frog will come on the arc IH. But as FQF'' will be greater than Q, the difference FQF' will be *negative*, indicating that the distance HF' is to be laid off *backwards* from H.

c. The middle track straight beyond F, *and tangent to the curve at F.* Fig. 74.

Let F'' be the required position of the frog F'. A tangent to the curve at F' makes an angle $(F' + F)$ with the main track, and a tangent at F'' makes an angle of F'' with the same; hence the angle they make with each other is

$(F' + F - F'')$, and this is the curvature of the arc $F''F'$, and equals the angle $F''C'F'$.

Produce the straight line $F'H$ backwards to G, and draw $F''G$ perpendicular to it. Then $F''G = FH - F''F \,.\, \sin F$, or

$$F''G = g - F''F \,.\, \sin F \tag{279}$$

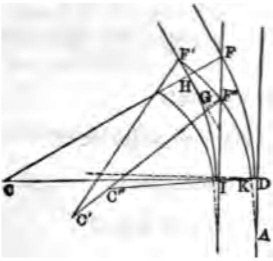

FIG. 74.

In the right-angled triangle $F'GF''$, the angle $F''F'G = F' - \frac{1}{2}(F' + F - F'') = \frac{1}{2}(F' + F'' - F)$.

$$\therefore \qquad F''F' = \frac{F''G}{\sin \frac{1}{2}(F' + F'' - F)} \tag{280}$$

and

$$GF' = F''F' \,.\, \cos \frac{1}{2}(F' + F'' - F) \tag{281}$$

Observe that GF' cannot be less than $GH = F''F \,.\, \cos F$.

193. *Given: a turnout with a frog angle* **F**, *and the perpendicular distance* **p** *between the centre lines of the main and side*

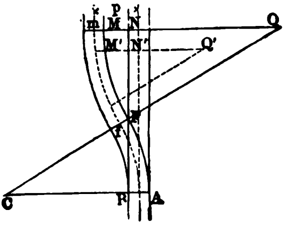

FIG. 75.

tracks; to find the radius **r** *of the* **curve connecting th turnout with the side track.** Fig. 75.

Let the reversing point be taken at F, and let Q on CF produced be the centre of the required curve, and draw QM perpendicular to the main track. Then $QM = QF = r - \frac{1}{2}g$; the point M is the point of tangent, and the angle $FQM = F$.

Now N being the intersection of the rail BF with the radius QM, we have $MN = QF \text{ vers } F$, but $MN = p - g$; hence

$$r - \tfrac{1}{2}g = \frac{p-g}{\text{vers } F} \tag{282}$$

The distance FN is evidently

$$FN = (r - \tfrac{1}{2}g) \sin F \tag{283}$$

and the chord to the centre line is

$$fm = 2r \sin \tfrac{1}{2}F \tag{284}$$

Should the distance FN consume too much of the track, it may be lessened by introducing a short tangent at F, denoted by k; then by eq. (48) the radius will be shortened by an amount equal to $k \,.\, \cot \frac{1}{2}F$, and the distance FN will be shortened by k.

Since the tangent k reduces the length of the tangent offset of the entire curve by $k \,.\, \sin F$, we have for the new radius r'

$$r' - \tfrac{1}{2}g = \frac{p - g - k \sin F}{\text{vers } F} \tag{285}$$

When r' is fixed by a limit, we obtain k by resolving eq. (285)

$$\therefore \quad k = \frac{p - g - (r' - \frac{1}{2}g) \text{ vers } F}{\sin F} \tag{286}$$

In case the main track is but *slightly* curved, we may at first assume it to be straight, and find r as above, eq. (282), and the degree of curve corresponding to r; but this degree of curve must then be *increased* or *diminished* by the degree of curve of the main track, according as the track is *concave* or *convex* toward Q.

194. *Given: the perpendicular distance* **p** *between the centre lines of a curved main track and a parallel side track, and the frog angle* **F** *of a turnout; to find the radius* **r** *of the connecting curve, and the length* **FN,** *or* **fm,** *of the curve.* Fig. 76.

Let FN be the rail of the main track, and GM the rail of the siding, adjacent to each other; let O be the centre of the main track, and Q the centre of the connecting curve. Then the connecting curve will terminate at m, on the line OQ produced.

In the diagram draw MF, and produce it to intersect the rail MG at G, and join GO, FO, and FQ.

Let R = radius of centre line of the main track; r = radius of centre line of the connecting curve; and θ = the angle FOM.

Case a.—*The siding* **outside** *the main track.* Fig. 76.

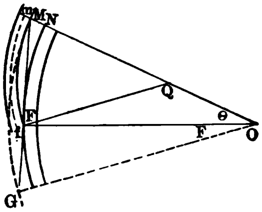

Fig. 76.

By similarity of the triangles GOM and FQM, GO is parallel to FQ, and the angle $GOF = F$; and by a process similar to that of § 186, we have

$$\tan \tfrac{1}{2}\theta = \frac{p - g}{2R + p} \cot \tfrac{1}{2}F \qquad (287)$$

$$r - \tfrac{1}{2}g = (R + \tfrac{1}{2}g) \frac{\sin \theta}{\sin (F + \theta)} \qquad (288)$$

$$FN = 2\,(R + \tfrac{1}{2}g) \sin \tfrac{1}{2}\theta \qquad (289)$$

$$fm = 2r \,.\, \sin \tfrac{1}{2}(F + \theta) \qquad (290)$$

Case b.—*The siding inside the main track.* Fig. 77.

By a process entirely similar to § 187, we have

$$\tan \tfrac{1}{2}\theta = \frac{p - g}{2R - p} \cot \tfrac{1}{2}F \qquad (291)$$

$$r - \tfrac{1}{2}g = (R - \tfrac{1}{2}g)\,\frac{\sin \theta}{\sin (F - \theta)} \qquad (292)$$

$$FN = 2(R - \tfrac{1}{2}g) \sin \tfrac{1}{2}\theta \qquad (293)$$

$$fm = 2r \sin \tfrac{1}{2}(F - \theta) \qquad (294)$$

When $\theta = F$ in the last equations, $\sin (F - \theta) = 0$, and $r - \frac{1}{2}g$ is infinite, and the curve FM becomes a straight line.

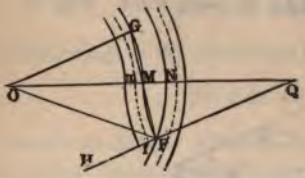

FIG. 77.

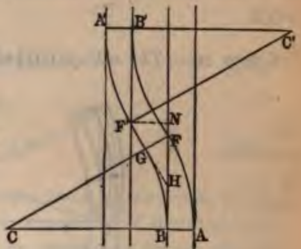

FIG. 78.

When $\theta > F$, $\sin (F - \theta)$ is negative, and the centre Q falls on the same side of the track as O, and we have

$$r + \tfrac{1}{2}g = (R - \tfrac{1}{2}g)\frac{\sin \theta}{\sin (\theta - F)} \qquad (295)$$

$$fm = 2r \,.\, \sin \tfrac{1}{2}(\theta - F) \qquad (296)$$

Equations (291) and (293) remain unchanged.

195. To locate a crossing *between parallel tracks.* Fig. 78.

When a turnout from one track enters a parallel track by means of another frog and switch, the whole is called a crossing. The frogs are alike, and the calculation for one end of the crossing answers for the other. §§ 180, 181. We have only to find the length of track between the two frogs.

In the diagram let AF be one turnout, and $A'F'$ the other, *connected* by the straight track $F'G$. It is required to determine the length $F'G$, or the distance FN measured on the main track from F to a perpendicular through F'. Producing the line $F'G$ to intersect the rail NF at H, we have two right-angled triangles GFH and $F'NH$, having the common angle at $H = F$. Let $p =$ the perpendicular distance between centre lines of main tracks, and $g =$ gauge. Then $GF = g$, and $F'N = (p - g.)$

$$F'G = F'H - GH = \frac{F'N}{\sin F} - GF \cot F$$

or
$$F'G = k = \frac{p - g}{\sin F} - g \cot F \qquad (297)$$

So
$$FN = NH - FH = (p - g) \cot F - \frac{g}{\sin F} \qquad (298)$$

When the main tracks are curved the distance $F'G$ may be calculated by the same formula (297) which gives a value only a fraction too small, but in laying the track the rail $F'G$ must be curved to a *radius* which is to R of the main track as $F'G : NF$.

196. *When* **p** *is large, or the tracks are very wide apart,* it will effect some saving of room *to lay the crossing in the form of a* **reversed curve;** and the frogs being alike, the two arcs will be equal, and the point of reversed curve P will be midway between F and F'. Fig. 79.

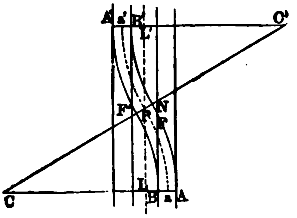

FIG. 79.

In the diagram we have aPa' the centre line of the crossing, and PL the centre line between tracks; $aL = \frac{1}{2}p$, and $aC = a'O' = r$. The radius r having been found by § 180 or § 181, we have

$$\text{vers } aCP = \frac{\frac{1}{2}p}{r} \qquad (299)$$

nd
$$PL = r \sin aCP \qquad (300)$$

The distance between frogs, FN, measured on the main track is evidently

$$FN = 2(PL - BF) \tag{301}$$

in which BF is determined by eqs. (209), (218), or by Tab. XI.

197. *To lay a* crossing *in the form of a* reversed curve, *when the parallel tracks are* on a curve. Fig. 80.

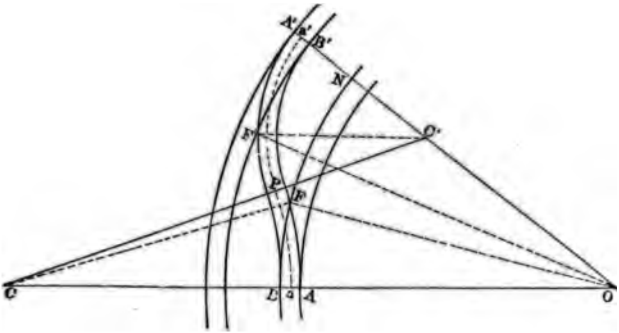

FIG. 80.

Let O be the centre of the main curve, C and C' the centres of the reversed curve.

Then in the triangle COC' we know all three sides; for $CO = R + r$; $CC' = r + r'$, and $C'O = R + p - r'$; and the half sum of the three sides is $s = R + r + \frac{1}{2}p$.

Denoting the angle COC' by φ, we have (Trig. Tab. II. 31)

$$\text{vers}\ \varphi = \frac{p\,(r - r' - \frac{1}{2}p)}{(R + r)\,(R + p - r')} \tag{302}$$

The angle φ determines the length of the arc BN described with the radius $(R + \frac{1}{2}g)$ and so fixes the position of the point A' from A.

By a formula similar to the above,

$$\text{vers}\ C'CO = \frac{p\,(R - r' + \frac{1}{2}p)}{(R + r)\,(r + r')} \tag{303}$$

The angle $C'CO$ determines the length of the arc aP described with the radius r; the angle $(\varphi + C'CO) = CC'A'$ determines the length of the arc Pa', and P is the point of reversed curve.

In this problem R is known, r is found by § 187, and r' is found by § 186, only observing that in this case the value of R must be increased by p. The frog angles F and F' may be equal or otherwise, only taking care that the point P shall be included between the radii $C'F'$ and CF.

The angle $FOC = \theta$ is given by eq. (257), and the angle $F'OC' = \theta'$ is given by eq. (252) (in which the value of R is to be increased by p); hence the angle $FOF' = \varphi - (\theta + \theta')$, which determines the distance between the frogs, measured on the main track.

198. *To find the* **middle ordinate** m, **for 1 station,** *or* 100 *feet, on any curve, in terms of the degree of curve* D.

Referring to Fig. 4 we have in the right triangle AGH

$$GH = GA \,.\, \tan GAH$$

But $GA = \frac{1}{2}AB = \frac{1}{2}C$, and (Tab. I. 18) $GAH = \frac{1}{2}AOB = \frac{1}{2}\Delta$; hence

$$M = \tfrac{1}{2}C \,.\, \tan \tfrac{1}{2}\Delta \qquad (304)$$

a general expression for the middle ordinate of any chord.

If in this equation we make $C = 100$, Δ becomes D; and denoting the corresponding value of M by m, we have

$$m = \tfrac{1}{2}100 \tan \tfrac{1}{2}D \qquad (305)$$

whence the **rule,** *Multiply the nat. tangent of* $\frac{1}{2}$ *the degree of curve by* 100 *and divide by* 2. Thus the values of m in the 5th column of Tab. IV. have been calculated

199. *To find the* **middle ordinate** *for any chord* **in terms of the chord** *and radius*

Referring to Fig. 4 we have

$$GH = OE - OG = OE - \sqrt{AO^2 - GA^2}$$

or

$$M = R - \sqrt{R^2 - \left(\frac{c}{2}\right)^2} \qquad (306)$$

When $C = 100$ we have for the middle ordinate of one station

$$m = R - \sqrt{R^2 - 2500} \tag{307}$$

For any subchord c, less than 100, we have for the middle ordinate,

$$\left.\begin{aligned} m_1 &= R - \sqrt{R^2 - \left(\frac{c}{2}\right)^2} \\ \text{or}\qquad m_1 &= R - \sqrt{\left(R + \frac{c}{2}\right)\left(R - \frac{c}{2}\right)} \end{aligned}\right\} \tag{308}$$

By adding $\frac{c^4}{64R^2}$ to the quantity under the radical in eq. (308) it becomes a perfect square, giving

$$m_1 = \frac{c^2}{8R} \text{ nearly,} \tag{309}$$

which is a very useful formula, although approximate. The error in m_1 does not exceed .002 for any subchord c when the radius is greater than 800. On a 20° curve the error will be .002 for a chord of 50 feet; and on a 40° curve the error in m_1 will be only .003 for a chord of 33 feet. Equation (309) is therefore practically correct in all cases *for finding the middle ordinates of rails.* Table XII. is calculated by eq. (308).

200. Curving Rails. Before any rail is spiked to its place in a curve, it must be evenly bent from end to end, so that it will assume the proper curvature when lying free. The bending may be done by using sledges, but is best accomplished, especially for turnouts and other sharp curves, by using a bending machine made especially for this purpose.

The proper curvature of a rail is tested by measuring its middle ordinate from a small cord stretched from end to end and touching the side of the rail-head. The cord should also be stretched from the middle point of the rail to either end, and the middle ordinate of each half length measured, *to test the uniformity* of curvature.

From the last equation it appears that, with a given radius, *the middle* ordinate varies nearly as the square of the chord

We may therefore find the middle ordinate of a rail whose length is c by the proportion

$$(100)^2 : c^2 :: m : m_1$$

or,
$$m_1 = \frac{c^2\, m}{10000} \text{ nearly,} \tag{310}$$

in which m is obtained from Tab. IV., col. 5, for the given radius or degree of curve.

Example.—What is the middle ordinate of a 30 ft. rail when curved for a 20° curve?

Eq. (310)
$$m_1 = \frac{900 \times 4.374}{10000} = .394 = 4\tfrac{3}{4} \text{ in.}$$

When a long rail is bent for a sharp curve, observe that c is the length of the chord of the rail—not of the rail itself.

For the chord of half a rail the middle ordinate is one-fourth the middle ordinate of the whole rail. Thus, in the above example it would be .099 or $1\tfrac{3}{16}$ inches.

Instead of using the chord of the whole rail, it may be more convenient to assume a chord shorter than the rail, especially when the chord is not an exact number of feet, knotting the string to the length assumed, and applying it to different portions of the rail successively.

201. Elevation of the outer rail on curves.

When a car passes around a curve, a centrifugal force is developed which presses the flanges of the wheels against the outer rail. This force acts horizontally, and varies as the square of the velocity, and inversely as the radius of the curve. Denoting the centrifugal force by f, we have from the theory of mechanics $f = \frac{wv^2}{32.166\,R}$, in which w = weight of loaded car in pounds, v = velocity in feet per second, and R = radius of curve in feet.

In Fig. 81, let ab represent a level line at right angles to the track, let a and c be the tops of rails on a curve, let $bc = e =$ elevation of outer rail c, and let the point d be the centre of gravity of the car. The force f acts in the direction ab, and $f' =$ the component of f in the direction ac, then

$$f' : f :: ab : ac.$$

The weight w, resting on the inclined plane ac, developes a component in the direction ca, and denoting this by w', we have by similar triangles,

$$w' : w :: bc : ac.$$

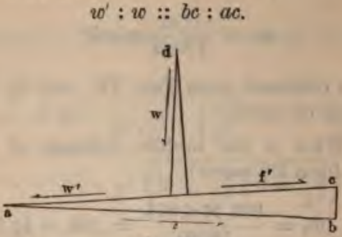

FIG. 81.

Since equilibrium requires that w' shall equal f', we have after dividing one proportion by the other $\frac{f}{w} = \frac{bc}{ab}$, or $f = \frac{w \cdot e}{ab}$. Equating this value of f with that given above we find,

$$e = \frac{ab \cdot v^2}{32.166\,R}$$

But $ab = \sqrt{\overline{ac}^2 - e^2}$, and ac = distance between rail centres = gauge + one rail head $= g + 0.188$. Also $v = \frac{5280}{3600} V$, if V denote the velocity in miles per hour. Making these substitutions and reducing, we have

$$e = (g + .188) \frac{.06688 \frac{V^2}{R}}{\sqrt{1 + \left(.06688 \frac{V^2}{R}\right)^2}} \qquad (311)$$

By this formula Table XIII. is calculated for the standard gauge $g = 4'\ 8\frac{1}{2}'', = 4.708$.

An approximate formula may be obtained by assuming that $ab = g$ for practicable values of e. Substituting this in the first value of e given above, and replacing v by $\frac{5280}{3600} V^2$ we have

$$\text{(approx.)} \qquad e = .06688 \frac{g V^2}{R} \qquad (312)$$

which is the formula generally employed.

In laying a new track, the transverse inclination is first given to the ballast by grade pegs driven either side of the centre line at a distance of $(g + .188)$ each side of the centre; the outside peg being set higher, and the inside peg lower than the grade of ballast on the centre line, by the proper elevation selected from Table XIII. But in re-surfacing an old track, the inner rail is taken as grade and the outer rail is raised the necessary amount.

202. The proper **elevation** may be **found mechanically** by the following method:

To find, on a curved track, the length of a chord whose middle ordinate shall equal the proper elevation of the outer rail for any velocity V in miles per hour.

By the conditions of the problem, we have m_1 in eq. (309) equal to e in eq. (312), or

$$\frac{c^2}{8R} = \frac{g V^2 .06688}{R}$$

$$\therefore \qquad c = .73144\, V \sqrt{g} \qquad (313)$$

When $g = 4.708$,

$$c = 1.587\, V \qquad (314)$$

Lay off the chord, c, upon the rail of the track, stretch a piece of twine between the points so found, and measure the middle ordinate; it will equal the proper elevation.

203. The velocity assumed in the preceding formulæ should be that of the fastest regular trains which will pass over the curve in question, since the flanges would be forced against the outer rail were there no centrifugal force developed, by reason of the wheels being rigidly attached to the axles, and the axles being parallel.

The rails on tangents should be level transversely, except near curves, where for 50 or 100 feet from the curve one rail is gradually raised, so that at the *P.C.* or *P.T.* it may have the full elevation due to the curve. At a *P.C.C.* the elevation should be an *average* of the elevations due to the two arcs. Owing to the difficulty of properly adjusting the elevation of rail, it is objectionable to have arcs of very dissimilar radii join each other; and the objection is much greater in the case of reversed curves unless separated by a short tangent. See § 82

On the other hand, a short tangent between arcs which curve in the same direction should be avoided, since it makes a "flat place" both in line and levels, at once unsightly and injurious to the rolling stock.

In the case of turnouts, however, no elevation of rail is possible (except when both tracks curve in the same direction); hence reversed curves are allowable, the speed of trains being usually quite low also.

204. The coning of the wheels, by which the wheel on the outer rail gains a diameter enough larger than the other to compensate for the superior length of the outer rail, although a theoretically perfect device, is gradually going into disuse. To be effective for the sharpest curves, the coning must be so great as to produce an unsteady motion on tangents, very objectionable at high speeds. Moreover, it is undesirable to seek for an equilibrium of lateral forces in a car on a curve, since the flanges are then sure to strike the inner and outer rails alternately with damaging force, as that equilibrium is momentarily disturbed. It is far better that the flange should press steadily against the outer rail, while that pressure is modified and reduced somewhat by the elevation of the rail. For these and other reasons, car-wheels are now made nearly cylindrical.

CHAPTER VIII.

LEVELLING.

205. The field operations with the Engineer's Level are of a more simple character than those performed with the transit, yet require equal skill and nicety of manipulation in order to produce trustworthy results. The transit is used to ascertain the relative horizontal position of points, the level to obtain their relative vertical position.

206. In order to express the elevation of points, they must be referred to some *level surface* of known (or assumed) elevation; and in order that the elevations may all be positive upward, this surface of reference should be selected below all the points to be considered. The level surface of reference is called **the datum.**

The elevation of the datum is always zero. The elevation of any point is its vertical height above the datum.

Near the coast the sea level is usually adopted as the datum; inland, the low water mark of a river or lake, etc.; but it is not necessary that the *datum* should coincide with a water surface. If any points whose elevations are to be ascertained are below the water surface, the latter may be assumed to have an elevation of 100 or 1000 feet instead of zero; that is, we remove the datum, in imagination, to 100 or 1000 feet below the level of the water surface.

207. In case of a survey commencing at a point quite remote from any important water surface, *any permanent point* may be selected as the original point of reference, and its elevation may be assumed at 100 or any other number of feet; that is, we fix the datum at the same number of feet below that point. The point of reference is called a *bench*, or **bench-mark,** and is designated by the initials B.M. Other benches re established at intervals during a survey, and their eleva- ons determined instrumentally. They are then convenien points of known elevation for future reference. We cannot *assume* the elevation of more than one bench on the same survey, else we should have more than one datum, and all the results would be thrown into confusion.

208. Having established the first bench and recorded its elevation, the next step is to set up the instrument firmly at a moderate distance from the bench, so that the telescope shall be somewhat higher than the bench, and in full view of a rod held vertically upon it. The instrument having been tested for its several adjustments, and found to be correct, the *line of sight* through the intersection of the cross-hairs is known to be horizontal when the bubble stands at the middle of its tube. Turning the line of sight upon the rod, the point of the rod covered by the horizontal cross-hair is known to be on a level with the cross-hair; and the latter is therefore *higher* than the bench by the distance intercepted on the rod from its lower end. Adding this distance to the elevation of the bench, we obtain the elevation of the cross-hair, known technically as the **"Height of Instrument,"** and designated by the initials *H.I.*

209. The distance intercepted on a rod from its lower end by the line of sight, when the rod is held vertically on any given point, is called the **reading** *of the rod* at that point.

210. Having obtained the height of instrument, the elevation of any point somewhat lower than the cross-hair is easily ascertained by taking a reading of the rod upon it. The reading subtracted from the height of instrument gives the elevation of the point above the datum. The elevation of any number of other points may be similarly obtained. But the elevation of points on the ground higher than the cross-hair, or farther below it than the length of the rod, cannot be determined, because in either case the line of sight will not cut the rod, and hence there can be no reading. In order to observe such points, the instrument must be removed to a new position, higher or lower than before, as the case may require.

211. Before the instrument is removed to a new position, *a temporary bench*, called a **Turning Point** (and designated by *T. P.* or "*Peg*") must be established, and its elevation asce tained as for any other point, but with more care. A turning point must be a *firm* and *definite* point whose position cannot readily be altered in the least, nor lost sight of. A small stake firmly driven, or a point of rock projecting upward, is frequently used. The reading having been taken on the turning point, the instrument is carried forward to a new position, levelled up properly, and the new Height of Instrument obtained by a new reading on the same turning point. Since the cross-hair is higher than the point (otherwise there could be no reading) the reading, added to the elevation of the point, gives the Height of Instrument.

212. In general, the intersection of the cross-hairs being higher than any point on which a reading is taken:

To find the Height of Instrument, **add** *the reading on a point to the elevation of the point; and*

To find the Elevation of a point, **subtract** *the reading on it from the Height of Instrument.*

A reading taken for the purpose of finding the Height of Instrument is called a **Backsight** (*B.S*). A reading taken for the purpose of finding the elevation of a turning-point (or of a bench used as such) is called a **Foresight** (*F.S*). Hence Backsights are always *plus*, and Foresights always *minus*.

213. The form of field-book used for the survey of a railroad, or other continuous line, is shown below. The *first* column contains the numbers of the stations on the line and of plus distances to other points on the line where readings are taken—also the initials of benches and turning points, in order, as they occur. The *second* column contains the backsights, taken on points of known elevation only. The *third* column contains the height of instrument, recorded on the same line as the elevation of the turning point (or bench) from which it is calculated. The *fourth* column contains the foresights, taken on new turning points, and benches used as such, only. The *fifth* column contains the readings taken on all other points noted in the first column. The *sixth* column contains the elevations of all points observed. The right-hand *page is reserved for* remarks, descriptive of the benches and *their location—of* objects crossed by the line, as roads, streams, *swamps, ditches, etc.*; the depths of streams, etc.

LEVEL BOOK.

Sta.	B.S.	H.I.	F.S.	Rod.	Elev.	Remarks.
B.M.	4.683	204.683			200.000	White oak, 115 R.
0				2.1	202.6	
1				3.4	201.3	
+ 50				5.2	199.5	
Peg	1.791	197.260	9.214		195.469	
2				3.7	193.6	
+ 25				7.0	190.3	Brook 5 wide; 1 deep
+ 50				3.1	194.2	
3				0.5	196.8	
Peg	11.750	208.574	0.436		196.824	
Peg	11.933	219.528	0.979		207.595	
+ 90				3.5	216.0	
4				2.6	216.9	
B.M.				2.075	217.453	Maple, 78 L.
5				1.7	217.8	
6				0.9	218.6	
Peg	9.005	227.801	0.732		218.796	
7				6.2	221.6	
	39.162		11.361			

When a bench is not used as a turning point, the reading on it is recorded in the fifth column.

The numbers in the second, fourth, and fifth columns come directly from the rod, those in the third are obtained by addition, those in the sixth by subtraction, according to the rule given above. The additions and subtractions made on each page should be *proved* before proceeding to the calculations of the next. When correct, the difference of the sums of the backsights and foresights on the page equals the difference of the first and last elevations on the page. Thus, in the form given

$$(39.162 - 11.361) = (227.801 - 200.000) = 27.801$$

In this proof we ignore all elevations except those of turning points, and benches used as such, and the height of instrument.

At the end of the survey, as well as at the end of each day's work, a bench is established from which the survey may be resumed at any future time See §§ 28, 29, and 80.

214. *The object of making such a survey with level and rod is to furnish a* **profile** or vertical section of the entire *line, showing in* **detail** the rise and fall of the surface over which it passes. The profile is plotted on profile-paper published for the purpose, the horizontal scale being usually 400 feet to an inch, and the vertical scale 30 feet to an inch. This distortion of scale magnifies the vertical measures so that slight changes in the elevation of the surface may be seen distinctly.

215. When only the difference of level of two extreme points is required, the survey is more simple. No readings are taken except on turning-points, the backsights and foresights being recorded in separate columns. No calculation is required until the survey is finished, when—the first reading having been taken on one of the given points, and the last on the other—the difference of the sums of the backsights and foresights is the difference in elevation of the two points, according to the method of proof mentioned in § 213. Thus the difference in level of any two benches established on a previous survey may be tested, and, if found correct, all the intermediate elevations on the line may be assumed to be correct also. The discrepancy should not exceed one tenth of a foot in any case, and is usually much less.

216. Any lack of adjustment in the instrument gives the line of sight a slight angle of elevation or depression, causing a slight error in every reading, proportional to the distance of the rod from the instrument. But the errors being equal for equal distances, and the backsights and foresights having opposite signs in our calculations, the errors cancel when the distances are equal. Hence, to avoid errors in elevation, each new turning-point should be as nearly as possible at the same distance from the instrument as the point on which the last backsight was taken. For precise reading, the rod should not be more than 400 feet from the instrument.

217. Another cause of error in readings is want of verticality in the rod. This may be avoided by the use of a disk-level, or in the absence of wind, by balancing the rod. The rod may be plumbed one way by the vertical cross-hair of the level, and to ensure a vertical reading in the plane of the line of sight, the rod may be gently waved each side of the vertical toward and from the instrument, the *shortest* reading being the correct one; or in case of a target rod, the target should rise *to*, but not *above* the horizontal cross-hair, as the rod is waved.

218. When very long sights are required to be taken with the level, another source of error must be considered, namely, the curvature of the earth.

A level line is parallel to a great circle of the earth, and is therefore an arc of a circle, or may be so considered.

A horizontal line is a straight line parallel to the plane of the horizon. Therefore the line of sight, being a horizontal line, is tangent to the circle of a level line passing through the instrument.

To find the correction in elevation due to **curvature of the earth** *for any distant station.* Fig. 82.

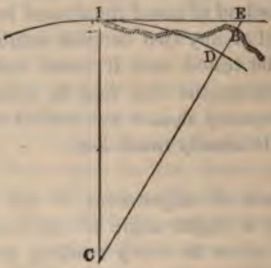

FIG. 82.

Let A be the station of the instrument I, and B the distant station observed.

Let $R_\circ = CI =$ the radius of curvature of the earth, or of the parallel arc ID. Let $L_\circ = ID =$ the level distance between A and B. Then IE, perpendicular to CI, is the line of sight, BE is the reading of the rod, and $DE = E_\circ =$ the correction due to curvature.

By Tab. I., 24, $IE^2 = DE\ (DE + 2R_\circ)$; but since DE is very small compared with $2R_\circ$, it may be omitted from the parenthesis, and since $IE = ID = L_\circ$ very nearly, because *the angle* ACB is very small, we have $L_\circ^2 = 2R_\circ E_\circ$.

$$\therefore \qquad E_\circ = \frac{L_\circ^2}{2R_\circ} \qquad (315)$$

$E_\circ$ *is to be added* to the apparent elevation of station B.

219. Refraction. In observing distant stations the line of sight passing through the atmosphere is refracted from the straight line *IE*, Fig. 82, and takes the form of a curve, which, for practical purposes, may be considered as the arc of a circle, concave downwards. Its radius, depending on the conditions of the atmosphere, varies from 5½ to 7½ times the radius of curvature of the earth. $7R_\circ$ is considered a good average value.

Refraction causes the observed object to appear too high, while the curvature of the earth causes it to appear too low;—the effects being contrary, the correction for curvature is reduced by the correction for refraction. If we let $H_\circ =$ the total correction for both curvature and refraction, to be *added* to the apparent elevation of the observed object, then

$$H_\circ = \frac{6}{7} E_\circ = \frac{3L_\circ^2}{7R_\circ} \tag{316}$$

Table XVII. is calculated by this formula, assuming a mean value of $R_\circ = 20{,}913{,}650$ feet.

220. The form of the earth is approximately an ellipsoid of revolution. Its meridian section at the mean level of the sea is an ellipse, the semi-axes of which are, according to Clarke,

at the equator $A = 6378206$ metres [6.8046985]
at the poles $B = 6356584$ " [6.8032238]

According to the same authority

1 metre $= 3.280869$ feet [0.5159889]

Therefore the semi-axes expressed in feet are

$A = 20\ 926\ 058$ feet [7.3206874]
$B = 20\ 855\ 119$ " [7.3192127]

Then the radius of curvature of the meridian

at the equator, $\frac{B^2}{A} = R_\circ = 20\ 784\ 422$ ft. [7.3177379]

at the poles, $\frac{A^2}{B} = R_\circ = 20\ 997\ 240$ " [7.3221622]

In latitude 40° the radius of curvature of the meridian is 20 871 900, and of a section at right angles to the meridian, 20 955 400; the mean value, or $R_0 = 20\,913\,650$ [7.320430], being adopted for general use. The error in the correction H_0 eq. (316) due to this assumption will usually be much less than that due to the assumed value of the radius of refraction.

221. Levelling by Transit or Theodolite. When a transit has a level-tube attached to the telescope, it may be used as a Theodolite for levelling, and for taking vertical angles. If the instrument be in perfect adjustment, the line of sight will be horizontal when the bubble stands at the middle point of the tube, and the reading of the vertical circle will be zero. Should there be a small reading when the line of sight is horizontal it is called the *index error*. When the line of sight is not horizontal, the angle which it makes with the plane of the horizon is called an angle of elevation, or of depression, according as the object upon which the line of sight is directed is above or below the telescope. This angle is measured on the vertical circle, being the *difference* of the reading and the index error, when both are on the same side of the zero mark, and their *sum*, when they are on opposite sides. When the distance to an observed object is known, and its angle of elevation or depression is measured, we may calculate its vertical height above or below the telescope.

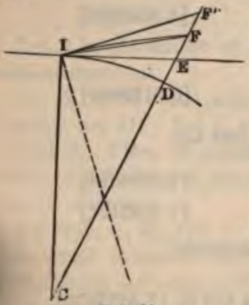

Fig. 83.

Let $\pm\,\alpha$ = angle of $\left\{\begin{array}{l}\text{elevation}\\ \text{depression}\end{array}\right.$
" L = the horizontal distance
" L' = the distance parallel to line of sight
" h = difference in elevation of object and instrument.

Then for short distances,

$$h = L \tan \alpha = L' \sin \alpha \qquad (317)$$

For long distances the curvature of the earth and refraction must be considered. Fig. 83.

Let I be the place of the instrument, and F the object observed.

Let $L_\circ$ = the distance, measured on the chord of the level arc ID, passing through the instrument; and let ψ = the number of seconds in the arc ID; hence, since for ordinary distances the chord and arc are sensibly equal,

$$\psi = \frac{L_\circ}{R_\circ}\, 206264''.8 \qquad [5.314425]$$

or giving to $R_\circ$ its mean value, § 220,

$$\psi = L_\circ \times .0098627 \qquad [7.993995]$$

or a fraction less than 1″ per 100 feet.

Let IF be the arc of the refracted ray, and assuming that its radius is $7R_\circ$, the arc will contain $\frac{1}{7}$th the number of seconds of the arc IF

IF', tangent to IF, is the direction of the telescope; IF is the chord of the arc IF, and IE is the horizontal.

Let $\alpha = EIF'$ = observed angle of elevation. Then EIF = true angle of elevation $= EIF' - F'IF = \alpha - \frac{1}{2} \,.\, \frac{1}{7}\psi = \alpha - .071\psi$.

The angle $EID = \frac{1}{2}\psi \;\therefore\; DIF = \frac{1}{2}\psi + \alpha - .071\psi$; and $IDF = 90° + \frac{1}{2}\psi \,.\; \therefore\; IFD = 90° - (\psi + \alpha - .071\psi)$.

We now solve the triangle IFD for the side $DF = h$, and find

$$h = L_\circ \frac{\sin\,(\frac{1}{2}\psi + \alpha - .071\psi)}{\cos\,(\psi + \alpha - .071\psi)} \qquad (318)$$

For an observed angle of depression make α negative in the formula.

The coefficient .071 is called the coëfficient of refraction, this being a fair average value, while its extreme range is from .067 to .100 under varying conditions of the atmosphere, and values of the angle α.

When the difference in elevation of two or more distant objects is required, we obtain the elevation of each separately, and subtract one elevation from another. The elevation of the observed object is given by (*H. I.*) ± h.

222. *To find the* **Height of Instrument** *of a transit or theodolite* **by an observation of the horizon.** Fig. 84.

Let I be the place of the instrument, and let $\alpha =$ observed angle of depression of the horizon.

Let F be the point where the refracted ray meets the level surface, and draw the chords IF and AF.

Let $\psi =$ the angle ACF, let $h = AI$, and let $k =$ the coefficient of refraction.

In the triangle IAF,

$$IAF = 90^\circ + \tfrac{1}{2}\psi,\ \ AFI = \tfrac{1}{2}\psi - k\psi,\ \ AIF = 90^\circ - (\psi - k\psi)$$

Hence $FIE = \psi - k\psi$. But $FIE = \alpha + k\psi$

$$\therefore \qquad \psi = \frac{\alpha}{1 - 2k} \qquad (319)$$

Let F'' be the tangent point of a right line drawn through I;

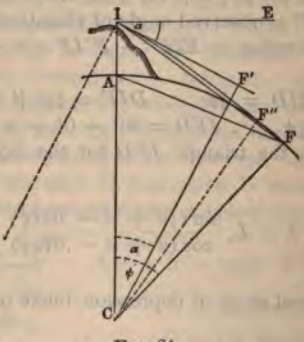

FIG. 84

then $AI = CF''$ exsec ACF'', but $CF'' = R_o$, and, since ψ is always very small, $ACF'' = \tfrac{1}{2}(\psi + \alpha)$ very nearly $= \dfrac{1 - k}{1 - 2k}\,\alpha$

$$\therefore \qquad h = R_o \operatorname{exsec} \frac{1 - k}{1 - 2k}\,\alpha \qquad (320)$$

Giving to R_o its mean value, § 220, and assuming $k = \frac{1}{14}$

$$\log h = 7.320430 + \log \operatorname{exsec} 1.0801\,\alpha \qquad (321)$$

Otherwise, we may solve the triangle AIF since

$$AF = 2R_\circ \sin \tfrac{1}{2}\psi = 2R_\circ \sin \frac{\alpha}{2(1-2k)}$$

and

$$h = AF \frac{\sin(\frac{1}{2}\psi - k\psi)}{\cos(\psi - k\psi)}$$

$$\therefore \quad h = 2R_\circ \sin \frac{\alpha}{2(1-2k)} \cdot \frac{\sin \frac{1}{2}\alpha}{\cos \dfrac{1-k}{1-2k}\alpha} \qquad (322)$$

When $k = \frac{1}{14}$

$$h = 2R_\circ \sin \tfrac{7}{12}\alpha \cdot \frac{\sin \frac{1}{2}\alpha}{\cos \frac{13}{12}\alpha} \qquad (323)$$

Example.—The observed dip of the sea horizon is $24' = \alpha$. What is the height of the instrument above the sea?

$$\begin{array}{llr}
\text{By eq. (321)} & 1.0801 \times \alpha \times 60 = 1555''.34 & 3.191825 \\
 & & 2 \\
\hline
 & & 6.383650 \\
 & \text{Table XXVI.} \quad (q - 2l) & 9.070130 \\
 & R_\circ & 7.320430 \\
\hline
\therefore \quad h = 594.58 & & 2.774210
\end{array}$$

Methods of determining heights by distant observations cannot be relied on for more than approximate results, since they necessarily involve the uncertain element of refraction, and usually a lack of precision in the vertical angle, the arc reading only to minutes in ordinary instruments. These methods, however, are useful where no great accuracy is required, as for a temporary purpose until levels can be taken in the regular way, or for interpolating between points of established elevation.

223. Stadia Measurements.

It is sometimes convenient to determine distances by instrumental observation For this purpose two additional cross-hairs may be placed in the telescope parallel to each other and equidistant from the central cross-hair. These are called stadia hairs, and distances determined by them are called stadia *measurements.* The stadia hairs are adjusted so as to intercept *a certain space* on a rod held at a certain distance from *he instrument* and perpendicular to the line of sight. For any other place of the rod, the distances and intercepted spaces are nearly proportional. The exact relation is given below. Fig. 85.

Let $l = AB$, the distance of the rod from the vertical axis of the instrument; $c =$ the distance from the axis to the object glass of the telescope; $a =$ the distance from the object-glass to the rod; $i =$ the space between the stadia hairs; $s = CD$ the space intercepted by them on the rod; and $f =$ the focal distance of the object-glass. We then have by optics, $\frac{s}{i} = \frac{a - f}{f}$, whence $a - f = \frac{f}{i}s$; and since $a = l - c \therefore l - (f + c) = \frac{f}{i}s$. Now in any given instrument the focal distance f, and the space between the stadia hairs i are constant, while s and c vary with l. For any other distance l', we then have $l' - (f + c') = \frac{f}{i}s'$, and combining the two equations

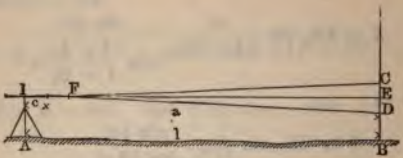

FIG. 85.

$$l - (f + c) = \frac{s}{s'}[l' - (f + c')] \qquad (324)$$

s' is usually assumed at 1 foot and $l' - (f + c')$ at 100 feet. and the stadia hairs are then adjusted accordingly. The focal distance f may be found by removing the object glass and exposing it to the rays of the sun and noting at what distance from the surface of the lens the rays form a perfect and minute image of the sun on a smooth surface; the distance c' is measured on the telescope when the rod is clearly in focus, at the assumed distance.

To measure any other distance, the rod is again observed at the desired point, and the space s noted, which, placed in eq. (324), gives $l - (f + c)$ at once. We then measure c on the telescope, and adding $(f + c)$, obtain l, the distance required.

But inasmuch as c has but a small range of values, it will usually be sufficient to assume for it a mean value, as a constant. In this case we may find the value of $(f + c) = IF$ for the instrument used. Making $c' = c$ in eq. (324), and solving for $(f + c)$, we have

$$f + c = \frac{sl' - s'l}{s - s'} \tag{325}$$

and by laying off on level ground any two distances from the instrument for l' and l, as 100 and 500, and observing the corresponding spaces s' and s intercepted on a rod, we insert them in eq. (325) and find $(f + c)$.

Having found $(f + c)$, lay off $(100 + f + c)$ from the instrument and adjust the stadia hairs to inclose just one foot on the rod at that distance. Any other distance is then found by the formula,

$$l = 100\,s + (f + c) \tag{326}$$

Example.—At $l' = 100$ we find $s' = 1.00$, and at $l = 500$ we find $s = 5.061$.

Hence, eq. (325) $\quad f + c = \dfrac{506.1 - 500}{4.061} = 1.502$

and eq. (326) $l = 100\,s + 1.5$; *provided* the stadia hairs be adjusted so as to intercept 1 foot at 101.5 feet distance from the centre of the instrument.

224. The foregoing formulæ are all that are necessary for horizontal sights, but since the line of collimation is generally inclined more or less to the horizon, it follows that the stadia hairs will intercept a larger space on the **vertical rod** than that due to the true horizontal distance. We therefore require **a formula for reducing inclined measurements to the horizontal.** Fig. 86.

Let $\alpha = EFG =$ the angle of inclination of the line of collimation IG;
" $\theta = CFD =$ the visual angle defined by the stadia hairs;
" $s = CD =$ space intercepted on a vertical rod.

Then (Fig. 85),

$$\tan \tfrac{1}{2}\theta = \frac{C'E}{EF} = \tfrac{1}{2} \cdot \frac{s'}{l' - (f + c)} \tag{327}$$

In Fig. 86

$$s = CE - DE = EF\,[\tan(\alpha + \tfrac{1}{2}\theta) - \tan(\alpha - \tfrac{1}{2}\theta)]$$

while the true value (for the same distance) would be

$$C'D' = 2EF \tan \tfrac{1}{2}\theta$$

Dividing one by the other we derive

$$\frac{C'D'}{s} = \frac{2 \tan \frac{1}{2}\theta}{\tan(\alpha + \frac{1}{2}\theta) - \tan(\alpha - \frac{1}{2}\theta)}$$

By giving to s' and $l' - (f + c)$ in eq. (327) their customary values, *viz.*, 1 and 100, we have $\tan \frac{1}{2}\theta = .005 \quad \therefore\ \theta = 34'\,22''.63$ and by Trig. Table II. 70,

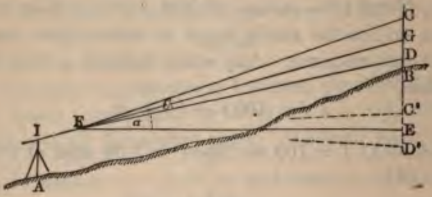

FIG. 86.

$$\tan(\alpha + \tfrac{1}{2}\theta) - \tan(\alpha - \tfrac{1}{2}\theta) = \frac{\sin\theta}{\cos(\alpha + \frac{1}{2}\theta)\cos(\alpha - \frac{1}{2}\theta)}$$

Since θ is small, we have sensibly

$$\sin\theta = 2 \tan \tfrac{1}{2}\theta, \text{ and } \cos(\alpha + \tfrac{1}{2}\theta)\cos(\alpha - \tfrac{1}{2}\theta) = \cos^2\alpha$$

and the last equation reduces sensibly to

$$\frac{C'D'}{s} = \cos^2\alpha \qquad (328)$$

which is the **coefficient of reduction** required by which multiply the observed space s in case of inclined sights. Hence the formula for distance (eq. 326) becomes in this case ithout sensible error

$$l = 100\,s\cos^2\alpha + (f + c) \qquad (329)$$

bles XVIII. and XIX. have been calculated by the exact mula for the coefficient.

Example.—Given: $\alpha = 8° 20'$ and $s = 9.221$; what is the horizontal distance to the rod?

Eq. (329)	100		log. 2.
s	9.221		" 0.964778
α		8° 20′ Tab. XIX.	" 9.990780
	902.7		2.955558
$f+c$	1.5	∴ *Ans.* 904.2 ft.	

The rodman should have a disk level to insure keeping the rod vertical.

225. Another method of procedure is that in which the rod is always held perpendicular to the line of collimation, however much inclined the latter may be. To secure this position of the rod, a small brass bar is attached, having sights upon it through which the rodman watches the instrument during an observation, the line of sight being at right angles to the rod. The distance thus obtained is of course parallel to the line of collimation, and requires to be reduced to the horizontal.

For this purpose, we have (Fig. 87).

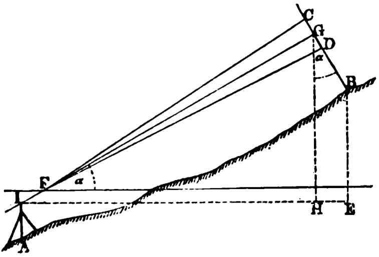

FIG. 87.

$$IE = IG \cos \alpha + BG \sin \alpha$$

or
$$IE = (100\, s + f + c) \cos \alpha + r \sin \alpha \qquad (330)$$

in which r is the reading of the rod by the line of collimation.

For the elevation of the point B above I,

$$EB = HG - GB \cos \alpha$$

or
$$EB = (100\, s + f + c) \sin \alpha - r \cos \alpha . \qquad (3$$

When the distances are sufficiently great, correction must be made for curvature of the earth and refraction, as already explained.

This method is employed by the topographical parties of the U. S. Coast Survey in connection with the plane table. Their instruments, however, are so constructed as to give distances in metres, and heights in feet, requiring a modification of the above formulæ.

CHAPTER IX.

CONSTRUCTION.

226. The engineer department of a railway company is usually reorganized for the construction of the road, as follows: Chief engineer, Division engineers, Resident engineers, Assistant engineers. On some roads the division engineers are styled "Principal Assistants;" the resident engineers, "Assistants;" and the assistant engineers are designated according to their duties, as "leveller," "rodman," etc.

A resident engineer has charge of a few miles of line, limited to so much as he can personally superintend and direct. He has one or more assistants and an axman in his party. All instrumental work is done and all measurements taken by the resident engineer and his assistants.

A division engineer has charge of several residencies, and inspects the progress of the work on his division once or twice a week. In his office, which should be centrally located, all maps, profiles, plans, and most of the working drawings required on his division are prepared. To him the resident engineers make detailed reports once a month, or oftener if necessary, which he passes upon as to their correctness, and from which he makes up a monthly report, or estimate, of the amount and value of the work done and materials provided by each contractor on his division. The estimates are forwarded about the first of each month to the chief engineer, who examines and approves them, returnin for modification any that seem to require it.

The chief engineer has charge of the entire work, and directs the general business of the engineer department. He occasionally inspects the work along the line.

227. Clearing and Grubbing. The first step in the work of construction is to clear off all growth of timber within the limits of the right of way. The resident engineer with his party passes over the line, making offsets to the right and left, and blazing the trees which stand on, or just within, the limits of the company's property. The blazed spot is marked with a letter *C*, as a guide to the contractor. After felling, the valuable timber should be piled near the boundary lines, to be saved as the property of the company. The brushwood is burned.

Where a deep cut is to be made, the stumps are left to be removed as the earth is excavated. In very shallow cuts and fills the contractor will generally prefer to tear up the trees by their roots at once, rather than to grub out the stumps after clearing. Where the embankments will be over three feet high, grubbing is not necessary; but the trees require to be *low-chopped*, leaving no stump above the roots. The engineer should indicate to the contractor the localities where each process is suitable.

228. While the clearing is in progress, the engineer should run a line of test levels touching on all the benches to verify their elevations; he may also rerun the centre line, replacing any stakes that may have disappeared, and setting guard plugs to any important transit points which may not have been previously guarded. If any changes in the alignment have been ordered, these may be made at the same time.

229. Cross Sections. The resident engineer is furnished with a profile of the portion of the line in his charge, upon which is plainly indicated by line and figures the established grade. From this he calculates the elevation of grade at each station, and by subtracting this from the elevation of the surface, he derives the depth of cut or fill (+ or −) to be made at each point. The grade given on the profile is that which is subsequently called the subgrade, being the surface of the road-bed. The final or true grade is the upper surface of the ties after the track is laid.

The base of a cross section is identical with the width of the road-bed. It is made wider in cuts than in fills to allow for the side ditches. Six feet should be allowed in earth, and four feet in rock cuts. **The ratio of the side slopes** depends upon the material. The usual slope ratio for earth is 1½ horizontal to 1 vertical for both excavation and embankment. Damp clay and solid gravel beds will stand for a time in cuts at 1 to 1, or an angle of 45°, but this cannot be permanently depended on. On the other hand, fine sand and very wet clay may require slopes of 1¾ to 1 or 2 to 1. Exceptional cases require slopes of 3 or 4 to 1. In rock work the slopes are usually made at ¼ to 1 for solid, ½ to 1 for loose, and 1 to 1 for very loose rock, liable to disintegrate. Rock embankments stand at 1 to 1.

230. All cross sections are taken in vertical planes at right angles to the direction of the centre line. Figs. 88, 89.

Formulæ.

Let $b = AB$, the base of section, or road-bed.

" $s = \frac{BH}{DH}$ = the slope ratio

" $d = CG$ = the cut (or fill) at the centre stake.

" $h = DH$ or EN = the cut (or fill) at the side stake.

" $x = CD$ = the "distance out" from centre to side stake.

" $y = h - d = KD$.

We have at once from the figures the general formula

$$x = \tfrac{1}{2}b + sh \qquad (332)$$

When the ground is level transversely;

$$h = d, \text{ and } x = \tfrac{1}{2}b + sd.$$

For embankment use the same formula, considering d or h as positive in this case also, the figure being simply inverted.

When the ground is inclined transversely;

$h = CG + DK = d + y$ on the *upper side in cuts;*

$$\therefore \qquad x = \tfrac{1}{2}b + sd + sy \qquad (333)$$

and $h = EN = d - y$ on the lower side in cuts

$$\therefore \qquad x = \tfrac{1}{2}b + sd - sy \qquad (334)$$

For embankments use the same formulæ, but apply eq. (333) to the *lower* side and eq. (334) to the *upper* side, the figure being inverted. The points D and E on the ground are usually found by trial, such that the corresponding values of x and y will verify the formulæ.

When the natural slope FD or LE is uniform its ratio s' may be found by measuring along the section the horizontal distance necessary to change the reading of the rod 1 foot (or half the distance necessary to change it 2 feet, etc.). Then, having found the depths of cut (or fill) at F and L, distant $\frac{1}{2}b$ from the centre C, we have

$$BH = sh = s'(h - BF)$$

and
$$AN = sh = s'(AL - h)$$

From these we have, for the *upper* side in cuts, and *lower* side in fills.

$$h = \frac{s'}{s' - s} BF \therefore x = \tfrac{1}{2}b + \frac{ss'}{s' - s} BF \qquad (335)$$

also, for the *lower* side in cuts, and *upper* side in fills,

$$h = \frac{s'}{s' + s} AL \therefore x = \tfrac{1}{2}b + \frac{ss'}{s' + s} AL \qquad (336)$$

We also have

$$\left.\begin{aligned} h - BF &= \frac{s}{s' - s} BF \\ \text{and} \qquad AL - h &= \frac{s}{s' + s} AL \end{aligned}\right\} \qquad (337)$$

whence the points D and E may be found by the level.

But points D and E thus calculated should have their positions verified by the general formula, eq. (332), lest the slope s' may not have been perfectly uniform.

When the natural surface intersects the base between the points A and B, the section is said to be in side hill work, Fig. 90. Both portions of the section are then determined by eq. (333), or where the slope s' is regular, by eq. (335) measuring in every case from the centre stake C; but observing that when the centre is in *cut* and one side in fill, or *vice versa*, that d must be considered *negative* for that side, whence eq. (333) becomes for this case

$$x = \tfrac{1}{2}b - sd + sy \qquad (333')$$

231. Staking out Earthwork. Beginning at a point on the centre line where the grade cuts the natural surface, the engineer drives a grade stake (marked 0.0) and notes the point in the cross-section book. If the line of intersection of the road-bed and surface would make an acute angle with the centre line, he also finds the points where the *edges* of the proposed road-bed will intersect the surface, drives grade stakes, and also stakes out a cross section through each of those points, if necessary.

Then advancing to the next point on the centre line where a section is required, he finds its elevation with the level (verifying or correcting the elevation taken on the location), calculates the depth of cut or fill CG, which is then marked upon the back of a stake there driven; a *cut* being designated by C and a *fill* by F.

If the ground is level transversely (Fig. 88), he calculates x by eq. (332) and lays off this distance at right angles to the centre line, driving slope stakes at the points D and E, marked with the depth of cut or fill. The marked side of slope stakes should face the centre line.

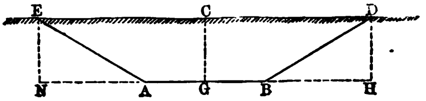

FIG. 88.

If the ground is inclined transversely (Fig. 89), he first measures the distance, $\frac{1}{2}b$, to F, and finds the depth BF for record. He then proceeds to find the point D. If the natural slope be uniform, D may be found by eq. (335) or (337), verifying the result by eq. (332). The point E of the other slope may be found similarly, using eq. (336) or eq. (337); verifying by eq. (332).

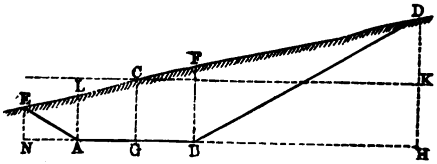

FIG. 89.

232. *If the ground be irregular,* the depth of cut or fill is found not only at the centre and edges of the road-bed, but also at every other *point* along the cross section *where the surface slope changes,* all of which depths are recorded, together with their respective distances from the centre. *To find the point D:* assume a point supposed to be near D, and there take a reading of the rod. The difference of the readings at that point and at C equals y' for that point, which inserted in eq. (333) gives a value x'. If x' agrees with the horizontal distance of the assumed point from C, the true position of D has been found. If x' be *greater* than this, by subtracting the eq. $x' = \frac{1}{2}b + sd + sy'$ from eq. (333) we derive

$$x = x' + s(y - y') \qquad (338)$$

the last term of which shows the correction to be added to x'. Now in advancing from the assumed point to the extremity of x', the rise of the surface is approximately $(y - y')$, and if, in going the additional distance, $s(y - y')$, a further rise is encountered, this last, multiplied by s, must also be added to x', and so on until the additional advance makes no change in the value of y. The point thus found, verified by eq. (332), is the point D required..

But if x' be *less* than the distance of the assumed point from C, we have

$$x = x' - s(y' - y) \qquad (338)'$$

the corrections being subtractive.

The point E on the other slope is found in a similar manner, using eq. (334) for the value of x'; if x' be *greater* than the assumed distance, we have

$$x = x' - s(y - y') \qquad (339)$$

the corrections being *subtractive;* but if x' be *less* than the assumed distance,

$$x = x' + s(y' - y) \qquad (339)'$$

the corrections being *additive.*

233. ***In side-hill* work** (Fig. 90) proceed in the same *manner, using eqs.* (333) or (333)′ and (338) in all cases of un*even ground.* When the surface slope s' is uniform, eq. (335 *may be used, if* preferred, on either side. In addition t centre and side stakes, a grade stake is driven at the point *O*, where the surface intersects the grade, the stake facing down hill.

To find a grade point, set the target to a reading equal to the height of instrument *less* the elevation of grade, and stand the rod at various points along the given line until the target coincides with the line of collimation.

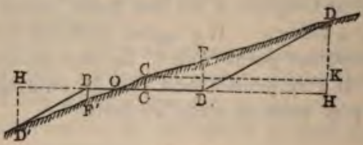

FIG. 90.

234. When two materials are found in the same section, as rock overlaid with earth, each material requires its own slope, and a **compound section** is the result. *To stake out work of this description,* the depth of earth to the rock must be known, and may be nearly ascertained by reference to an adjacent section already excavated. Fig. 91.

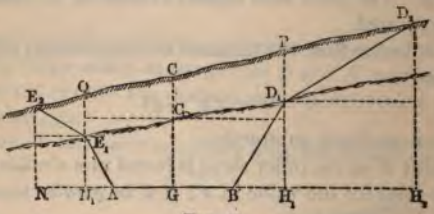

FIG. 91.

Let a_1 be the depth of earth at C
" a_2 " " " P or Q
" s_1 be the ratio of rock slope
" s_2 " " earth slope

Then $$x = \tfrac{1}{2}b + s_1(d - a_1 \pm y_1) + s_2(a_2 \pm y_2) \qquad (340)$$

in which y_1 = difference of rod readings on the *rock* at C_1 and D_1, or C_1 and E_1; and y_2 = difference of rod readings on the *surface* at P and D_2, or at Q and E_2. The upper sign applies to the upper side, the lower sign to the lower.

It is better, however, to make an indefinite cross profile at first, driving two reference stakes quite beyond the section limits; and when the contractor has removed the earth from between D_1 and E_1, indicate to him those exact points by marks on the rock, and also set the slope stakes at D_2 and E_2.

235. The frequency with which cross sections should be taken depends entirely upon the form of the surface; where this is regular, a section at each station is sufficient. A cross section should be taken, not only at every point on the centre line where there is an angle in the profile, but also wherever an angle would be found in the profile of a line joining a series of slope stakes on either side, even though the profile of the centre line may be quite regular at the corresponding point:—the object being, not only to indicate the proper outlines of the earthwork, but to furnish the data necessary to calculate correctly the quantities of material removed. Rockwork will generally require more frequent sections than earthwork.

236. Vertical Curves.—The grades as given on the profile are right lines, which intersect each other with angles more or less abrupt. These angles require to be replaced by *vertical curves*, slightly changing the grade at and near the point of intersection. A vertical curve rarely need extend more than 200 feet each way from that point. Fig. 92.

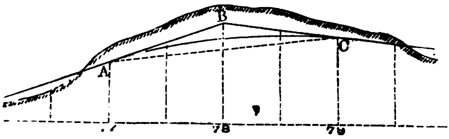

FIG. 92.

Let AB, BC, be two grades in profile, intersecting at station B, and let A and C be the adjacent stations. It is required to join the grades *by a vertical* curve extending from A to C. *Suppose a chord drawn* from A to C;—the elevation of the *middle point of* the chord will be a mean of the elevations of *rade at A and* C; and one half of the difference between t and the elevation of grade at B will be the middle ordinate of the curve. Hence we have

$$M = \tfrac{1}{2}\left(\frac{\text{grade } A + \text{grade } C}{2} - \text{grade } B\right) \qquad (341)$$

in which $M =$ the correction in grade for the point B. The correction for any other point is proportional to the square of its distance from A or C. Thus the correction at $A + 25$ is $\frac{1}{16}M$; at $A + 50$ it is $\frac{1}{4}M$; at $A + 75$ it is $\frac{9}{16}M$; and the same for corresponding points on the other side of B. The corrections in the case shown are *subtractive*, since M is negative. They are *additive* when M is positive, and the curve concave upward.

These corrections are made at the time the cross sections are taken, and the corrected grades are entered in the field-book opposite the numbers of the respective stations.

237. Form of Field-book.—A complete record of all cross-section work is kept in the cross-section book. On the left-hand page is recorded, in the *first column*, the numbers of the stations and other points where sections are taken; *in the second*, the elevations of those points, copied in part from the location level-book, but verified or corrected at the time the section is taken; *in the third*, the elevation of the grade for the same points; *in the fourth*, the width of base b; *in the fifth*, the slope ratios, s; and *in the sixth*, the surface ratio s' when uniform. The right-hand page has a central column, in which, and opposite the number of the station, is recorded the centre depth of the section, marked + or −, to indicate cut or fill, as the case may require. To the right of this are recorded the notes of that portion of the section which lies on the right of the centre line, as the line was run, and to the left, the notes of the left side. The distance from the centre to each point noted is recorded as the numerator of a fraction, and the cut or fill at the point as the denominator, prefixed by a + or − as the case may require. The denominator for a grade point is zero. The numbers of the stations should increase up the page, as in a transit book, so that there may be no confusion as to the right and left side of the line. The several points being noted in order as they occur from the centre outwards, the notes farthest from the centre of the page usually appertain to the slope stakes; but in case the cross profile is extended beyond the slope stake, the note of the latter should be surrounded by a circle to distinguish it. The following form is a specimen of a right-hand page, with the first column only of the left-hand page:

Sta.	Cross					Sections.			
83	$\frac{22.9}{+8.6}$	$\frac{16.5}{+14}$	$\frac{10}{+17.7}$	$\frac{5}{+21}$	$\frac{0}{+21.5}$	$\frac{10}{+20.8}$	$\frac{20}{+25.6}$	$\frac{32}{+28.3}$	$\frac{55.6}{+30.4}$
+ 60		$\frac{17.5}{+5.0}$	$\frac{10}{+10}$	$\frac{4}{+13.2}$	$\frac{0}{+14}$	$\frac{10}{+14.7}$	$\frac{24}{+20.1}$	$\frac{42.6}{+21.7}$	
82			$\frac{14.2}{+2.8}$	$\frac{10}{+5.4}$	$\frac{0}{+9.4}$	$\frac{6}{+8.5}$	$\frac{10}{+11.6}$	$\frac{31.6}{+14.4}$	
+ 38				$\frac{10}{0}$	$\frac{0}{+2.8}$	$\frac{10}{+3.8}$	$\frac{19.3}{+6.2}$		
+ 27					0				
+ 19			$\frac{21.7}{-9.8}$	$\frac{7}{-5.6}$	$\frac{0}{-4.7}$	$\frac{10}{0}$			
81			$\frac{25.9}{-12.6}$	$\frac{7}{-11.2}$	$\frac{0}{-12}$	$\frac{7}{-10.6}$	$\frac{15}{-5.3}$		
80			$\frac{33.4}{-17.6}$	$\frac{7}{-16.4}$	$\frac{0}{-17.6}$	$\frac{7}{-19.6}$	$\frac{18}{-19.1}$	$\frac{25.6}{-12.4}$	

238. In case there is a liability to land-slips, the profiles of cross sections should be carried beyond the slope stakes, on the upper side of the cut, to any distance thought necessary to reach firm ground, and stakes driven for future reference. When a number of consecutive cross profiles are to be considerably extended, it is well to first run, instrumentally, a line parallel to the centre line, and set stakes opposite the stations, taking their elevations. The intermediate surface of the sections may then be taken with cross-section rods if more convenient. See § 37.

239. In case of **inaccessible ground,** preventing a regular staking out, an indefinite profile of the section may generally be *obtained, referred* to the *datum* for elevation and *to the centre line for* position, which being plotted on cross-*section paper, and* the grade line and side slopes added, shows *ale where the* slope stakes should be.

240. Any isolated mass of rock or earth which occurs within the limits of the slope stakes, but not included in the regular notes, is separately measured and noted, so that its contents may be computed and added to the sum of the same material found in the cross sections.

241. Borrow-pits.—When the excavations will not suffice to complete the embankments, material may be taken from other localities, termed *borrow-pits.* These should be staked out by the engineer and their contents calculated, unless the contractor is to be paid for work by embankment measurements. A number of cross profiles are taken of the original surface, and (on the same lines) of the bottom of the pit after it is excavated, which furnish the depth of cutting at each required point. Borrow-pits should be regularly excavated, so that they may not present an unsightly appearance when abandoned. Borrow-pits may be avoided by widening the cut uniformly at the time it is staked out, so that it may furnish sufficient material; *provided* the material is suitable, the embankment accessible, and the distance not too great. When the excavation is in excess, the surplus material should be uniformly distributed by widening the adjacent embankments, if possible; otherwise it is deposited at convenient places indicated by the engineer and is said to be *wasted.*

242. Shrinkage.—In estimating the relative amounts of excavation and embankment required, allowance must be made for difference in the spaces occupied by the material before excavation and after it is settled in embankment. The various earths will be *more* compact in embankment, rock *less* so. The difference in volume is called *shrinkage* in the one case, and *increase* in the other.

Material.	*Shrinkage* in 1000 cu. yds. Of excavation.	 Of settled embkt.
Sand and gravel	80 C. Yds.	87 C. Yds.
Clay	100 "	111 "
Loam	120 "	136 "
Wet soil	150 "	200 "
	Increase in 1000 cu. yds.	
Rock, large fragments	600 C. Yds.	375 C. Yds.
" *medium fragments*	700 "	413 "
" *small* "	800 "	444 "

Thus, an excavation of sand and gravel measuring 1000 cubic yards will form only about 920 cubic yards of embankment; or an embankment of 1000 cubic yards will require 1087 cubic yards of sand or gravel measured in excavation to fill it; but will require only 587 cubic yards of rock excavation, the rock being broken into medium-sized fragments; while 1000 cubic yards of the latter, measured in excavation, will form 1700 cubic yards of embankment.

The lineal settlement of an earth embankment will be about in the ratio given above, therefore the contractor should be instructed in setting his poles to guide him as to the height of grade on an earth embankment, to add 10 per cent (average) to the fill marked on the stakes. In rock embankments this is not necessary. The engineer should see that all embankments are made full width at first, out to the slope stakes, and by measure at or above grade, so that the whole may settle in a compact mass. Additions to the width made subsequently are likely to slide off.

243. The cross-section notes should be traced in ink at the first opportunity to secure their permanence. An office copy should also be made to serve in case of loss or damage to the original.

244. Alteration of Line.—Inasmuch as the centre line at grade is the base of reference for all measurements and calculations in earthwork, any change made in it after the work of grading has begun should be most carefully recorded and explained. The centre stakes of the old line should be left standing until after the new line is established, so that the perpendicular offset from the old line to the new, at each station, may be measured, as also the distance that the new station may be in advance of, or behind the old one. The *date* of the change should be recorded. The original cross sections are extended any amount requisite, the distance out being still reckoned from the *old* centre, while a marginal note states the amount by which the centre has been shifted.

The difference in length of the lines will make a long or short station at the point of closing. The exact length of such a station should be recorded, so that it may be observed in re-racing the line at any time, and in calculating the quantity of earthwork. The original transit notes of the altered line should be preserved, but marked as "abandoned," with a reference to the notes of the new line on another page.

245. Drains and Culverts.—The engineer should examine the nature and extent of each depression in the profile with reference to the kind of opening required for the passage of water. For small springs, and for a limited surface of rainfall, cement pipes, in sizes varying from 12 to 24 inches diameter, serve an excellent purpose as drains. These are easily laid down, and if properly bedded, with the earth tamped about them, are very permanent; but their upper surface should be at least 2½ feet below grade. The embankment is protected at the upper end of the drain by a bit of vertical wall, enclosing the end of the pipe. If necessary, a paved gutter may lead to it.

Where stone abounds, the bed of a dry ravine may be partly filled with loose stone, extending beyond the slopes a few feet, which will prevent the accumulation of water.

When the flow of water is estimated to be too great for two lines of the largest cement pipe, or when the embankment is too shallow to admit them safely, a culvert is required. A pavement is laid one foot thick, protected by a curb of stone or wood 3 feet deep at each end, and wide enough to allow the walls to be built upon it. It should have a uniform slope, usually between the limits of 50 to 1 and 100 to 1 to ensure the ready flow of water. In firm soils the foundation pit is excavated one foot below the bed of the stream, but if mud is found this must be removed and the space filled with riprap, the upper course of which is arranged to form the pavement at the proper level. In a V-shaped ravine, requiring too much excavation at the sides, and where the fall is considerable, riprap may be used to advantage, the bed of the stream above the culvert being graded up by the same material to meet the pavement. In some cases a curtain, or cross wall, is necessary on the lower end to retain the riprap.

Culverts should be laid out at right angles to the centre line whenever practicable, the bed of the stream being altered if necessary. The length of an open culvert is the entire distance between slope stakes, the walls being parallel throughout, or the length may be taken somewhat less than this, and the walls turned at right angles on the upper end, forming a facing to the foot of the slope. The walls are carried up to grade for the width of the road-bed, and are stepped down to suit the slopes. A course is afterwards added to retain the ballast.

In box culverts the span varies from 2 to 5 feet, the height in the clear from 2 to 6 feet; the thickness of walls from 3 to 4 feet; the thickness of cover from 12 to 18 inches, and its length at least 2 feet greater than the span. The walls terminate in short head-walls built parallel to the centre line, the top course being a continuation of the cover. The length of a head-wall, measured on the outer face, is equal to the height of the culvert in the clear multiplied by the slope ratio of the embankment. The perpendicular distance from the centre line to the face of a head-wall is equal to one half the road-bed, *plus* the depth of the top of the wall below grade multiplied by the slope ratio, or $\frac{1}{2}b + sk$. A coping is sometimes added.

246. Arch culverts are used when the span required is more than 5 feet, and the embankment too high to warrant carrying the walls up to grade as an open culvert. The span varies from 6 to 20 feet; the arch is a semicircle, the thickness varying from 10 or 12 inches to 18 or 20 inches. The height of abutments to the springing line varies from 2 to 10 feet, the thickness at the springing line from 3 to 5 feet, and at the base from 3 to 6 feet, the back of the abutment receiving the batter. The foundations are laid broader and deeper than in box culverts, each abutment having its own pit, carried to any depth found necessary. The half length of the culvert is $\frac{1}{2}b + sk$, in which k is the depth of the crown of the arch below grade. The abutments are carried up half way from the spring to the level of the crown of the arch, and thence sloped off toward the crown. The face walls are carried up to the crown, and coped. The wing walls stand at an angle of 30° with the axis of the culvert; they receive a batter on the face, and are stepped (or sloped) down to suit the embankment. Their thickness, at the base, is the same as that of the abutment; at the outer end 3 feet. They stop about 3 feet short of the foot of the slope. They need not be curved in plan.

Any stone structure of dimensions greater than those given above, scarcely comes under the head of culverts, and should be made the subject of a special design by the engineer.

247. Staking out Foundation Pits.—For box culverts.—The engineer having decided upon the size of culvert required, makes a diagram of it in plan, on a page of his masonry book, recording all the dimensions, stating the station and *plus* at which its centre is taken, the span and height of the opening, etc. He then sets the transit at the centre *A*, Fig. 93, measures the angle between the centre line and axis, (making it 90° if practicable); on the axis he lays off the distances to the ends of the culvert and drives stakes at *B* and *C*. Perpendicular to *BC* he lays off the half widths of the pit, setting stakes at *D* and *E*, and laying off *DF* and *EH* = *AB*; and *DG* and *EI* = *AC*. On *IG* produced he lays off *CJ* = *CK*, and perpendicular to this *JM* and *KL*, and finds the intersections *O* and *N*. A stake is driven at each angle, and upon it is marked the cut required to reach the assumed level for the oundation. These cuts are recorded on the corresponding ngles of the diagram. The pit is thus no larger than the lan of the proposed masonry, and the sides are vertical, which nswers the purpose for shallow pits.

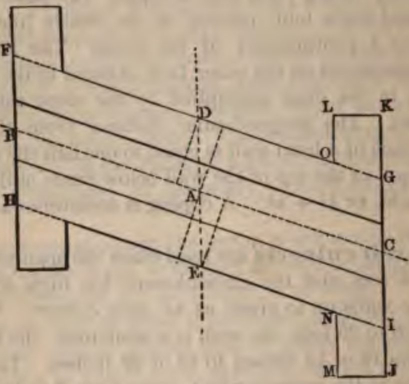

FIG. 93.

For arch culverts.—The pit for each abutment when shallow may be of the same dimensions as the lower foundation course: if more than five feet deep, it should be enlarged by an extra space of one foot all around. In Fig. 94 the inside lines show the plan of the abutments at the neat-lines; the outside lines represent the pits. Having prepared a plan of the structure suited to the locality, and made a diagram of the same in the masonry book, set the transit at *A*, and drive stakes at *D*, *E*, *N* and *O* on the centre line. Then turning to the axis *BC*, lay off *AC*, and set stakes at *G* and *I*. With *G* as a centre, and a radius equal to 2*DE*, describe on the ground an arc cutting *EI* in *X* or ($IX = DE \cdot \cot 30°$) may be calculated; and on *XG* produced lay off *GK*, and perpendicular to this, *KL*. From *N* lay off *NP*, parallel to *AC*, and measure *PL* as a check. Drive a stake at each angle, marked with the proper cutting, and record the same on the diagram. The locality may require the wings to be of different lengths and angles, of which the engineer will judge. Guard-plugs should be driven in line with the intended face of one or both abutments, so that the neat-lines can be readily given when required. In case the material is not likely to stand vertically, the pit must be staked out with sloping sides, as described below.

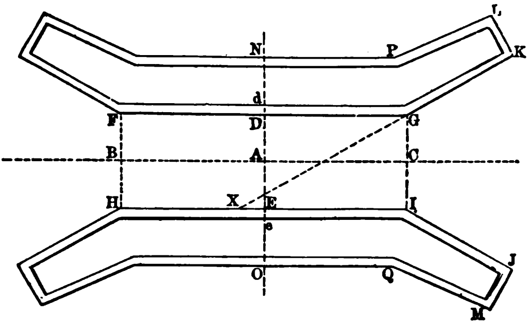

FIG. 94.

For bridge abutments.—A design for every important structure is usually prepared in the office after a survey of the site. The foundation pit is then laid out from dimensions furnished on a tracing, but a diagram of the pit should be made *in the masonry book* as usual. *When the bridge is on a tangent, Fig. 95, set the transit at A* on the centre line at its *intersection with the axis BC* of the abutment *at the level of* the seat.

Deflect from the tangent the angle giving the direction of BC, and lay off AC, AB, setting plugs at B and C, and reference plugs (two on each side) on BC produced. After staking out the sides of the pit parallel to BC, set the transit at C, and deflect the angle for the wing, laying off CD, and driving stakes at the corners E and F. Two reference points are then set on the line CD produced. The other wing being staked out in the same manner, the cut is found at each stake and marked and recorded. Cross sections are then taken near each corner, perpendicular to each side, and slope stakes (marked "slope") are driven where the slope runs out. Intermediate sections are taken when the unevenness of the ground makes it necessary, and the lines joining the slope stakes are produced to intersect, and other stakes are driven at the intersections. The position of each stake is shown on the diagram, and the cut recorded.

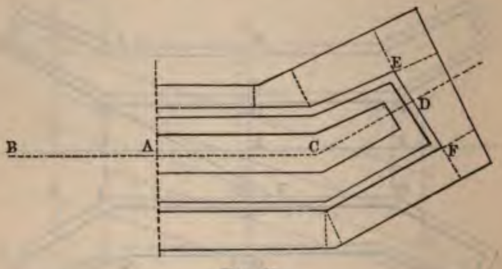

FIG. 95.

A slope of 1 to 1 is usually sufficient for pits. If the material will not stand at 1½ to 1, or if space cannot be spared for the slope, the sides may be carried down vertically, supported by sheet piling braced from within.

The reference points should be so chosen that the points A, B and C may be found by intersection, on any course of the masonry, during the progress of construction.

When the bridge is on a curve, the bridge-chord *should be found* and the abutments laid out from this. Fig. 96. *The bridge-chord* is a line AB, midway between the chord of *the curve* CD, *joining* the centres of the abutments, and a tangent *to the curve* at the middle point of the span. Hence

$CA = DB = \frac{1}{2}MN$, which may be laid off, and A and B are the true centres of the abutments, from which the foundations are staked out as before.

The distance $CE = DF$ to the points where the bridge-chord cuts the curve is $0.147CD$.

Should an abutment site on a curve be inaccessible, as when under water, from any transit point P on the curve lay off PX perpendicular to the tangent at M, observing that

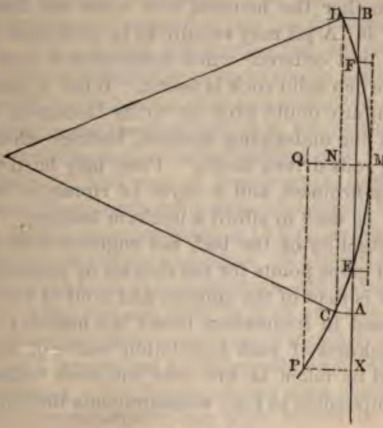

FIG. 96.

$$PX = MQ - AC = R\left(\text{vers}\,PM - \tfrac{1}{2}\,\text{vers}\,CM\right)$$

and $$AX = PQ - \tfrac{1}{2}AB = R\left(\sin PM - \tfrac{1}{2}CD\right)$$

The point A may then be found by intersection, or by direct measurement with a steel tape or wire, driving a long stout stake to show the point above the water. Other points may then be approximately found, sufficient to begin operations.

In case of a bridge of several spans, the piers are laid out in the same manner, from a centre point and axis. If on a curve, each span has its own bridge-chord, but for convenience, the centre of a pier may be taken on the centre line during its construction, and the bridge-chord only found for the purpose of placing the bridge; the piers being long enough to allow of the shift.

To locate the centres of piers, a base line is required on one or both shores, and two transits are used to give the intersections by calculated angles. When practicable the spans should also be measured with a steel tape or wire.

The bed of a pit for any sort of structure should receive the closest scrutiny of the engineer, it being his duty to judge whether the material will resist the load to be imposed upon it. A pit may require to be excavated to a greater depth than first ordered, while sometimes a less depth will answer, as when solid rock is found. When a good material is reached, if any doubt exist as to its thickness, or as to the character of the underlying stratum, borings should be made or sounding rods driven down. Piles may be driven to gain the requisite firmness, and a layer of riprap, of beton, or of timber may be used to afford a uniform bearing. When satisfied of the stability of the bed, the engineer finds the original centres, and gives points for the courses of masonry. A complete record is kept of the amount and kind of excavation, the materials used in foundation under the masonry, and of the size and thickness of each foundation course of masonry; the notes should be taken at the time the work is done, it being generally impossible to take measurements thereafter.

248. Cattle-guards are shallow pits placed at right angles across the road at the fence lines to prevent the passage of cattle. They are either entirely open, in which case they should be at least 4 feet deep, or they are covered in part with wooden rails laid a few inches apart. The open guard is preferred. It is built like an open culvert except that no pavement is required. The stringers carrying the rails over any opening should be no longer than the span *plus* the thickness of the walls.

249. Trestle Work.—No wooden culverts should ever be used. If stone cannot be had at first, two trestle bents may be erected, leaving between them a space sufficient to contain the stone structure to be built when the material for it can be brought by rail. The bents may be backed by plank to retain the embankment, and the stringers are then notched down an inch on the caps to receive the pressure of the earth, and render the bents mutually sustaining. The sills are prevented from yielding to the pressure of the earth by being sunk in a trench, or by sheet piling. Should the span be too long, a central bent may be used, so as not to interfere with building the wall. Sometimes pile-bents may be used with greater advantage, the piles being driven in rows of four each, and capped to receive the stringers. In districts where suitable stone is entirely wanting, pile or trestle abutments and piers are used for the support of bridges, the piles or posts being arranged in groups and capped to receive the direct weight of the trusses. They should not sustain the embankment, but should be connected with it by a short trestle work.

Trestle work is frequently used as a substitute for embankment, either to lessen the first cost, or to hasten the completion of the line, or for lack of suitable material with which to form an embankment. The cost of trestle work, however, is not less than that of an earth embankment formed from borrow pits, unless its height exceeds about 15 feet, depending on the relative prices of materials and labor. When not exceeding 30 feet in height, the *bents*, for single track, are usually composed of two posts, a cap and sill, each 12 × 12, and two batter posts, 10 × 12, inclined at ¼th to 1, all framed together. Two lengths of 3-inch plank are spiked on diagonally on opposite sides of the bent as braces. The length of the caps should equal the width of the embankment; the posts should be 5 feet from centre to centre, and the batter posts 2 feet from the posts at the cap. The sill should extend about two feet beyond the foot of the batter post. A masonry foundation for the bent is preferable, though pile foundations are not uncommon, and some temporary structures are placed directly on a firm soil, supported only by mudsills laid crosswise under the sill. The spans, or distance between bents, may vary from 12 to 16 feet. The stringers should consist of 4 pieces, 2 under each rail, bolted together, with packing blocks to separate them 2 or 3 inches. Over each bent and at the centre of each span a piece of thick plank about 4 feet long should be placed on edge between the two pair of beams to preserve the proper distance between them, while rods pass through the beams and strain them up to the ends of the plank, to increase the stability of the beams and prevent their buckling under a load. The stringers should be able to carry safely the heaviest load without bracing against the posts. The bents, however, if high, must be braced against each other. The stringers should be continuous, the two pieces breaking joints with each other at the bents, to which they are firmly bolted. They may rest directly on the caps, or corbels may intervene. The spans on a curve should be shorter than on a tangent. The ties should be notched down to fit the stringers closely, and guard rails, either wood or iron, secured to them firmly. Unless the spans are very short, horizontal bracing should be employed consisting of 3-inch plank, extending from the centre of each span to the ends of the caps, which are notched down to receive the plank.

For trestles much higher than 30 feet **the cluster bent** is preferable, so termed because each vertical post is composed of a cluster of four pieces, 8 × 8, standing a little apart to allow the horizontal members to pass between them. The verticals are continuous, breaking joints, two and two, while the horizontals pass the posts and are bolted to them at the joints; the framing is accomplished entirely by packing blocks and bolts. The batter posts consist each of two pieces 8 × 8; the horizontals may be 4 × 10, and extend not only across the bent, but from one bent to another. Proper bracing is also used in every direction. When very high, a secondary pair of batter posts may be introduced in the lower part of the structure. The batter need not exceed ⅕th to 1. In some instances two adjoining bents are strongly braced together, forming a tower or pier, and the piers placed from 50 to 100 feet apart, the roadway being carried on trussed bridges. The cluster bent admits of any piece being removed and a new one inserted when necessary.

Iron trestles are now adopted where a permanent structure is desired. Owing to the expansion of the metal by heat, the bents cannot be continuously connected with each other as in a wooden trestle; hence the pier form is resorted to, having spans varying from 30 to 150 feet, covered by trussed bridges, and the whole structure is more properly styled a viaduct.

250. Tunnels. Tunnels are adopted in certain cases to avoid excessive excavations, steep grades, high summits, and circuitous routes. Their disadvantages are the increased time nd cost of their construction compared with an open line, and their lack of light and fresh air when in use. It is desirable that they should be on a tangent throughout, both for the admission of light and for convenience of alignment. Many tunnels, however, have been built with a curve at one or both ends.*

The location of a tunnel, other things being equal, should be such as to make not only the tunnel proper, but also its immediate approaches by open cut as short as possible; and the latter should be selected so as not to be subject to overflow, nor liable to land slides. The material to be encountered may frequently be determined with tolerable accuracy by a study of the geological formation in the vicinity, or by actual borings. The most favorable material for tunnelling is a homogeneous self-supporting rock, devoid of springs, which does not disintegrate on exposure to the atmosphere. The worst materials are saturated earth and quicksands. The presence of water in any material increases the cost considerably.

The alignment of a tunnel is made the subject of special survey, after the general location is decided, and this is more or less elaborate according to the length of tunnel. A permanent station is established at the highest point crossed by the tunnel tangent, from which, if possible, monuments are set in each direction at points beyond the ends of the tunnel. If there are two principal summits, stations on these will define the tangent, which may then be produced. The monuments established beyond the tunnel should be sufficiently distant to afford a perfect backsight from the ends of the tunnel, where other monuments are also established. The first quality of instruments only should be used, and these perfectly adjusted, and the observations should be repeated many times until it is certain that all perceptible errors are eliminated. Since the line of collimation will be frequently inclined to the horizon at a considerable angle, it is important that it should revolve in a vertical plane; and to secure this, a sensitive bubble tube should be attached to the horizontal axis, at right angles to the telescope of the transit. The distance may be obtained by triangulation, though direct measurement is to be preferred. A steel tape is convenient and accurate, providing that allowance be made for variations due to temperature, from an assumed standard. The rods described in § 43 may be used instead of plumb lines, the tape being held at right angles to them, and therefore horizontal. A plug should be driven for each rod to stand on, and a centre set to indicate the line and measurement.

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* *The Mont Cenis tunnel*, requiring a curve at each end, was first *opened on the tangent* produced, giving a straight line through, and the *curves were excavated* subsequently.

As the excavation of the tunnel proceeds, the centre line is given at short intervals by points either on the floor or roof. Overhead points are generally preferred, from which short plumb lines may be hung, constantly indicating the line, with little danger of being disturbed. When a new transit point is required in the tunnel, it should be established directly under an overhead point, which serves as a check upon its permanence, and as a backsight when needed.

Shafts are sometimes opened to give access to several points of the tunnel at the same time, thus facilitating the work, though at an increased cost. They also serve for ventilation during the progress of the work, though they are worse than useless for this purpose afterward, except possibly in the case of a single shaft near the centre of the tunnel. Some of the longest tunnels have been formed without shafts, while many shorter ones have had several, which have generally been closed after the tunnel was completed. Shafts are either vertical, inclined, or nearly horizontal; in the latter case they are called adits. Inclined shafts should make an angle of at least 60° with the vertical. Vertical shafts may be either rectangular, round, or oval. Their dimensions vary, depending on their depth and the material encountered, between 8 and 25 feet. They are usually sunk on the centre line of the tunnel, though sometimes at one side. When over the tunnel the alignment below is obtained directly from two plumb lines of fine wire suspended on opposite sides of the shaft from points very carefully determined at the surface. The plummets are suspended in water to lessen their vibrations, and as soon as the transit can be set up at a sufficient distance to bring the lines into focus, it is shifted by trial into exact line with the mean of their oscillations, the latter being very limited. Permanent points may hen be set, but should be repeatedly verified. As soon as the vorkings from a shaft communicate with those from either nd, or from another shaft, the alignment thus found is ested, and revised if necessary. These operations require the greatest nicety of observation and delicacy of manipulation to obtain satisfactory results.

From plumb lines in the central shaft of the Hoosac tunnel, the line was produced three tenths of a mile, and met the line produced 2.1 miles from the west end with an error in offset of five sixteenths of an inch. In the Mont Cenis tunnel the lines met from opposite ends with "no appreciable" error in alignment, while the error in measurement was about 45 feet in a total length of 7.6 miles.

When **a curve** occurs in a tunnel it is usually near one end. The tunnel tangent is produced and established as before described, and a second tangent from some point on the curve outside the tunnel is produced to intersect it, the intersection being precisely determined and the angle measured with many repetitions. The tangent distances are then calculated, and the position of the tangent points corrected by precise measurements, and permanent monuments are established. As the tunnel advances, points may be set at short intervals on the curve in the usual manner; but at intervals of 100 feet the regular stations should be defined with finely centred monuments, using a 100-foot steel tape carefully supported in a horizontal position. When it is necessary to use a subchord, its exact length should be calculated as shown in § 107. When the curve has advanced so far as to render a new transit point necessary, this should be established at a full station. The subtangents from the two transit points should then be produced to intersect, and measured for equality with each other and with their calculated length. The distance from their intersection to the middle of the long chord should also be measured as a check on the deflections. When no perceptible errors remain, the curve may be produced as before until the *P.T.* is reached. It is evident that correct measure is indispensable to correct alignment on curves. Should obstacles on the surface necessitate triangulation, more than ordinary care must be exercised, and as many checks introduced as possible. The triangles should be so arranged that all of the angles and most of the sides may be measured.

Test levels are carried over the surface with great care, each turning point being made a permanent bench, and its elevation determined with a probable error not exceeding 0.005 foot. Levels may be carried down a shaft on a series of bolts or spikes about 12 feet apart in the same vertical line, the distances being measured by the same level-rod as that with which the benches are determined. The measures should be taken between two graduations of the rod, not using the end of the rod, which may be slightly worn. Fine horizontal lines on the heads of the bolts may be used to mark the exact distances. After the shaft reaches the level of the tunnel, the depth may be measured more directly with a steel tape, the entire length of which has been corrected at the given temperature, by comparison with the same rod.

If the **grade of a tunnel** is to be continuous, it should be assumed at something less than the maximum of the road, but not less than 0.10 per station, which is required for drainage. If a summit is to be made in the tunnel, the grade from the upper end should not exceed 0.10 per station. Grades are given in the tunnel from day to day, or as often as required by the progress of the work, the marks being made on the sides at some arbitrary distance above grade. Turning points should be taken on permanent benches.

The least width of a tunnel in the clear should be, for single track about 15 feet, and for double track 26 feet. **The least height** in the clear above the tie should be 18.5 feet for single track, and 16.5 feet at the outside rails for double track, allowing for tie and ballast; the roof at the centre of the section should be at least 20 feet above subgrade, and with a full centred arch 22 or 23 feet for double track. **The form** of section depends somewhat on the material traversed. In perfectly solid rock a nearly rectangular section may be used, the roof being slightly rounded. In dry clay, and stratified rock, a flat arch may be used, and in other cases a full-centred arch. The latter form is rather to be preferred on account of the better ventilation afforded. The sides are made vertical, battered or curved, as necessity or taste may dictate. In wet and infirm soil an invert floor may be required, otherwise it is made level transversely. When a lining is required the original section must of course be made large enough to allow for the masonry, and the temporary timber supports behind it. Hard burned brick is usually adopted for arching, being durable and easily handled. In loose rock the arching may be from 13 to 26 inches thick, in wet and yielding soil a thickness of from 26 to 39 inches may be necessary. The walls may be from 2½ to 6 feet thick.

In forming a tunnel, **a heading** or gallery of small cross section is first driven and afterwards enlarged to the full size required. In firm clay or loose rock which will temporarily support itself until the masonry can be put in, it is better to drive the heading along the floor (at subgrade) of the tunnel, the remaining material being then easily thrown down in sections as the arching is advanced. In solid rock, or wet earth, a top-heading (along the roof) is generally preferred. The dimensions of a heading driven by hand are usually 8 feet high by 8 or 10 feet wide, but in solid rock where drilling machinery is introduced, it is advantageous to make the heading as wide as the tunnel at once. By drilling holes into the face at points about five feet each side of the centre, and converging on the centre line at a depth of about ten feet, a triangular mass of rock may be blown out, and the space thus gained facilitates the blasting of the adjacent rock on either side. An advance of about 10 feet in each day of 24 working hours may thus be made, using nitroglycerine in some form as the explosive agent. Owing, however, to unavoidable delays from various causes, this rate of progress cannot always be maintained. At the Hoosac tunnel the greatest advance in one week was 50 feet; in one month 184 feet at one heading. At the Musconetcong tunnel a heading 8 × 22 feet in syenitic gneiss was advanced at the average rate of 137 feet per month for 6 months, the maximum being 144 feet —the enlargement of the tunnel to full size going on at the same time, a few hundred feet behind. At the St. Gothard tunnel the north heading 2.5 × 3 metres was advanced in mica gneiss, during the year 1875 at the average daily rate of 3.71 metres, with a maximum of about 4 metres, but the enlargement was not made. The south heading advanced at the rate of 2 metres a day, timbering being at times necessary.

In ordinary clay a heading may be driven at from 75 to 180 ft. per month, according to circumstances, where timbering is put in. The enlargement, including timbering and masonry, may be advanced at from 20 to 60 ft. per month. Small tunnels for water conduits are driven through dry clay at the rate of 10 ft. per day, the masonry following at once without timbering.

The compressed air used to drive the drilling machinery *serves to supply* ventilation also. When this is wanting or *proves insufficient,* exhaust fans are used. At Mont Cenis a horizontal *brattice* or partition was built in the tunnel, dividing it so as to secure a circulation of air. When foul gases are encountered, ventilation becomes a serious question, and in one instance an important work was abandoned for this cause.

Cross sections of the heading, and also of the tunnel en largement, should be measured at intervals of about 20 feet, as soon as opened, to see that the sides, roof, and floor are taken out to the prescribed lines, at the same time that the latter are exceeded as little as possible. In solid rock, since some material outside of the true section will necessarily be thrown down, leaving an irregular outline, it is well to take two cross sections at the same point, one following the projections and the other the recesses of the rock, from which an average section may be estimated. A daily, or at least a weekly, record of operations should be kept in tabular form, and the progress indicated by a profile and cross sections drawn on a sufficiently large scale to show details.

The drainage of a tunnel is best secured by a line of stoneware or cement pipe laid in a trench along each side, and covered with ballast or other loose material. The entire floor is thus made available for the use of the trackmen. When an invert is used, the drain is placed in the centre between tracks. If the amount of water is large, drain pipe may be laid behind the walls, and the back of the arch may be covered with asphaltum, or coal tar, to prevent a constant dripping on the track.

251. Retracing the Line.—As the grading progresses, in either excavation or embankment, the principal transit points are established on the road-bed from the points of reference, and the centre line is retraced, setting stakes at every 50 feet. Transit points on grade should be fixed upon stout, durable posts firmly set in the ground, and standing high enough to be easily reached after the ballast is laid. To recover the old line, any discrepancies in measurement must be left between the transit points where they occur, and not carried forward. In retracing a curve, if the transit *is placed at* the forward point, allowing the chain to advance *toward it*, slight differences in measurement will not affect the position of the curve. If any short or long stations have been introduced on the location, their position on the line must not be changed in retracing. The chain may be adjusted so that its measures will agree with the recorded distances between transit points. Offsets are made right and left from the new stakes to see that the road-bed is of the full width at all points. The levels are also carried over the grade, and any remaining cut or fill found necessary is marked on the back of the stakes, due allowance being made for the probable settlement of embankments.

252. As the work approaches completion the contractor goes over the line dressing it to grade and opening the side ditches if this has not been previously done.

Drain-tile should be laid at the bottom of these ditches and lightly covered with earth, particularly if the cut be wet. These not only prevent the water from reaching the ballast, but by keeping the foot of the slope comparatively dry prevent the earth from sliding down and filling up the cut. There is also a marked economy in their use, as the cost is trifling, and all further excavation of mud and water from the cut is generally obviated. Should any springs appear in the slope a branch line of smaller tile may be laid to meet it. If the slope is liable to be overflowed from the surface above, an open ditch should be dug a few feet beyond the slope stakes, leading the surface water to discharge elsewhere.

253. The road-bed being prepared, **ballast stakes** are driven at every half station, giving the width of the ballast at its base, while the tops of the stakes indicate the proper level of its upper surface, which is the under side of the tie. These stakes should be set so as to give the proper elevation to the outer rails on curves when the ballast is graded to them. The ballast should be about one foot deep before the ties are laid. Broken stone or a mixture of coarse and fine gravel is the best material, affording elasticity and good drainage. The side slopes of the ballast are made 1 to 1; its width at the under side of the tie should be one foot greater than the length of the tie.

254. Track-laying.—After the ballast has been laid and graded, the centre line is retraced upon it; short stakes are used, each of which is centred. On long tangents, one stake in every 200 feet is sufficient, on ordinary curves one in every 50 feet, and on very sharp curves one in every 25 feet. The ties are then spaced evenly according to the number prescribed per mile, or per rail length; but a tie should not be allowed to cover a transit point. Ties for the standard gauge are 8 or 9 feet long; they should be sawed off square at the ends and in uniform lengths for appearance sake when laid. Specifications usually call for ties having a thickness of 6 inches and a width of from 7 to 10 inches. The ends of the ties are aligned on one side of the road, though if cut into uniform lengths both ends will be equally well aligned. The rails are then laid on, and spiked to gauge. The first spikes are driven in the ties near a centre stake, the centre mark of the gauge bar being kept over the centre on the stake. Upon curves the rails must be sprung to the proper arc before they are laid (§ 199). *All* the ties required in a given distance should be laid before the rails are brought upon them. The practice of laying only joint and middle ties at first subjects the rails to the danger of bending from passing loads.

Owing to the **expansion of the rails** by heat, a space must be left at the rail-joints. The highest temperature of a rail in the summer sun is about 130° Fah. The expansion of iron or steel per 100° is .0007 per foot; or for a 30-foot rail .021 foot or .252 inch. Therefore when 30-foot rails are laid at a temperature near the freezing point, or 100° below the maximum, the space allowed must be at least a quarter of an inch. At 80° Fah. or 50° below the maximum, it need be only half as much. The space required is also proportional to the length of rail used. The exact space should be given, as less would result in the rails being forced up by expansion, while more than necessary space gives a rough road, and hastens the destruction of the rail.

Wherever **sidings** are required, the necessary frogs and long switch-ties should be provided in advance, so that they may be put in place at the time of laying the main track. For every road crossing at grade, heavy oak plank should be provided, and laid upon the ties as soon as the rails are spiked, so that the highway travel may not be impeded.

CHAPTER X.

CALCULATION OF EARTHWORK.

254. The first step toward finding the cubical content of an excavation is to divide it into a number of prismoids by several cross sections.

A prismoid is a solid having plane parallel bases or ends, and bounded on the sides either by planes, or by such surfaces as may be generated by a right line moving continuously along the edges of the bases as directrices.

The positions of the cross sections must be so selected that the solid included between any two consecutive sections may be a prismoid as nearly as possible. Upon a tangent the road-bed and side-slopes are planes, so that the prismoidal character of a given solid depends upon the shape of the natural surface. When the natural surface is a plane, the sections are taken only at the regular stations, 100 feet apart; when it is curved, warped, irregular, or broken, the sections must be more numerous, so that the surface limited by any two shall be composed substantially of right-lined elements extending from one section to the other.

If two end sections of a prismoid are somewhat similar, we infer that the corresponding points are connected by right-lined elements, forming in each case the axis of a ridge or of a hollow. If one section has less breaks than the next, some of these ridges or hollows must vanish; and in order that the solid may be a prismoid, they must vanish in the section of least breaks; therefore a cross section must be taken on the ground through the point where each ridge or hollow vanishes, and the distance of that point from the centre line noted, so that it may be coupled with the proper point in the next section for exact calculation of content.

When ridges or hollows run diagonally across the line of road, *cross sections must* be taken where they are intersected *not only by the centre line* but also by the side slopes; that is, *sections must be taken* so that a side stake may stand on top of each ridge and at bottom of each hollow. In case the centre line intersects at right angles a retaining wall or other vertical surface, two cross sections are required at the same point, one at top and the other at base of wall, in order to furnish the data necessary to calculate the content each way from the vertical surface. (See Art. 235.)

Every thorough cut terminates in either side-hill cutting, a pyramid, or a wedge; the latter happens only when the contour of the natural surface is at right angles to the line of road. Sections should always be taken through the points where the edges of the road-bed meet the surface, as these are the points of separation between thorough and side hill-work. Such sections also serve to define terminal pyramids when they occur as is illustrated by Fig. 97. In side-hill work the foregoing rules apply as well, but sections will generally be more numerous than in thorough cuts. The same rules apply also to embankment, but as grading is preferably paid for in excavation, the same precision in determining the quantities in embankment is not usually necessary.

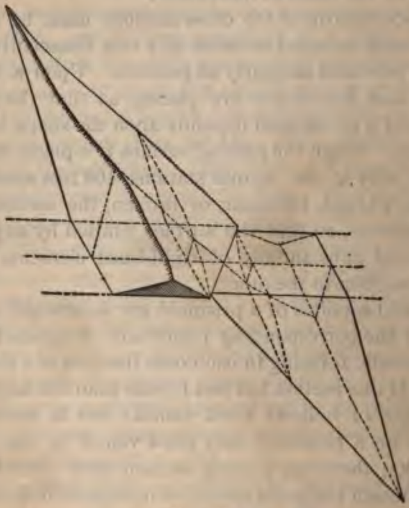

FIG. 97.

255. Formulæ for Sectional Areas.

Let b = base of section or width of road-bed,

" s = slope ratio $= \dfrac{\text{horizontal}}{\text{vertical}}$.

" d = depth at centre stake.

" h, k = depths at side stakes.

" m, n = horizontal distances from centre to side stakes.

For ground level transversely, the section is a parallelogram, and the area is evidently

$$A = bd + sd^2 \qquad (342)$$

or directly from the field notes,

$$A = \tfrac{1}{2}(b + m + n)d \qquad (343)$$

For ground of uniform transverse slope between slope stakes, Fig. 98, the section consists of the parallelogram $ABOE$ and the triangle EOD. Hence

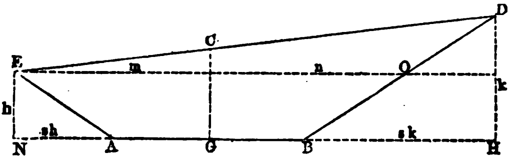

Fig. 98.

$$A = \tfrac{1}{2}(AB + EO)EN + \tfrac{1}{2}EO(DH - EN)$$

or

$$A = \tfrac{1}{2}(AB \,.\, EN + EO \,.\, DH)$$

also

$$\left.\begin{array}{l} A = \tfrac{1}{2}[bh + k(b + 2sh)\,] \\ A = \tfrac{1}{2}[bk + h(b + 2sk)\,] \end{array}\right\} \qquad (344)$$

From which also and

$$\left.\begin{array}{l} A = \tfrac{1}{2}bh + mk \\ A = \tfrac{1}{2}bk + nh \end{array}\right\} \qquad (345)$$

These formulæ are independent of the centre depth. They *are convenient for* calculating the area of a plotted sectio

[illegible]

having an irregular surface after the surface line has been averaged by stretching a silk thread over it. The points where the thread intersects the slope lines determine the values of h, k, m, and n respectively.

When the ground has uniform slopes transversely from the centre to the side stakes: Fig. 99: If in the diagram we draw

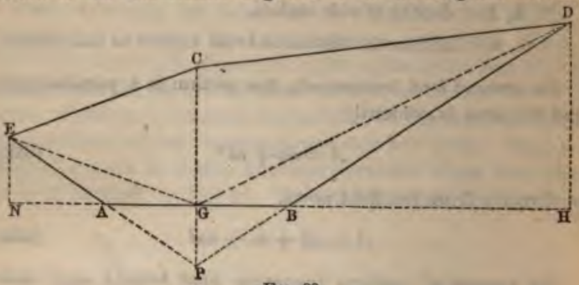

FIG. 99.

EG and DG, the section will be divided into four triangles, *two* having the common base $CG = d$ and respective heights $GN = m$ and $GH = n$, and *two* having the equal bases $AG = GB = \frac{1}{2}b$ and the respective heights $EN = h$ and $DH = k$. Hence we have for the area of section

$$A = \tfrac{1}{2}d(m+n) + \tfrac{1}{4}b(h+k) \qquad (346)$$

Otherwise, if the slope lines are produced to meet below grade at P, then $GP = \frac{AG}{s} = \frac{b}{2s}$. The area of $CEPD$ is $\frac{1}{2}CP \times NH = \frac{1}{2}\left(d + \frac{b}{2s}\right)(m+n)$. The area of ABP is $AG \times GP = \frac{b^2}{4s}$ Hence we have for the area of the section

$$A = \tfrac{1}{2}\left(d + \frac{b}{2s}\right)(m+n) - \frac{b^2}{4s} \qquad (347)$$

Both these formulæ are convenient, and as the values of the several letters can be substituted directly from the field notes, it is unnecessary to plot such sections.

When the surface of the ground is irregular, verticals are conceived to be drawn to the grade line through the slope stakes, and through each break in the surface line, giving a number of trapezoids, the areas of which are severally calculated, and from their sum is subtracted the area of the two triangles *ENA* and *DHB*. The remainder is the area of section required. This calculation may be made directly from the data furnished by the field notes without plotting; but if the ground has a number of small breaks, it is generally better to plot the sections and stretch an averaging line over them, finding the areas by eq. (345). Or two averaging lines may be employed extending from the centre stake, each way, when the area may be calculated by eq. (346) or (347).

256. Prismoidal Formulæ for Solid Contents.—The content of a prismoid may be exactly calculated by means of the Prismoidal Formula, which is

$$S = \frac{l}{6 \times 27}[A + 4M + A'] \qquad (348)$$

S = cubic yards, l = length in feet, A, A' = the areas at the two parallel ends, and M = the area of a section midway between the ends. This area is not a mean of the other two, but the linear dimensions of the mid-section are means of the corresponding dimensions severally of the end sections; from which therefore the area of the mid-section may be computed.

The labor of calculating the middle area may be avoided in many instances by substituting in the prismoidal formula, eq. (348), for A, A', and M, their values as given in eq. (342) for ground level transversely:

$$A = bd + sd^2 \quad A' = bd' + sd'^2 \quad M = b\frac{d + d'}{2} + s\frac{d^2 + 2dd' + d'^2}{4}$$

$$\therefore \quad S = \frac{l}{6 \times 27}[2sd^2 + (3b + 2sd')d + (3b + 2sd')d'] \qquad (349)$$

in which S is expressed in terms of the end dimensions.

257. Tables of cubic yards may be constructed upon this formula which are very convenient in practice. The constant values in *any one table* are l which is taken at 100, and b and s *which are given values corresponding to the road-bed and slope ratio. The variables* are d and d'. The columns in the table will be headed by the successive values of d', while each horizontal line will be headed by a value of d. For any one column therefore d' is constant, and the only variable is d. Assuming any value for d', the values of S in that column may be computed, letting d take a series of values differing by unity from zero upwards, and the corresponding values of S will be placed in the column d' opposite the several values of d.

But instead of solving the eq. (349) for each value of S required, the process of filling the table may be much abbreviated by observing that since the equation is of the second degree with respect to the variable d, the second difference of the values of S will be a constant and equal to *twice* the coefficient of d^2, or $\delta'' = \frac{4sl}{6 \times 27}$ Also the first term in the series of first differences of S in the column d' (*i.e.* between $d = 0$ and $d = 1$) is expressed by the *sum* of the coefficients of d^2 and d; or

$$\delta_0' = \frac{l}{6 \times 27} [3b + 2s(1 + d')]$$

The first value of S in any column d' is found by solving eq. (349) after making $d = 0$; or,

$$S_0 = \frac{l}{6 \times 27} [(3b + 2sd')d']$$

Starting with these values we may fill any column d' simply by successive additions. The values of d' for the several columns should also differ by unity. The final value of S in each column should be calculated by formula as a check; or since all the final quantities in the same line d of the table form a series of which the second difference is δ'', if on taking their differences this result is obtained, the quantities are proved to be correct.

Example.—Given a base of 18 feet and slopes $1\frac{1}{2}$ to 1, to fill the column of $d' = 6$ in a table of cubic yards for level cross sections. Here $l = 100$, $b = 18$, $s = \frac{3}{2}$, $d' = 6$. Hence $\delta'' = 3.7037+$, $\delta_0' = 46.2962+$, and $S_0 = 266.6666+$. It is not necessary to go beyond the fourth decimal place, since that figure will always be the same as the first decimal (a result due to dividing by 27), and may be corrected by it after every addition. The process is as follows:

d	S	δ'	δ''
0	266.6666		
		46.2962	
1	312.9629		3.7037
		50.0000	
2	362.9629		3.7037
		53.7037	
3	416.6666		3.7037
		57.4074	
4	474.0740		3.7037
		61.1111	
5	535.1851		3.7037
		64.8148	
6	600.0000		3.7037
		68.5185	
7	668.5185		3.7037
		72.2222	
8	740.7407		3.7037
		75.9259	
	etc.	etc.	etc.

In copying into the table, the quantities are taken to the nearest unit only, and the decimals are otherwise neglected.

The completed table furnishes values of S corresponding to any values of d and d' in even feet. The correction for the decimal parts of the depths, when there are such, is made by adding to S (found opposite the even feet) the product of the half sum of the decimals by the difference between S as found and the next value of S diagonally below to the right. These differences may for convenience be inserted originally in the table under each quantity in small figures.

If the length of the solid differs from 100 feet, multiply the corrected quantity by the length and divide by 100, since S varies directly as l. Such tables are published in separate sheets for a variety of bases and slopes, so that usually one may be purchased to suit the case in hand.

258. These tables may be used to find quantities when the ground is not level transversely, by finding, first, the area of the actual sections, and second, the depths of level sections having equal areas, and then using the depths so found as the values of d and d' in the table of quantities. The depths of **equivalent level sections** are called equivalent depths. They may be calculated by the formula

$$d = -\frac{b}{2s} + \sqrt{\frac{A}{s} + \frac{b^2}{4s}} \qquad (350)$$

which *is derived directly* from eq. (342). The more convenient *method, however, is to* construct a table on eq. (342), giving to *s a series of values* varying by one tenth of a foot from zero upward. The values of b and s in this table must agree with those of the road and of the table of cubic yards.

259. When the transverse slope is uniform between slope stakes **the equivalent depth** may be expressed **in terms of the centre depth and slope of surface** without reference to the area. Fig. 100.

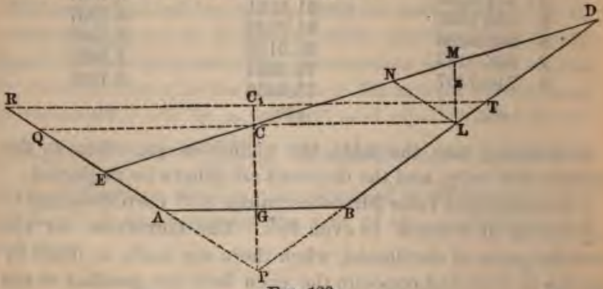

FIG. 100.

Let $EABD$ be the given section.

" $RABT$ be the equivalent level section.

Produce the side slopes to meet at P, and let $c = CP$ and $c_1 = C_1P$.

Through C draw the horizontal line QL, and at L erect the perpendicular $LM = z$, and draw LN parallel to PE.

The area $RPT =$ area EPD; and $QEC = LNC$; hence area $EPLN = QPL = EPD - NLD$.

Since NLD is similar to EPD, we have

$$EPD : NLD :: c^2 : z^2$$

or
$$EPD - NLD : EPD :: c^2 - z^2 : c^2$$

Since QPL and RPT are similar,

$$QPL : RPT :: c^2 : c_1^2$$

$$\therefore \qquad c^2 - z^2 : c^2 :: c^2 : c_1^2 \text{ or } c_1^2 = \frac{c^4}{c^2 - z^2}$$

Let $s' =$ slope ratio of surface $= \frac{CL}{ML} = \frac{cs}{z}$. Then $z^2 = \frac{c^2 s^2}{s'^2}$ which substituted gives

$$c_1 = \frac{cs'}{\sqrt{s'^2 - s^2}} \tag{351}$$

If d_1 = the equivalent depth C_1G, then

$$d_1 = d + (c_1 - c)$$

A table may be prepared giving $(c_1 - c)$ for various inclinations of surface with given base and side slopes. It is then only necessary to add this correction to the centre depth to obtain the equivalent depth. Such tables of correction usually accompany the published tables of cubic yards. This method of obtaining quantities is particularly applicable to preliminary estimates, where the ground has not been cross-sectioned, and only the centre depth and transverse inclination is known.

260. The use of the earthwork tables described gives **correct results**;—

1st. When the surface of the prismoid is a plane, however much inclined; provided it does not intersect the road-bed within the limits of the prismoid.

2d. When with regular, or three-level end sections, generally similar to each other, the surface is regularly warped from one end to the other; provided that the side lines and centre line of the surface are straight, and that no two of them are inclined to grade in opposite directions.

3d. When the ridges or hollows of an undulating surface are parallel to the line of the road.

4th. When a surface of numerous irregularities may be averaged by planes or warped surfaces so as to comply with one of the preceding conditions.

But the method **fails** on undulating ground when the ridges or hollows run obliquely to the line of road, even though the sections may appear quite regular.

In general, the method of equivalent depths holds good when the mid-section of the equivalent level end sections equals in area the actual mid-section of the prismoid; otherwise it fails.

261. The content of a prismoid may be approximately obtained by the method of **mean areas,** the formula for which is

$$S = \frac{l}{2 \times 27} A + A' \qquad (352)$$

Although approximate, this method is much employed or account of its convenience. It is approved by statute to be used upon the public works of the State of New York.

If the values of A and A' derived from eq. (342) be substituted in eq. (352), and then eq. (349) be subtracted from it there remains

$$\frac{ls}{6 \times 27}(d - d')^2$$

which is the correction by which S obtained by eq. (352) must be diminished to make it equal to S obtained by eq. (349) when the ground is level transversely.

Again, for three-level sections, if the values of A, A', and M derived from eq. (347) be substituted in both eq. (348) and (352), and one subtracted from the other, there remains

$$\frac{l}{12 \times 27}\left[(d - d')\left(m + n - (m' + n')\right)\right]$$

which is the correction by which S obtained by eq. (352) must be diminished to make it equal to S obtained by eq. (349). Hence we may write at once, for three level-sections, the correct formula:

$$S = \frac{l}{2 \times 27}\left[A + A' - \frac{d - d'}{6}[m + n - (m' + n')]\right] \qquad (353)$$

This formula gives results identical with eq. (349), is applicable to the same cases, and gives correct results or fails to do so according to the conditions stated in the previous section.

262. When the conditions of the surface are such that eq. (349) or eq. (353) will not give correct results, **the area of the mid-section** may be derived from its calculated linear dimensions as stated in § 256. The contents of the prismoid are then given by eq. (348).

Example 1. (Fig. 101.)—Base 20. Slopes $1\frac{1}{2} : 1$.

$$\begin{array}{llllll}
A'. + \frac{22}{8} & + \frac{0}{8} & & & + \frac{47.5}{25} & \therefore A' = 433 \text{ sq. ft.} \\
A + \frac{34}{16} & + \frac{0}{4} & & & + \frac{16}{4} & \therefore A = 200 \text{ sq. ft.} \\
\therefore M + \frac{28}{12} & + \frac{11}{6} & + \frac{0}{6} & + \frac{8}{6} & + \frac{31.75}{14.5} & \therefore M = 241.75 \text{ sq. ft.}
\end{array}$$

If $l = 100$, eq. (348)

$$S = \frac{100}{6 \times 27}(443 + 967 + 200) = 993 \text{ c. yds.}$$

Had this been calculated by eq. (349) or eq. (353) or by the

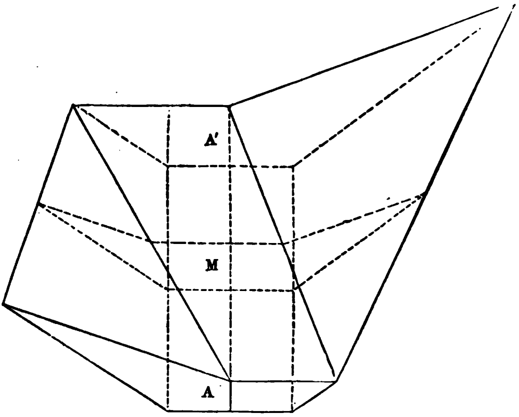

FIG. 101.

tables, the result would be 1167 c. yds., showing an error of 174 cubic yards in *excess*.

Example 2. (Fig. 102.)—Base 20. Slopes 1½ : 1.

$$A' + \frac{22}{8} + \frac{0}{8} \qquad + \frac{19}{6} \qquad 234 \text{ sq. ft.}$$

$$A + \frac{13}{2} + \frac{0}{4} \qquad + \frac{16}{4} \qquad 88 \text{ sq. ft.}$$

$$\therefore M + \frac{17.5}{5} + \frac{11}{6} + \frac{0}{6} + \frac{8}{6} + \frac{17.5}{5} \qquad 104.5 \text{ sq. ft.}$$

If $l = 100$, eq. (348)

$$S = \frac{100}{6 \times 27}[88 + 658 + 234] = 605 \text{ c. yds.}$$

Had this been calculated by eq. (349) or eq. (353) or by the tables, the result would be 584 c. yds., showing an error of 21 cubic yards in *deficit*.

263. At the termination of a cut or fill we have usually either **a wedge** or **a pyramid.** To a wedge the preceding formulæ and tables based on them apply by making one end depth equal zero. In the case of a pyramid, the content is equal to the area of the section forming the base multiplied by one third the length of the solid, and divided by 27; or

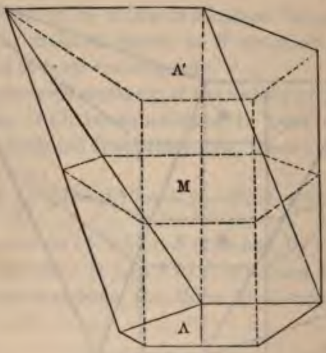

FIG. 102.

$$S = \frac{lA}{3 \times 27} \qquad (354)$$

264. Side-hill Work.—When the natural surface has a regular transverse slope and intersects the road-bed, the cross section is reduced to a triangle. If w = the intercepted portion of the road-bed, and k = the side height, then $A = \frac{1}{2}wk$. Similarly $A' = \frac{1}{2}w'k'$ and $4M = \frac{1}{2}(w + w')(k + k')$, which substituted in eq. (348) give

$$S = \frac{l}{12 \times 27}\left[(2w + w')k + (2w' + w)k'\right] \qquad (355)$$

which is convenient for direct calculation from the field notes. It is not adapted to the construction of tables, since it contains four independent variables.

If the slope of the natural surface is given, let s' be the surface slope ratio at one section, and s'' that at the other, and s the ratio of the side slope. Then $w = k(s' - s)$ and $w' = k'(s'' - s)$, which substituted in eq. (355) give

$$S = \frac{l}{6 \times 27}\left[(s' - s)k^2 + \left(\frac{s'' + s'}{2} - s\right)kk' + (s'' - s)\,k'^2\right]$$

If the surface is a *plane*, then $s'' = s'$, and we have for this case

$$S = \frac{l\,(s' - s)}{6 \times 27}\,[k^2 + kk' + k'^2] \qquad (356)$$

which is a formula of quite limited application; yet it is the one on which tables and diagrams are usually constructed. Consequently the latter will not give correct results, except when the surface is a *plane*.

265. When the natural surface is broken the sections may be plotted, and the values of w and k taken from the points where **an averaging line** intersects the grade and side slope respectively. Finding values for w' and k' in the same way, the content may then be obtained by eq. (355) as before. The averaging line should not only cut off the same area as the original section, but should also have in each case a slope agreeing as nearly as possible with the general slope of the natural surface. The slope is determined simply by inspection of the diagram, but the area may be had precisely, for, taking w from the averaging line, and knowing A, we may calculate k by the formula $k = \frac{2A}{w}$; or k may be taken from the plot and w calculated.

Otherwise, the actual mid-section may be calculated and the cubic contents determined by the method illustrated in § 262.

266. To express side-hill areas and cubic yards in terms of the centre depth, d, and transverse slope-ratio s′. Fig. 103.

When $d = 0$, $A = \frac{1}{4}bk = \frac{b^2}{8(s' - s)}$

For any depth d, add to this area

$$s'd\left(k+\frac{x}{2}\right)=s'd\left(\frac{b}{2(s'-s)}+\frac{s'd}{2(s'-s)}\right)$$

and there results,

$$\left.\begin{aligned} A &= \frac{(\frac{1}{2}b+s'd)^2}{2(s'-s)} \\ S &= \frac{l(\frac{1}{2}b+s'd)^2}{2\times 27(s'-s)} \end{aligned}\right\} \qquad (357)$$

Observe that d may be *plus* or *minus*, and that its limits are $d=\pm\frac{b}{2s'}$

Tables of cubic yards may be constructed on this formula, making d and s' the variables, which would be extremely convenient for making up estimates upon preliminary lines on which the profile of centre line and angle of transverse slope only are known. Since s' is the cotangent of the slope angle the columns of the table may be headed by the angles in a series of degrees, while the corresponding values of s' are used in the formula. The values of d should vary by tenths of a foot. The results obtained by eq. (356) and eq. (357) will be identical for the same sections.

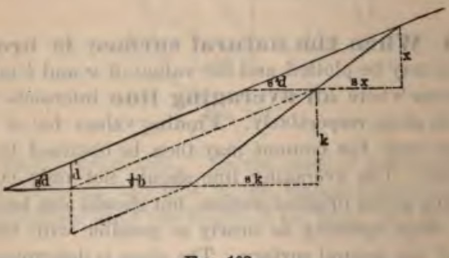

FIG. 103.

267. *Several different systems of* **diagrams** *have been devised and published for determining quantities in earthwork by a sort of* graphical method. These diagrams, which are *substitutes for* tables are preferred by some engineers. They are based on the same principles, and are constructed on modifications of the same formulæ.

268. Correction of Earthwork for Curvature.

—The preceding calculations are based on the assumption that the centre line is straight, with cross sections at right angles to it. When an excavation is on a curve, the cross sections, being in radial planes, are inclined to each other, so that the condition of a prismoid is not exactly fulfilled. But by the property of Guldinus, if any plane area is made to revolve about an axis in the same plane, the volume of a solid generated by the area is equal to that of a prism having a base equal to the given area, and a height equal to the length of path described by the centre of gravity of the area. The path, being the arc of a circle, is proportional to the radius drawn to the centre of gravity. If therefore a cross section is symmetrical with respect to the centre line, the path of the centre of gravity is equal to the measured length of the centre line, and no correction for curvature is required.

But when the ground is inclined transversely, the centre of gravity is one side of the centre line, and its path, if we conceive it to sweep around the curve, from one end of a prismoid to the other, is longer or shorter than the distance measured on the centre line, according as the centre of gravity is outside or inside of the centre line curve.

Let C = correction in cubic yards due to curvature.
" S = cubic yards as obtained by prismoidal formula.
" R = radius of centre line.
" e = eccentricity of centre of gravity of section.
= horizontal distance from centre line to centre of gravity.

We then have the proportion,

$$S \pm C : S :: R \pm e : R$$

$$C = \frac{Se}{R}$$

As the sections of a solid are seldom similar and equal, we *shall usually have a* different value of e for every section, from which, however, a mean average value may be deduced, and used in the above formula. But it will be more convenient to correct the areas themselves for eccentricity before finding S, which will then require no correction. For the same result will ensue whether we multiply S by $\frac{e}{R}$, or multiply one of the component factors of S by the same ratio.

If then c = correction of area in square feet due to eccentricity, we have at once

$$c = \frac{Ae}{R}$$

and the corrected area equals $A \pm c$ according as the cut is deeper on the outside or inside of the curve. Each area used in determining the solid contents should, on a curve, be first corrected in this manner.

To find the value of e for any three-level section. Fig. 104.

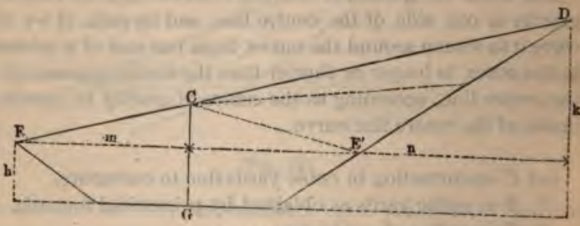

FIG. 104.

Find the areas either side of the centre line separately, calling them H and K, and take their sum and difference. Using the same notation as in § 255, $H = \frac{1}{2}md + \frac{1}{4}bh$, $K = \frac{1}{2}nd + \frac{1}{4}bk$, and $H + K = A$.

$$K - H = \tfrac{1}{2}d\,(n - m) + \tfrac{1}{4}b\,(k - h)$$

In the figure draw CE' equal to CE, and the triangle $CE'D$ will represent the area $(K - H)$. Bisect the side E'D, and draw a line from C to the middle point. Then the centre of gravity of the triangle will be on this line at two thirds its length from C, and the horizontal distance of the centre of gravity from C is $\frac{2}{3} \times \frac{1}{2}(m + n) = \frac{1}{3}(m + n)$. The centre of gravity of the remainder of the section is on the centre line CG, so that the value of e is found from the proportion

$$e : \tfrac{1}{3}(n + m) :: K - H : A$$

$$\therefore \qquad e = \frac{n + m}{3A}(K - H)$$

Hence $$c = \frac{Ae}{R} = \frac{n + m}{3R}\left[\tfrac{1}{2}d(n - m) + \tfrac{1}{2}b(k - h)\right] \qquad (358)$$

Sections which are more irregular may be plotted and reduced by averaging lines to three-level sections, in order that the formula may be applied. If the ground is so irregular as to require the computation of the middle section, the correction c should be found and applied to this area (M) also before introducing it into the prismoidal formula. As the correction for curvature is always relatively small, it is usually ignored in practice for thorough cuts, except where deep cuttings with steep transverse slope occur on sharp curves.

The correction is of more importance relatively **in side-hill work** as the centre of gravity of the section is more remote from the centre line. Let the section be reduced to a triangle by an averaging line (Fig. 105), and w be the base of the triangle formed by the averaging line. The centre of gravity is at one third the horizontal distance from the middle point of w to the side stake D, while the distance of this middle point from the centre stake C is evidently $\frac{1}{2}b - \frac{1}{2}w$

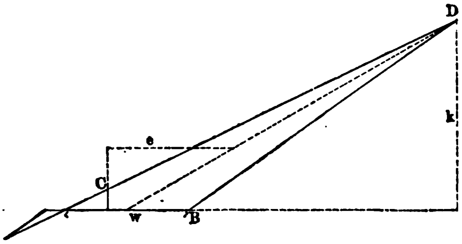

Fig. 105.

Hence $e = \frac{1}{2}b - \frac{1}{2}w + \frac{1}{3}[n - (\frac{1}{2}b - \frac{1}{2}w)]$

or $e = \frac{1}{3}(b + n - w)$

and $$c = \frac{Ae}{R} = \frac{b + n - w}{3R} \times \frac{wk}{2} \qquad (359)$$

The correction c will be *plus* or *minus* as before explained. This formula applies to all side-hill triangular sections, whether there be cut or fill at the centre stake.

Example 1.—Thorough cut; base 20; slopes $1\frac{1}{2} : 1$.

$l = 100$; 8° curve, left; $R = 716.78$

Notes. $A . + \frac{16}{4} + \frac{12}{0} + \frac{58}{32}$

$A' . + \frac{13}{2} + \frac{8}{0} + \frac{40}{20}$

Then $K = \frac{1}{2} \times 58 \times 12 + \frac{1}{4} \times 20 \times 32 = 508$

$H = \frac{1}{2} \times 16 \times 12 + \frac{1}{4} \times 20 \times 4 = 116 \therefore A = 624$

$K - H = \overline{392}$

Eq. (358) $c = \frac{16 + 58}{3 \times 716.78}\ 392 =$ 13.49

$(A + c)$ $\overline{637.49}$

$K' = \frac{1}{2} \times 40 \times 8 + \frac{1}{4} \times 20 \times 20 = 260$

$H' = \frac{1}{2} \times 13 \times 8 + \frac{1}{4} \times 20 \times 2 = 62 \therefore A' = 322$

$K - H = \overline{198}$

$c' = \frac{13 + 40}{3 \times 716.78}\ 198 =$ 4.87

$(A' + c') = \overline{326.87}$

From which we obtain $S = 1758$ cub. yds.—*Ans.*
Without correction we have 1726 " "
Showing a difference of $\overline{32}$ " "

Had the curve been to the *right* with same notes, c would have been *minus*, and S would $= 1694$.

Example 2.—Side-hill cut; base 20; slopes 1½ : 1

$l = 60$; 10° curve, right; $R = 573.69$

Notes.

$$\frac{6}{0} + \frac{0}{2.8} + \frac{40}{20}$$

$$-\frac{0}{0.8} + \frac{2}{0.0} + \frac{37}{18}$$

$$A = \tfrac{1}{2} \times 16 \times 20 = \qquad 160$$

Eq. (359) $$c = \frac{20 + 40 - 16}{3 \times 573.69}\, 160 = \qquad 3.58$$

$$(A - c) = 156.42$$

$$A' = \tfrac{1}{2} \times 8 \times 18 = \qquad 72$$

$$c' = \frac{20 + 37 - 8}{3 \times 573.69}\, 72 = \qquad 2.05$$

$$(A' - c') = 69.95$$

Hence $S = 248$ cub. yds.

Without correction S would $= 255$ " "

Difference 7 " "

269. Haul.—The cost of removing excavated material, when the distance does not exceed a certain specified limit, is included in the price per cubic yard of the material as measured in the cutting. But when the material must be carried beyond this limit, the extra distance is paid for at a stipulated price per cubic yard, per 100 feet. The extra distance is known by the name of *haul*, and is to be computed by the engineer with respect to so much of the material as is affected by it.

The contractor is entitled to the benefit of all short hauls (less than the specified limit), and material so moved should not be averaged against that which is carried beyond the limit. Therefore, in all cuts, the material of which is all deposited within the limiting distance, no calculation of haul is to be made.

On the other hand, the company is entitled, in cases of long haul, to free transportation for that *portion* of the cutting, no one yard of which is carried beyond the specified limit. Therefore, this *portion is first to* be determined in respect to its *extent; and the number of* cubic yards contained in it is to be deducted from the total content of the cutting, before estimating the haul upon the remainder. Find on the profile of the line two points, one in excavation, and the other in embankment, such, that while the distance between them equals the specified limit, the included quantities of excavation and embankment shall just balance. These points are easily found by trial, with the aid of the cross sections and calculated quantities, and become the starting points from which the haul of the remainder of the material is to be estimated.

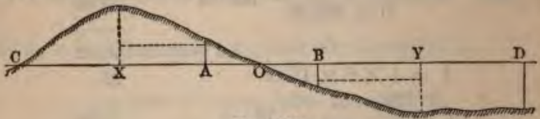

FIG. 106.

Fig. 106 represents a cut and fill in profile. The distance *AB* is the limit of free haul. The materials taken from *AO* just make the fill *OB* and without charge for haul; but the haul of every cubic yard taken from *AC*, and carried to the fill *BD*, is subject to charge for the distance it is carried, less *AB*. It would be impossible to find the distance that each separate yard is carried, but we know from mechanics that the average distance for the entire number of yards is the distance between the centres of gravity of the cut *AC*, and of the fill *BD* which is made from it. If, therefore, *X* and *Y* represent the centres of gravity, the actual average haul is the sum of the distances ($AX+BY$), and this (expressed in stations) multiplied by the number of cubic yards in the cut *AC*, gives the product to which the price for haul applies.

But the product of *AX* by the number of cubic yards in *AC* is equal to the sum of the products obtained by multiplying the contents of each prismoid in *AC* by the distance of its own centre of gravity from *A*. The distance of the centre of gravity of a prismoid from its mid-section is expressed by the formula

$$x = \frac{l^2\,(A - A')}{12 \times 27\,S} \qquad (360)$$

If we replace S by its approximate value, $\frac{l\,(A + A')}{2 \times 27}$, which will produce no important error in this case, we have

$$x = \frac{l}{6} \cdot \frac{A - A'}{A + A'} \qquad (361)$$

in which A should always represent the more remote end area from the starting point A, fig. 106. Hence, x may be + or −, and it must be applied, with its proper sign, to the distance of the mid-section from the starting point A, before multiplying by the contents S. Each partial product is thus obtained.

By a similar process with respect to the prismoids composing the mass BD, and using the point B as the starting point, we obtain finally a sum of the products representing this portion of the haul.

If a cut is divided, and parts are carried in opposite directions, the calculation of each part terminates at the dividing line. If a portion of the material in AC is wasted, it must be deducted, and the haul calculated only on the remainder.

The specified limit is sometimes made as low as 100 feet, sometimes as high as 1000 feet. A limit of about 300 feet, however is usually most convenient, as it includes the wheelbarrow work, and a large part of the carting, while it protects the contractor on such long hauls as may occur.

270. The Final Estimate is a complete statement in detail, of the amount of work done and materials provided, in the construction of the road, and is the basis of final settlement between the company and contractor. Its preparation should be begun as soon as possible after the work is in progress, and should be continued, as fast as the necessary data are accumulated, while the circumstances are still fresh in mind, and when any omissions in the field notes may be readily supplied. The content of each prismoid, the classification of its material, and the length of haul to which it is subject, should be matters of special record in a book provided for that purpose. These results having been carefully computed by exact methods form a standard of comparison for those approximate results which must be had from time to time during the progress of the work, and furnish a limit to the amounts of the monthly estimates. The same remark applies to all other items of labor and material. The notes and record of the final estimate should be particularly full and exact in respect to all such items as will be *inaccessible to measurement at the completion of the work, such as foundation pits,* foundation courses of masonry, *culverts, and works* under water.

271. Monthly Estimates.—On or before the last day of every month during the progress of construction, measurements are taken to determine the total amount of work done and material provided up to that date. The estimates based on these measurements are called Monthly Estimates. It is frequently necessary to take measurements for both monthly and final estimates at other times than the end of the month, as in the case of foundations which are not long accessible. With respect to each piece of work satisfactorily completed, the monthly estimate should be exact, and identical in amount with the final estimate. With respect, however, to items of work in progress at the time of measurement, the monthly estimate is only approximate, yet should be as precise as the nature of the case will allow; and the quantities stated should not be in excess of fair proportion of the total quantities given on the final estimate for the same piece of work.

A special field book is devoted to monthly estimate notes. Each page should be dated with the day on which the notes upon it were taken. The notes consist of measurements of all sorts, principally of cross sections partially excavated. These sections should be at the same points on the line as the original sections, so that comparisons may be made. Wherever the excavation is finished to grade, it is only necessary to write "completed" opposite such stations, and the quantities may be taken from the final estimate or computed from the original notes. It is frequently necessary to retrace portions of the centre line in taking estimate notes, so that all the field instruments are required, but a party of three or four men is usually sufficient.

If the contractor has provided materials, such as stone, lumber, etc., which are not as yet put into any structure when the estimate is taken, these should be measured and entered under the head of **temporary allowance,** an arbitrary price be- ng used somewhat below the actual value of the material as elivered. Such allowances should never be copied from one onth's estimate to the next, but made anew on such material may be found that seems to require it. But all *completed* *ms of contract* work, and of extra work when ordered by *engineer,* are necessarily copied from one monthly esti- *te to the next* during the continuance of the contract.

A blank form is used by the resident engineer in report ing monthly estimates, on which a column is provided for each class of material and work required by the contract, while the several lines, headed by the numbers of the proper stations, are devoted to the different cuttings, structures, etc., in consecutive order as they occur on the line of road. The estimates are made out and reported separately for the several sections into which the line of road is divided for letting.

These reports are reviewed by the division engineer, and the footings copied upon another blank, which is the monthly estimate proper; the prices are attached to the items, and the amounts extended and summed up. This sum indicates approximately the total amount earned by the contractor up to date, from which is deducted a certain percentage (usually 15 per cent.), which is retained by the company until the completion of the contract. From the remainder is deducted the amount of previous payments, which leaves the amount due the contractor on the present estimate. A blank form of receipt is appended, to be signed by the contractor.

CHAPTER XI.

TOPOGRAPHICAL SKETCHING.

272. Topographical sketches taken on preliminary surveys are usually of great value in projecting a line for location; they should be made therefore as accurate and complete as possible. In too many instances sketches are presented having a picturesque appearance, but conveying little information, if not tending to mislead the map-maker. The aim of the topographer should be to record the topographical features either side of the line with as much precision as those directly upon the line, without taking actual measurements, except in rare instances. The eye and the judgment must be usually depended on for distances and dimensions. The sketch of a tract extending to 400 feet each side of the line ought to be accurate enough to warrant its being copied literally upon the map. If a much wider range is required it may be advisable to use the plane-table; but an approximation to plane-table methods may be employed in ordinary sketching.

273. As **artificial features** are the most readily defined and located these should first receive attention in making a sketch. When recorded they form a skeleton upon which the natural features can be drawn with more precision than if the order were reversed. The point where each fence crosses the line and the angle between the two may be sketched exactly. The distance along the fence to any object may be estimated, and checked (in case of an oblique angle) by observing where a line from the object perpendicular to the centre line would intersect the latter. The book may be rested on any support, the centre-line of the page coinciding with the line of survey, and the direction of objects defined by a small ruler laid on the page. This operation being repeated from another point gives intersections which locate the several objects on the sketch. If the bearings are taken they may be plotted on the page as well as recorded, giving the same results. The eye may be trained to estimate distances correctly by observing the appearance of objects along the measured line, the distances to which are therefore known.

274. After the artificial objects the more distinct **natural features** are to be sketched, as streams, shores, margins of swamps, forests, etc., ravines, ridges, and bluffs, taking care that all these outlines intersect the features of the sketch already delineated at the proper points. The correct representation of **contours** is the most difficult part of sketching, since these lines are quite imaginary, yet for railroad maps they are usually as important as any others. It is desirable to know not only the locality of a hill or slope, but also its shape, steepness, and height. This information is best given by contour lines. A contour is the intersection of the surface of the ground by an imaginary level surface. When the surface is real, like that of a lake, the intersection is called a shore. If the water should rise a certain height a new shore would be defined, and rising double that height still another shore would result, each of which, on the subsidence of the water, would be a contour. A practiced eye is able to follow on the round the course of a contour with all its windings; but in etching them due allowance must be made for the foreortening effect of distance. All contours on the same sketch ould have the same vertical interval, so that by counting them the height of the hill may be known. The spaces on the sketch between contours vary as the cotangent of the slope angle, so that the width of the spaces indicates the degree of steepness. The contours nearest the topographer should generally be sketched first, although if there be a shore that is apt to be the best guide to the shape of the slopes. If the height of the hill is known and the upper contour located, the other contours can be spaced between with less difficulty, the proper number being ascertained by dividing the height by the assumed vertical interval. A special line of levels up an inclined ravine or sloping ridge to fix the contour points is often of the greatest service in obtaining correct results. Other random lines are sometimes run to locate the contours more definitely. These should be made to cross several contours rather than to trace a single one. Old preliminary lines which have proved useless in themselves often furnish by their profiles valuable information in respect to contours.

The use of hatchings should be avoided in the sketch-book, except to represent precipitous banks, or slight terraces, which would not be sufficiently defined by the contour system. Hatchings freely used consume too much time, and fail to give an accurate idea of either slope or height, while they obscure the page for the representation of other objects.

275. The centre line on the page is straight, and for sketching purposes the surveyed line on the ground is assumed to be so also. Slight deflections in the course of a preliminary line may be ignored in the sketch; but if a large angle occurs it is better to terminate the sketch with the course, and begin again, leaving a few blank lines between the two sketches. On a located line with curves, the sketch is continuous. The curved line in the field is represented by the straight line on the page, and the radial lines through the stations are represented by the parallel lines ruled across the page. All objects are sketched at the proper offset distance by scale from the centre line; but longitudinally the sketch is necessarily diminished outside of the curve, and magnified inside of the curve. Consequently topographical lines which are straight in fact appear *curved in the sketch*, concave to the centre line if inside *the curve, and* convex if outside of it. Such features are correctly *sketched* by means of offsets estimated or measured from each station of the curve on the radial lines. This kind of distortion creates no confusion if properly done, for in making the map, after drawing the curve and the radial lines, the same offsets will give the correct positions of the objects delineated. This method is preferable to drawing a curved line on the page to represent the centre line, as it is difficult to draw it correctly; it will cross the ruled lines obliquely, rendering them of no service for offsets or scale, and moreover is likely to run off the page altogether.

CHAPTER XII.

ADJUSTMENT OF INSTRUMENTS.

Every adjustment consists of two processes: first *the test,* and second *the correction.* Inasmuch as the amount of correction is made by estimation, the test must always be repeated until no further lack of adjustment is observable.

276. THE TRANSIT.

The level tubes should be parallel to the vernier plate.

Test: Place the tubes in position over the levelling screws, and turn the latter till the bubbles are centred; revolve the plate 180°. The bubbles should remain centred; if they have retreated—

Correction: Bring them half way back to the centre by turning the adjusting screws which attach the tubes to the plate.

The line of collimation should be perpendicular to the horizontal axis.

Test: Clamp the limb, and by the tangent screws bring he intersection of the cross-hairs to cover a well-defined point about on a level with the telescope; plunge the telescope to *ook in the opposite* direction, and note any point about on a *evel with* the telescope and about equidistant with the first *point, which* the intersection of the cross-hairs now happens to *cover.* Now unclamp the limb and turn through 180°, and *repeat the* above operation, using the same first point as before.

The *third* point obtained should be identical with the *second;* if not—

Correction : Move the vertical cross-hair over *one fourth* of the apparent distance from the third to the second point, by turning the adjusting screws at the side of the telescope.

The horizontal axis should be parallel to the vernier plate.

Test : After completing the above adjustments level the limb, clamp it, and bring the intersection of the cross-hairs to cover some high point so that the telescope may be elevated to a large angle; depress the telescope and note some point on the ground now covered by the intersection of the cross-hairs. Now unclamp the limb, turn it through 180°, and repeat the above operation, using the same high point as before. The third point found should be identical with the second; if not—

Correction : Raise the end of the axis opposite the second point (or lower the other end) by a small amount, by turning the adjusting screws in the standard. The amount of motion required is only determined by repeated trials until the test is satisfied.

The intersection of the cross-hairs should appear in the centre of the field of view.

Test : Bring the cross-hairs into focus and direct the telescope toward the sky, or hold a sheet of blank paper in front of it. If the intersection appear eccentric—

Correction : Turn the screws (by pairs) which support the end of the eyepiece until the desired result is obtained.

If there be **a level on the telescope** it **should be parallel to the line of collimation.**

Drive two stakes equidistant from the instrument in exactly opposite directions, and having perfected the previous adjustments, level the plate carefully, clamp the telescope in about a horizontal position, and observe a rod placed on each stake. Have the stakes driven by trial until the rod reads alike on both. The heads of the stakes are then on a level. Remove the instrument beyond one stake, and set it up in line *with the two, level the* plate, and elevate or depress the *telescope to a position* which will again give equal readings *on the stakes.* The line of collimation is now level—

Test: While in this position the bubble of the attached level should stand centred; if not—

Correction: Bring the bubble to the centre by turning the nuts at one end of the tube, while the cross-hair continues to give equal readings.

277. THE Y LEVEL.

The line of collimation should coincide with the axis of the telescope.

Test: Clamp the spindle, and bring the intersection of the cross-hairs to cover a well-defined point by the tangent and levelling screws; revolve the telescope half over in the Ys, so that the level tube is on top. The intersection of the cross-hairs should still cover the point. If either hair has departed—

Correction: bring it half way back by means of the pair of adjusting screws at the extremities of the other hair.

The attached level should be parallel to the axis of the telescope.

Test: Bring the telescope over one pair of levelling screws, clamp the spindle, open the clips, and bring the bubble to the centre. Then gently remove the telescope from the Ys, and replace it end for end. If the Ys have not been disturbed, the bubble should return to the centre. If it does not—

Correction: bring the bubble half way back by turning the nuts at one end of the tube.

But as now the level tube and telescope may only lie in parallel planes, and yet not be parallel to each other—

Test: bring the bubble to the centre as before, and turn the telescope on its axis so as to bring the level tube out to one side. The bubble should remain centred. If it has departed—

Correction: bring it back to the centre by the adjusting screws at one end.

The axis of the telescope should be at right angles to the spindle.

Test: Having completed the above adjustments (and not before), fasten down the clips, unclamp the spindle, and bring the bubble to the centre over each pair of levelling screws in *succession, then* swing the telescope end for end on the spindle. *The bubble* should settle at the centre. If it do not—

Correction: bring it half way back by the large nuts at one end of the bar.

278. THE THEODOLITE.

This instrument being a combination of Transit and Level, its several adjustments are to be made according to the rules already given for those instruments.

CHAPTER XIII.

EXPLANATION OF TABLES.

TABLE I.—Contains concise statements of such geometrical truths as are applicable to the various discussions in this volume. References are given to Davies' Geometry, in which the demonstrations of the propositions may be found.

TABLE II.—Contains all the formulæ necessary to the solution of any plane triangle; also, a select list of miscellaneous formulæ. A few formulæ with respect to the versed sine and external secant are new.

TABLE III.—Contains a complete list of formulæ expressing the relations between the radius, tangent, chord, versed sine, external secant, and central angle of a railway curve; also, the relations between the radius, degree of curve, length of curve, and central angle. The notation is identical with that used elsewhere in the book.

TABLE IV.—Contains the radius, and its logarithm, for every degree of curve to single minutes up to 10 degrees, and thence by larger intervals up to 50 degrees. With the radius is given also the perpendicular off-set, t, from the tangent to a point on the curve at the end of the first 100-foot chord from the tangent-point, and the middle ordinate, m, of a 100-foot chord. See eqs. (16, 34, 37, 40, and 305).

TABLE V.—Contains the corrections to be added to the tangents and externals of any railroad curve, as obtained by reference to Table VI., according to the degree of the given curve (found at head of columns), and its central angle, (found in th

first column.) If the given degree of curve, or central angle, does not appear in the table, the exact value of the correction may be easily obtained by interpolation.

TABLE VI.—Contains the exact values of the tangents, T, and externals, E, to a 1 degree curve, for every 10 minutes of central angle, from 1° to 120° 50′. Approximate values of the tangent and external to any other degree of curve may be had by simply dividing the tabular values opposite the given central angle by the given degree of curve, expressed in degrees. These approximations may be made exact by adding the proper corrections taken from Table V. See eqs. (21) and (24).

TABLE VII.—Contains the value of Long Chords of from 2 to 12 stations, for every 10 minutes of degree of curve from 0° to 15°, and of a less number of stations for degrees of curve between 15° and 30°. As the chord of one station is always 100 feet, the column of the first station gives instead the length of arc subtended by the chord of 100 feet. See §§ 121, 122, 123, 124, 125.

TABLE VIII.—Contains the values of Middle Ordinates to long chords of from 2 to 12 stations, for every 10 minutes of degree of curve from 0° to 10°, and of from 2 to 6 stations for every curve from 10° to 20°, at 10-minute intervals. The table may be used, not only to fix the middle point of an arc, but also, in conjunction with the table of long chords, to locate intermediate stations. See §§ 121, 122, 123, 124, 125.

TABLE IX.—Contains the chords of a series of angles varying by half degrees up to 30° for radii varying by 100 feet up to 1000 feet. It shows, therefore, the linear opening between the extremities of two equal lines at any given number of hundred feet from their intersection, when the angle does not exceed 30°. For any distance exceeding 1000 we have only to add to the value found in that column, the value found in the *column headed* by the excess of distance over 1000 feet. *Conversely, the table* gives the angular deflection required between *wo equal lines,* in order that at a given distance from the point *f intersection* they may be separated a given amount.

TABLE X.—1. Contains values of the ratio $u = \frac{i}{\Delta}$, according to the notation of § 147 for finding the angle i (Fig. 34) between the radius PO of the curve at any point P, and the tangent PK to the valvoid arc PX by the simple formula eq. (80) $i = u\Delta$. The table embraces lengths of curve from 300 to 2000 feet, and central angles from 10° to 120°.

When $\frac{L\Delta}{1000} = 60°$ $u = \frac{1}{3}$, and for hasty approximation this value of u may be assumed in any case without consulting the table.

2. Contains values of the ratio $v = \frac{r}{L}$ for finding the radius of the valvoid arc at the point P (Fig. 35) in terms of the length of curve $L = AP$ by the simple formula, eq. (82), $r = vL$.

3. Contains values of the length $l_{,}$ of a valvoid arc limited by two curves of equal length laid out from the same tangent and same *P.C.*, but whose central angles differ by 1°. The length L of each curve is given in the first column, and the half sum of their central angles $\left(\frac{\Delta' + \Delta''}{2}\right)$ is given at the head of the other columns.

When the central angles of two curves of equal length differ by x degrees the length l of the valvoid arc joining their extremities is expressed by the simple formula, Fig. 36,

eq. (86) $$l = P'P'' = (\Delta' - \Delta'')l_{,}$$

in which $l_{,}$ is taken from the column headed by $\frac{\Delta' + \Delta''}{2}$ and opposite the given value of L; or $l_{,}$ is found by interpolation if necessary. See § 150 and example.

TABLE XI.—Contains the measurements necessary to lay down a turnout with frogs of given numbers or angles for both a standard and a three-foot gauge. The distance BF is measured on the rail of the given track from the *heel* of the switch to the *point* of the frog, while af is the chord of the centre line *of the turnout* between the same points. The *radius r applies to* the centre line of the turnout. The *distance aF'' is measured* on the centre line of the straight track

from the *heel* of the switch to the point of the middle frog. The length of switch AD should conform to the tabular values unless the throw is to be different from that assumed in the table. See §§ 180, 181, 182.

TABLE XII.—Contains the middle ordinates of chords varying in length from 10 to 32 feet, and for degrees of curve varying from 1° to 50°. The use of the table is obvious. See § 199.

TABLE XIII.—Gives the proper difference in elevation of rails on curves of various degrees from 1° to 50° for velocities varying from 10 to 60 miles per hour. See § 201.

TABLE XIV.—Gives the rise of grades in feet per mile and their angle of inclination corresponding to a rise per station (100 feet) varying from 0.01 foot to 10 feet.

TABLE XV.—Contains values of the formula $(\log h - 1)$ 60384.3 in which $h =$ reading of the barometer in inches. The inches and tenths of the readings are in the left-hand column, while the hundredths are found at the top of the other columns. The *difference* of any two values corresponding to two readings taken simultaneously at any two stations is the difference in elevation in feet of those stations. But the difference in height so found is subject to a correction for temperature given in the next table. See § 10.

TABLE XVI.—Contains coefficients of correction for atmospheric temperature, by which the approximate heights obtained by Table XV. are to be multiplied for a correction of these heights, which correction is to be added or subtracted according as the coefficient given in the table is marked + or −. See § 11.

TABLE XVII.—Contains corrections in feet, required by the curvature of the earth and the refraction of the atmosphere, to be applied to the elevation of a distant object as obtained by a level or theodolite observation for distances ranging from 300 feet to 10 miles. See § 119.

TABLE XVIII.—Contains the coefficients for reducing the space on a vertical rod intercepted by the stadia hairs when

the line of collimation is inclined to the horizon, to the space that would be intercepted were the line of collimation horizontal; *provided*, that the visual angle θ defined by the stadia hairs is such that $\tan \frac{1}{2}\theta = .005$ or $\theta = 0^\circ\ 34'\ 22''.63$, which is its customary value in surveying instruments. The angle of inclination α is taken at every 10 minutes through half a quadrant.

TABLE XIX.—Contains the logarithms of the coefficients given in Table XVIII.

TABLE XX.—Gives the lengths of circular arcs to a radius = 1.

To find the length of any arc expressed in degrees, minutes, and seconds, take from the table the lengths of the given number of degrees, minutes, and seconds respectively, and multiply their sum by the length of the radius. The product is the length of arc required.

TABLE XXI.—Contains the values of minutes and seconds expressed in decimals of a degree, for every 10 seconds of arc, and also for quarter minutes up to one degree.

TABLE XXII.—Contains the values of inches and fractions expressed in decimals of a foot for every 32d of an inch up to one foot.

TABLE XXIII.—Contains the squares, cubes, square roots, cube roots, and reciprocals of numbers from 1 to 1054. Its use may be greatly extended by observing that if any number is multiplied by n its square is multiplied by n^2, its cube by n^3, and its reciprocal by $\frac{1}{n}$.

TABLE XXIV.—The logarithm of a number consists of two parts, a whole number called the *characteristic*, and a decimal called the *mantissa*. All numbers which consist of the same figures standing in the same order have the same mantissa, regardless of the position of the decimal point in the number, or of the number of ciphers which precede or follow *the significant figures of* the number. The value of the char*acteristic depends entirely* on the position of the decimal point *in the number. It is* always one less than the number o

figures in the number to the left of the decimal point. The value is therefore diminished by one every time the decimal point of the number is removed one place to the left, and *v. co versa.* Thus

Number.	*Logarithm.*
13840.	4.141136
1384.0	3.141136
138.40	2.141136
13.84	1.141136
1.384	0.141136
.1384	—1.141136
.01384	—2.141136
.001384	—3.141136
etc.	etc.

The mantissa is always positive even when the characteristic is negative. We may avoid the use of a negative characteristic by arbitrarily adding 10, which may be neglected at the close of the calculation. By this rule we have

Number.	*Logarithm.*
1.384	0.141136
.1384	9.141136
.01384	8.141136
.001384	7.141136
etc.	etc.

No confusion need arise from this method in finding a number from its logarithm; for although the logarithm 6.141136 represents either the number 1,384,000, or the decimal .0001384, yet these are so diverse in their values that we can never be uncertain in a given problem which to adopt.

The table XXIV. contains the mantissas of logarithms, carried to six places of decimals, for numbers between 1 and 9999, inclusive. The first three figures of a number are given in the first column, the fourth at the top of the other columns. The first two figures of the mantissa are given only in the second column, but these are understood to apply to the remaining four figures in either column following, which are comprised between the same horizontal lines with the two.

If a number (after cutting off the ciphers at either end) consists of not more than four figures, the mantissa may be taken direct from the table; but by interpolation the logarithm of a number having six figures may be obtained. The last column contains the average difference of consecutive logarithms on the same line, but for a given case the difference needs to be verified by actual subtraction, at least so far as the last figure is concerned. The lower part of the page contains a complete list of differences, with their multiples divided by 10.

To find the logarithm of a number having six figures:—Take out the mantissa for the four superior places directly from the table, and find the difference between this mantissa and the next greater in the table. Add to the mantissa taken out the quantity found in the table of proportional parts, opposite the difference, and in the column headed by the fifth figure of the number; also add $\frac{1}{10}$ the quantity in the column headed by the sixth figure. The sum is the mantissa required, to which must be prefixed a decimal point and the proper characteristic.

Example.—Find the log of 23.4275.

For 2342 mantissa is 369587
" diff. 185 col. 7 129.5
" " " " 5 9.2

Ans. For 23.4275 log is 1.369726

The decimals of the corrections are added together to determine the nearest value of the sixth figure of the mantissa.

To find the number corresponding to a given logarithm.—If the given mantissa is not in the table find the one next less, and take out the four figures corresponding to it; divide the difference between the two mantissas by the tabular difference in that part of the table, and annex the figures of the quotient to the four figures already taken out. Finally, place the decimal point according to the rule for characteristics, prefixing or annexing ciphers if necessary. The division required is facilitated by the table of proportional parts, which furnishes by inspection the figures of the quotient.

Example.—Find the number of which the logarithm is 8.263927

8.263927
First 4 figures 1836 from 263873
Diff. 54.0
Tabular diff. = 236 ∴ 5th fig. = 2 47.2
6.80
6th fig. = 3 7.08

Ans. No. = *.0183623* or 183,623,000.

The number derived from a six-place logarithm is not reliable beyond the sixth figure.

At the end of table XXIV. is a small table of logarithms of numbers from 1 to 100, with the characteristic prefixed, for easy reference when the given number does not exceed two digits. But the same mantissas may be found in the larger table.

TABLE XXV.—**The logarithmic sine, tangent, etc.** of an arc is the logarithm of the natural sine, tangent, etc. of the same arc, but with 10 added to the characteristic to avoid negatives. This table gives log sines, tangents, cosines, and cotangents for every minute of the quadrant. With the number of degrees at the left side of the page are to be read the minutes in the left-hand column; with the degrees on the right-hand side are to be read the minutes in the right-hand column. When the degrees appear at the top of the page the top headings must be observed, when at the bottom those at the bottom. Since the values found for arcs in the first quadrant are duplicated in the second, the degrees are given from 0° to 180°. The differences in the logarithms due to a change of one second in the arc are given in adjoining columns.

To find the log. sin, cos, tan, or cot of a given arc.: Take out from the proper column of the table the logarithm corresponding to the given number of degrees and minutes. If there be any seconds multiply them by the adjoining tabular difference, and apply their product as a correction to the logarithm already taken out. The correction is to be *added* if the logarithms of the table are increasing with the angle, or *subtracted* if they are decreasing as the angle increases. In the first quadrant the log sines and tangents increase, and the log. cosines and cotangents decrease as the angle increases.

Example.—Find the log sin of 9° 28′ 20″.

Log sin of 9° 28′ is	9.216097
Add correction 20 × 12.62	252
Ans.	9.216349

Example.—Find the log cot of 9° 28′ 20″.

Log cotan of 9° 28′ is	10.777948
Subtract correction 20 × 12.97	259
Ans.	10.777689

To find the angle or arc corresponding to a given logarithmic sine, tangent, cosine, or cotangent.—If the given logarithm is found in the proper column take out the degrees and minutes directly; if not, find the two consecutive logarithms between which the given logarithm would fall, and adopt that one which corresponds to the least number of minutes; which minutes take out with the degrees, and divide the difference between this logarithm and the given one by the adjoining tabular difference for a quotient, which will be the required number of seconds.

With logarithms to six places of decimals the quotient is not reliable beyond the tenth of a second.

Example.—9.383731 is the log tan of what angle?
Next *less* 9.383682 gives 13° 36′
Diff. 49.00 ÷ 9.20 = 05″.3
Ans. 13° 36′ 05″.3

Example.—9.249348 is the log cos of what angle?
Next *greater* 583 gives 79° 46′
Diff. 235 ÷ 11.67 = 20″.1
Ans. 79° 46′ 20″.1

The above rules do not apply to the first two pages of this table (except for the column headed cosine at top) because here the differences vary so rapidly that interpolation made by them in the usual way will not give exact results.

On the first two pages, the *first* column contains the number of seconds for every minute from 1′ to 2°; the minutes are given in the *second*, the log. sin. in the *third*, and in the *fourth* are the last three figures of a logarithm which is the difference between the log sin and the logarithm of the number of seconds in the first column. The first three figures and the characteristic of this logarithm are placed, once for all, at the head of the column.

To find the log sin of an arc less than 2° given to seconds.—Reduce the given arc to seconds, and take the logarithm of the number of seconds from the table of logarithms, *and add to this* the logarithm from the fourth column *opposite the same* number of seconds. The sum is the log sin *required.*

The logarithm in the fourth column may need a slight inte polation of the last figure, to make it correspond closely to the given number of seconds.

Example.—Find the log sin of 1° 39′ 14″.4.

1° 39′ 14″.4 = 5954″.4 log 3.774838
add $(q-l)$ 4.685515

Ans. log sin 8.460353

Log tangents of small arcs are found in the same way, only taking the last four figures of $(q-l)$ from the *fifth* column.

Example.—Find the log tan of 0° 52′ 35″.

52′ 35″ = (3120″ + 35″) = 3155″ log 3.498999
add $(q-l)$ 4.685609

Ans. log tan 8.184608

To find the log cotangent of an angle less than 2° given to seconds.—Take from the column headed $(q+l)$ the logarithm corresponding to the given angle, interpolating for the last figure if necessary, and from this *subtract* the logarithm of the number of seconds in the given angle.

Example.—Find the log cotan of 1° 44′ 22″.5.

$q+l$ 15.314292
6240″ + 22″.5 = 6262.5 log 3.796748

Ans. 11.517544

These two pages may be used in the same way when the given angle lies between 88° and 92°, or between 178° and 180°; but if the number of degrees be found at the *bottom* of the page, the title of each column will be found there also; and if the number of degrees be found on the *right hand* side of the page, the number of minutes must be found in the right hand column, and since here the minutes increase upward, the number of seconds on the same line in the first column must be *diminished* by the odd seconds in the given angle to obtain the number whose logarithm is to be used with $(q \pm l)$ taken from the table.

Example.—Find the log cos of 88° 41′ 12″.5

$(q-l)$ 4.685537
4740″ − 12″.5 = 4727.5 log 3.674631

Ans. 8.360168

Example.—Find the log tan of 90° 30′ 50″.

$1800'' + 50'' = 1850''$ $\qquad$ $q + l$ 15.314413
log 3.267172
Ans. 12.047241

To find the arc corresponding to a given log sin, cos, tan, or cotan which falls within the limits of the first two pages of Table XXV.

Find in the proper column two consecutive logarithms between which the given logarithm falls. If the title of the given function is found at the top of that column read the degrees from the top of the page; if at the bottom read from the bottom.

Find the value of $(q - l)$ or $(q + l)$, as the case may require, corresponding to the given log (interpolating for the last figure if necessary). Then if $q =$ given log and $l =$ log of number of seconds, n, in the required arc, we have at once $l = q - (q - l)$ or $l = (q + l) - q$, whence n is easily found.

Find in the first column two consecutive quantities between which the number n falls, and if the degrees are read from the *left hand* side of the page, adopt the *less*, take out the minutes from the second column, and take for the seconds the difference between the quantity adopted and the number n. But if the degrees are read from the *right hand* side of the page, adopt the *greater* quantity, take out the minutes on the same line from the right-hand column, and for the seconds take the difference between the number adopted and the number n.

Example.—11.734268 is the log cot of what arc?

$q + l$ 15.314376
q 11.734268
$\therefore\ n =$ 3802.8 $\qquad$ 3.580108
For 1° adopt 3780. giving 03′

Difference 22″.8
Ans. 1° 03′ 22″.8 or 178° 56′ 37″.2.

Example.—8.201795 is the log cos of what arc?

$q - l$ 4.685556
q 8.201795
$\therefore\ n =$ 3282″.8 $\qquad$ 3.516239
For 89° adopt 3300. giving 05′

Difference 17″.2
Ans. 89° 05′ 17″.2 or 90° 54′ 42″.8.

TABLE XXVI.—Contains logarithmic versed sines and external secants for every minute of the quadrant, with the differences of the same corresponding to a change of 1 second in the arc or angle. Interpolation for seconds is made in the same manner as with log sines of the preceding table, except on the first two pages. For angles less than 4° the differences vary so rapidly that interpolation by direct proportion will not give exact values.

On the first two pages the column headed $q - 2l$ contains the difference between the log versed sine (or log ex secant) of an arc and *twice* the logarithm of the number of seconds in the same arc. The characteristic, and first three decimals (9.070) are common to all the logarithms in these columns up to 3° 19′ for log vers sines, where it changes to (9.069), as shown at the foot of the column; and up to 4° for log ex secants, where it changes to (9.071). At the point of change a cipher is replaced by the mark ✦ to call attention.

To find the log vers sin, or log ex sec of an angle given to seconds.—Reduce the angle to seconds, take the logarithm of this number, multiply it by 2, and add the product to the logarithm in the column $(q - 2l)$ found opposite the given angle. The log $(q - 2l)$ should be corrected by interpolation for the fractional part of a minute in the given angle.

Example.—What is the log ex secant of 2° 14′ 43″.7?

$$
\begin{array}{lrr}
2^\circ\,14'\,43''.7 = 8040'' + 43.7 = 8083''.7 & \log & 3.907610 \\
 & & 2 \\
\hline
 & 2l & 7.815220 \\
 & (q - 2l) & 9.070064 \\
\hline
\textit{Ans.} & \therefore\ q & 6.885284
\end{array}
$$

To find the arc corresponding to a given log vers, or log ex sec.—Find in the column of log vers, or log ex sec the two values between which the given log falls, and take out from the column $(q - 2l)$ the logarithm corresponding to the given log (interpolating for the value of the last figure if necessary). Subtract this from the given logarithm and divide by 2. The quotient is the logarithm of the number of seconds in the required arc.

Example.—7.344728 is the log vers of what arc?

q 7.344728

3° 48′ + $(q - 2l)$ 9.069960

2)8.274768

13720″.9 ∴ l 4.137384

13680.

Ans. 3° 48′ 40″.9

To find the log ex sec of an arc greater than 88° given to seconds.—Take from the column $(q + l)$ the logarithm corresponding to the given arc, interpolating for the fraction of a minute. From this subtract the logarithm of the number of seconds in the *complement* of the given arc.

Example.—What is the log ex sec of 88° 24′ 20″.5?

For 88° 24′ $q + l$ 15.302183

Correction $129 \times \frac{20.5}{60} =$ 44

$q + l$ 15.302227

Comp. 88° 24′ 20″.5 = 5739″.5 log 3.758874

Ans. log ex sec 11.543353

To find the angle corresponding to a given log ex sec when the angle is greater than 88°.—Find in the table the two consecutive log ex secants between which the given one falls, and then find by interpolation the value of the log $(q + l)$ corresponding to the given log ex sec and subtract the latter from it. The difference will be the logarithm of the number of seconds in the *complement* of the required angle, which is then easily found.

Example.—11.924368 is the log ex sec of what arc?

Given log ex sec 11.924368

Next less 11.918290 $q + l$ 15.309225

Diff. 6078

$$\text{Correction} = \frac{9352 - 9225}{29141 - 18290} \times 6078 = 71$$

$q + l$ 15.309296

Given *log ex sec* 11.924368

∴ log 3.384928

0° 40′ 26″.2 = 2426″.2

Ans. 89° 19′ 33″.8.

TABLE XXVII.—Contains natural sines and cosines, to five places of decimals for every minute of the quadrant. Corrections for fractions of a minute are made directly proportional to the difference of consecutive values in the table; positive for sines, negative for cosines.

TABLE XXVIII.—Contains natural tangents and cotangents to five places of decimals for every minute of the quadrant. Corrections for fractions of a minute are made directly proportional to the difference of consecutive values in the table; positive for tangents, negative for cotangents.

TABLE XXIX.—Contains natural versed sines and external secants to five places of decimals for every minute of the quadrant. Corrections for fractions of a minute are made directly proportional to the difference of consecutive values. They are positive in every case.

TABLE XXX.—Contains the number of cubic yards contained in prismoids of various side slopes, bases, and depths, as indicated by the title and the numbers in the first column. Each prismoid is supposed to have a uniform level cross section throughout. These tables are chiefly useful in making up preliminary estimates from the profile, or in other cases where only approximate results are required. For monthly and final estimates more elaborate tables are required, such as are described in § 257.

To make an approximate estimate of quantities from a profile by use of Table XXX.—Select the proper column for base and slopes, and if the outline of a cut on the profile is roughly a four-sided figure, stretch a fine silk thread over the surface line to average it, note the depth from thread to grade line midway of the cutting, and multiply the tabular number opposite this depth by the average length of the cutting in stations of 100 feet. (By average length is meant the half sum of the length of the grade line in the cutting and of so much of the surface line as is covered by the thread.) If *the area of a cutting* as seen on the profile is approximately *triangular, stretch* an averaging line over each incline, and *note the depth* from the intersection of these lines to grade, *and multiply* the tabular number opposite this depth by one-half the length of the cut measured on the grade line in stations. The resulting quantities will be slightly in excess if the ground is level transversely, but may be too small if the transverse slope is steep, and cutting on the centre line is small. In general they furnish a good approximation. Quantities in embankments may, of course, be found similarly. A cut or fill may be divided on the profile into several portions, and the contents of each portion found separately if preferred.

The content of a prismoid, level transversely, but having different end depths, may be found correctly by this table thus: add together the quantities opposite each end-depth and 4 times the quantity opposite the half sum of the depths; multiply the sum by the length in feet, and divide by 600.

TABLE XXXI.—Contains a variety of useful numbers and formulæ. The logarithms are here given to seven places of decimals.

TABLES.

TABLE I.—GEOMETRICAL PROPOSITIONS.

The References are to Davies' Legendre, Revised Edition.

No.	Reference.	Hypotheses.	Consequences.
1	IV., XI.	If a triangle is right angled,	The square on the hypothenuse is equal to the sum of the squares on the other two sides.
2	I., XI., Cor. 1.	If a triangle is equilateral,	It is equiangular.
3	I., XI.	If a triangle is isosceles,	The angles at the base are equal.
4	I., XI., Cor. 2.	If a straight line from the vertex of an isosceles triangle bisects the base,	It bisects the vertical angle. And is perpendicular to the base.
5	I., XXV., Cor. 6.	If one side of a triangle is produced,	The exterior angle is equal to the sum of the two interior and opposite angles.
6	IV., XX.	If two triangles are mutually equiangular,	They are similar. And their corresponding sides are proportional.
7	I., XXVII.	If the sides of a polygon are produced in the same order,	The sum of the exterior angles equals four right angles.
8	I., XXVI., Cor. 1.	If a figure is a quadrilateral,	The sum of the interior angles equals four right angles.
9	I., XXVIII. I., XXXI.	If a figure is a parallelogram,	The opposite sides are equal. The opposite angles are equal. It is bisected by its diagonal. And its diagonals bisect each other.
10	III., VII.	If three points are not in the same straight line,	A circle may be passed through them.
11	III., XVII.	If two arcs are intercepted on the same circle,	They are proportional to the corresponding angles at the centre.
12	V., XIII., Cor. 2.	If two arcs are similar,	They are proportional to their radii.
13	V., XIII.	If two areas are circles,	They are proportional to the squares on their radii.
14	III., VI.	If a radius is perpendicular to a chord,	It bisects the chord. And it bisects the arc subtended by the chord.
15	III., IX.	If a straight line is tangent to a circle,	It meets it in only one point. And it is perpendicular to the radius drawn to that point.
16	III., XIV., Cor. I., XI.	If from a point without a circle tangents are drawn to touch the circle,	There are but two. They are equal. And they make equal angles with the chord joining the tangent points.

TABLE I.—GEOMETRICAL PROPOSITIONS.

The References are to Davies' Legendre, Revised Edition.

No.	Reference.	Hypotheses.	Consequences.
17	III., X.	If two lines are parallel chords or a tangent and parallel chord,	They intercept equal arcs of a circle.
18	III., XVIII.	If an angle at the circumference of a circle is subtended by the same arc as an angle at the centre,	The angle at the circumference is equal to half the angle at the centre.
19	III., XVIII., Cor.2	If an angle is inscribed in a semicircle,	It is a right angle.
20	III., XXI.	If an angle is formed by a tangent and chord,	It is measured by one half of the intercepted arc.
21	IV., XXVIII., Cor.	If two chords intersect each other in a circle,	The rectangle of the segments of the one, equals the rectangle of the segments of the other.
22	IV., XXIII., Cor. 2	And if one chord is a diameter, and the other perpendicular to it,	The rectangle of the segments of the diameter is equal to the square on half the other chord. And the half chord is a mean proportional between the segments of the diameter.
23	IV., XXIX., Cor..	If two secants meet without the circle,	The rectangles of each secant and its external segment are equal.
24	IV., XXX.	If a secant and tangent meet,	The rectangle of the secant and its external segment is equal to the square on the tangent. And the tangent is a mean proportional between the secant and its external segment.
25	IV., XIV.	If a straight line from the vertex of a triangle bisects its base,	The sum of the squares on the two sides is equal to twice the square of half the base increased by twice the square of the bisecting line.
26	IV., XII.	If a perpendicular be drawn from the vertex of a triangle to the base,	The square of a side opposite an acute angle is equal to the sum of the squares of the other side and the base, diminished by twice the rectangle of the base and the distance from the vertex of the acute angle to the foot of the perpendicular.

TABLE II.—TRIGONOMETRIC FORMULÆ.

TRIGONOMETRIC FUNCTIONS.

Let A (Fig. 107) = angle BAC = arc BF, and let the radius $AF = AB$: $AH = 1$.

We then have

$\sin A = BC$
$\cos A = AC$
$\tan A = DF$
$\cot A = HG$
$\sec A = AD$
$\operatorname{cosec} A = AG$
$\operatorname{versin} A = CF = BE$
$\operatorname{covers} A = BK = HL$
$\operatorname{exsec} A = BD$
$\operatorname{coexsec} A = BG$
$\operatorname{chord} A = BF$
$\operatorname{chord} 2A = BI = 2BC$

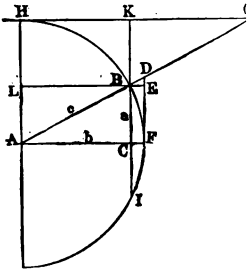

FIG. 107.

In the right-angled triangle ABC (Fig. 107)

Let $AB = c$, $AC = b$, and $BC = a$.

We then have :

1. $\sin A = \frac{a}{c} = \cos B$
2. $\cos A = \frac{b}{c} = \sin B$
3. $\tan A = \frac{a}{b} = \cot B$
4. $\cot A = \frac{b}{a} = \tan B$
5. $\sec A = \frac{c}{b} = \operatorname{cosec} B$
6. $\operatorname{cosec} A = \frac{c}{a} = \sec B$
7. $\operatorname{vers} A = \frac{c-b}{c} = \operatorname{covers} B$
8. $\operatorname{exsec} A = \frac{c-b}{b} = \operatorname{coexsec} B$
9. $\operatorname{covers} A = \frac{c-a}{c} = \operatorname{versin} B$
10. $\operatorname{coexsec} A = \frac{c-a}{a} = \operatorname{exsec} B$
11. $a = c \sin A = b \tan A$
12. $b = c \cos A = a \cot A$
13. $c = \frac{a}{\sin A} = \frac{b}{\cos A}$
14. $a = c \cos B = b \cot B$
15. $b = c \sin B = a \tan B$
16. $c = \frac{a}{\cos B} = \frac{b}{\sin B}$
17. $a = \sqrt{(c+b)(c-b)}$
18. $b = \sqrt{(c+a)(c-a)}$
19. $c = \sqrt{a^2+b^2}$
20. $C = 90° = A + B$

21. area $= \frac{ab}{2}$

TABLE II.—TRIGONOMETRIC FORMULÆ.

SOLUTION OF OBLIQUE TRIANGLES.

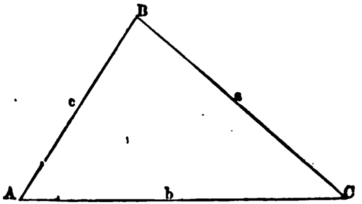

FIG. 108.

	GIVEN.	SOUGHT.	FORMULÆ.
22	A, B, a	C, b, c	$C = 180° - (A + B)$, $b = \frac{a}{\sin A} \cdot \sin B$, $c = \frac{a}{\sin A} \sin (A + B)$
23	A, a, b	B, C, c	$\sin B = \frac{\sin A}{a} \cdot b$, $C = 180° - (A + B)$, $c = \frac{a}{\sin A} \cdot \sin C$.
24	C, a, b	$\frac{1}{2}(A + B)$	$\frac{1}{2}(A + B) = 90° - \frac{1}{2} C$
25		$\frac{1}{2}(A - B)$	$\tan \frac{1}{2}(A - B) = \frac{a - b}{a + b} \tan \frac{1}{2}(A + B)$
26		A, B	$A = \frac{1}{2}(A + B) + \frac{1}{2}(A - B)$, $B = \frac{1}{2}(A + B) - \frac{1}{2}(A - B)$.
27		c	$c = (a + b) \frac{\cos \frac{1}{2}(A + B)}{\cos \frac{1}{2}(A - B)} = (a - b) \frac{\sin \frac{1}{2}(A + B)}{\sin \frac{1}{2}(A - B)}$
28		area	$K = \frac{1}{2} a b \sin C$.
29	a, b, c	A	Let $s = \frac{1}{2}(a + b + c)$; $\sin \frac{1}{2} A = \sqrt{\frac{(s - b)(s - c)}{b c}}$
30			$\cos \frac{1}{2} A = \sqrt{\frac{s (s - a)}{b c}}$; $\tan \frac{1}{2} A = \sqrt{\frac{(s - b)(s - c)}{s (s - a)}}$
31			$\sin A = \frac{2 \sqrt{s (s - a)(s - b)(s - c)}}{b c}$; $\text{vers}\ A = \frac{2 (s - b)(s - c)}{b c}$
32		area	$K = \sqrt{s (s - a)(s - b)(s - c)}$
33	A, B, C, a	area	$K = \frac{a^2 \sin B \cdot \sin C}{2 \sin A}$

TABLE II.—TRIGONOMETRIC FORMULÆ.

GENERAL FORMULÆ.

34 $\sin A = \frac{1}{\operatorname{cosec} A} = \sqrt{1-\cos^2 A} = \tan A \cos A$

35 $\sin A = 2 \sin \tfrac{1}{2} A \cos \tfrac{1}{2} A = \operatorname{vers} A \cot \tfrac{1}{2} A$

36 $\sin A = \sqrt{\tfrac{1}{2} \operatorname{vers} 2A} = \sqrt{\tfrac{1}{2}(1-\cos 2A)}$

37 $\cos A = \frac{1}{\sec A} = \sqrt{1-\sin^2 A} = \cot A \sin A$

38 $\cos A = 1 - \operatorname{vers} A = 2\cos^2 \tfrac{1}{2} A - 1 = 1 - 2\sin^2 \tfrac{1}{2} A$

39 $\cos A = \cos^2 \tfrac{1}{2} A - \sin^2 \tfrac{1}{2} A = \sqrt{\tfrac{1}{2} + \tfrac{1}{2}\cos 2A}$

40 $\tan A = \frac{1}{\cot A} = \frac{\sin A}{\cos A} = \sqrt{\sec^2 A - 1}$

41 $\tan A = \sqrt{\frac{1}{\cos^2 A} - 1} = \frac{\sqrt{1-\cos^2 A}}{\cos A} = \frac{\sin 2A}{1+\cos 2A}$

42 $\tan A = \frac{1-\cos 2A}{\sin 2A} = \frac{\operatorname{vers} 2A}{\sin 2A} = \operatorname{exsec} A \cot \tfrac{1}{2} A$

43 $\cot A = \frac{1}{\tan A} = \frac{\cos A}{\sin A} = \sqrt{\operatorname{cosec}^2 A - 1}$

44 $\cot A = \frac{\sin 2A}{1-\cos 2A} = \frac{\sin 2A}{\operatorname{vers} 2A} = \frac{1+\cos 2A}{\sin 2A}$

45 $\cot A = \frac{\tan \tfrac{1}{2} A}{\operatorname{exsec} A}$

46 $\operatorname{vers} A = 1 - \cos A = \sin A \tan \tfrac{1}{2} A = 2\sin^2 \tfrac{1}{2} A$

47 $\operatorname{vers} A = \operatorname{exsec} A \cos A$

48 $\operatorname{exsec} A = \sec A - 1 = \tan A \tan \tfrac{1}{2} A = \frac{\operatorname{vers} A}{\cos A}$

49 $\sin \tfrac{1}{2} A = \sqrt{\frac{1-\cos A}{2}} = \sqrt{\frac{\operatorname{vers} A}{2}}$

50 $\sin 2A = 2 \sin A \cos A$

51 $\cos \tfrac{1}{2} A = \sqrt{\frac{1+\cos A}{2}}$

$\cos 2A = 2\cos^2 A - 1 = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A$

GENERAL FORMULÆ.

53. $\tan \frac{1}{2} A = \dfrac{\tan A}{1 + \sec A} = \operatorname{cosec} A - \cot A = \dfrac{1 - \cos A}{\sin A} = \sqrt{\dfrac{1 - \cos A}{1 + \cos A}}$

54. $\tan 2 A = \dfrac{2 \tan A}{1 - \tan^2 A}$

55. $\cot. \frac{1}{2} A = \dfrac{\sin A}{\operatorname{vers} A} = \dfrac{1 + \cos A}{\sin A} = \dfrac{1}{\operatorname{cosec} A - \cot A}$

56. $\cot 2 A = \dfrac{\cot^2 A - 1}{2 \cot A}$

57. $\operatorname{vers} \frac{1}{2} A = \dfrac{\frac{1}{2} \operatorname{vers} A}{1 + \sqrt{1 - \frac{1}{2} \operatorname{vers} A}} = \dfrac{1 - \cos A}{2 + \sqrt{2 (1 + \cos A)}}$

58. $\operatorname{vers} 2 A = 2 \sin^2 A$

59. $\operatorname{exsec} \frac{1}{2} A = \dfrac{1 - \cos A}{(1 + \cos A) + \sqrt{2 (1 + \cos A)}}$

60. $\operatorname{exsec} 2 A = \dfrac{\tan^2 A}{1 - \tan^2 A}$

61. $\sin (A \pm B) = \sin A . \cos B \pm \sin B . \cos A$

62. $\cos (A \pm B) = \cos A . \cos B \mp \sin A . \sin B$

63. $\sin A + \sin B = 2 \sin \frac{1}{2} (A + B) \cos \frac{1}{2} (A - B)$

64. $\sin A - \sin B = 2 \cos \frac{1}{2} (A + B) \sin \frac{1}{2} (A - B)$

65. $\cos A + \cos B = 2 \cos \frac{1}{2} (A + B) \cos \frac{1}{2} (A - B)$

66. $\cos B - \cos A = 2 \sin \frac{1}{2} (A + B) \sin \frac{1}{2} (A - \mathrm{B})$

67. $\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin (A + B) \sin (A - \mathrm{B})$

68. $\cos^2 A - \sin^2 B = \cos (A + B) \cos (A - B)$

69. $\tan A + \tan B = \dfrac{\sin (A + B)}{\cos A . \cos B}$

70. $\tan A - \tan B = \dfrac{\sin (A - B)}{\cos A . \cos B}$

TABLE III.—CURVE FORMULÆ.

	GIVEN.	SOUGHT.	FORMULÆ.
	R, M	*T*	$T = \frac{R \sqrt{M(2R - M)}}{R - M}$
	"	*C*	$C = 2\sqrt{M(2R - M)}$
	"	*E*	$E = \frac{RM}{R - M}$
	R, E	*T*	$T = \sqrt{E(2R + E)}$
	"	*C*	$C = \frac{2R \sqrt{E(2R + E)}}{R + E}$
	"	*M*	$M = \frac{RE}{R + E}$
	T, C	*R*	$R = \frac{CT}{\sqrt{(2T + C)(2T - C)}}$
	"	*M*	$M = \frac{1}{2} C \sqrt{\frac{2T - C}{2T + C}}$
	"	*E*	$E = T \sqrt{\frac{2T - C}{2T + C}}$
	T, E	*R*	$R = \frac{(T + E)(T - E)}{2E}$
58	"	*C*	$C = \frac{2T(T^2 - E^2)}{T^2 + E^2}$
59	"	*M*	$M = \frac{E(T^2 - E^2)}{T^2 + E^2}$
60	*C, M*	*R*	$R = \frac{M^2 + (\frac{1}{2}C)^2}{2M}$
61	"	*T*	$T = \frac{C(C^2 + 4M^2)}{2(C^2 - 4M^2)}$
62	"	*E*	$E = M \frac{C^2 + 4M^2}{C^2 - 4M^2}$
63	*M, E*	*R*	$R = \frac{EM}{E - M}$
64	"	*T*	$T = E \sqrt{\frac{E + M}{E - M}}$
65	"	*C*	$C = 2M \sqrt{\frac{E + M}{E - M}}$
66	*T, M*	*R*	$R^3 - R^2 \frac{M^2 + T^2}{2M} + RT^2 - \frac{1}{2} MT^2 =$
67	"	*E*	$E^3 + E^2 M - ET^2 + MT^2 = 0$
68	"	*C*	$C^3 + 2TC^2 + 4M^2 C - 8M^2 T = 0$
69	*C, E*	*R*	$R^3 + R^2 \frac{4E^2 - C^2}{8E} - R \frac{C^2}{4} - \frac{C^2}{8}$
70	"	*T*	$2T^3 - T^2 C - 2TE^2 - CE^2 = 0$
	"	*M*	$M^3 + M^2 E + M \frac{C^2}{4} - \frac{C^2 E}{4} = 0$

Deg. D.	Radius. R.	Logarithm. log. R.	Tan. Off. t.	Mid. Ord. m.	Deg. D.	Radius. R.	Logarithm. log. R.	Tang. Off. t.	Mid. Ord. m.
0° 0′	Infinite	Infinite	.000	.000	1° 0′	5729.65	3.758128	.873	.218
1	343775.	5 536274	.015	.001	1	5635.72	.750950	.887	.222
2	171887.	5.235244	.029	.007	2	5544.83	.743888	.902	.225
3	114592.	5.059153	.044	.011	3	5456.82	.736939	.916	.229
4	85943.7	4.934214	.058	.015	4	5371.56	.730100	.931	.233
5	68754.9	.837304	.073	.018	5	5288.92	.723367	.945	.236
6	57295.8	.758123	.087	.022	6	5208.79	.716737	.960	.240
7	49110.7	.691176	.102	.025	7	5131.05	.710206	.974	.244
8	42971.8	.633184	.116	.029	8	5055.59	.703772	.989	.247
9	38197.2	.582031	.131	.033	9	4982.33	.697432	1.004	.251
10	34377.5	4.536274	.145	.036	10	4911.15	3.691183	1.018	.255
11	31252.3	4.494881	.160	.040	11	4841.98	3.685023	1.033	.258
12	28647.8	.457093	.175	.044	12	4774.74	.678949	1.047	.262
13	26444.2	.422331	.189	.047	13	4709.33	.672959	1.062	.265
14	24555.4	.390146	.204	.051	14	4645.69	.667051	1.076	.269
15	22918.3	.360183	.218	.055	15	4583.75	.661221	1.091	.273
16	21485.9	.332154	.233	.658	16	4523.44	.655469	1.105	.276
17	20222.1	.305825	.247	.062	17	4464.70	.649792	1.120	.280
18	19098.6	.281002	.262	.065	18	4407.46	.644189	1.134	.284
19	18093.4	.257521	.276	.069	19	4351.67	.638656	1.149	.287
20	17188.8	4.235244	.291	.073	20	4297.28	3.633194	1.164	.291
21	16370.2	4.214055	.305	.076	21	4244.23	3.627799	1.178	.295
22	15626.1	.193852	.320	.080	22	4192.47	.622470	1.193	.298
23	14946.7	.174547	.335	.084	23	4141.96	.617206	1.207	.302
24	14323.6	.156064	.349	.087	24	4092.66	.612005	1.222	.305
25	13751.0	.138335	.364	.091	25	4044.51	.606866	1.236	.309
26	13222.1	.121302	.378	.095	26	3997.49	.601787	1.251	.313
27	12732.4	.104911	.393	.098	27	3951.54	.596766	1.265	.316
28	12277.7	.089117	.407	.102	28	3906.54	.591803	1.280	.320
29	11854.3	.073877	.422	.105	29	3862.74	.586896	1.294	.324
30	11459.2	4.059154	.436	.109	30	3819.83	3.582044	1.309	.327
31	11089.6	4.044914	.451	.113	31	3777.85	3.577245	1.324	.331
32	10743.0	.031125	.465	.116	32	3736.79	.572499	1.338	.335
33	10417.5	.017762	.480	.120	33	3696.61	.567804	1.353	.338
34	10111.1	4.004797	.495	.124	34	3657.29	.563160	1.367	.342
35	9822.18	3.992208	.509	.127	35	3618.80	.558564	1.382	.345
36	9549.34	.979973	.524	.131	36	3581.10	.554017	1.396	.349
37	9291.29	.968074	.538	.135	37	3544.19	.549517	1.411	.353
38	9046.75	.956493	.553	.138	38	3508.02	.545063	1.425	.356
39	8814.78	.945212	.567	.142	39	3472.59	.540654	1.440	.360
40	8594.42	3.934216	.582	.145	40	3437.87	3.536289	1.454	.364
41	8384.80	3.923493	.596	.149	41	3403.83	3.531968	1.469	.367
42	8185.16	.913027	.611	.153	42	3370.46	.527690	1.483	.371
43	7994.81	.902808	.625	.156	43	3337.74	.523453	1.498	.375
44	7813.11	.892824	.640	.160	44	3305.65	.519257	1.513	.378
45	7639.49	.883065	.654	.164	45	3274.17	.515101	1.527	.382
46	7473.42	.873519	.669	.167	46	3243.29	.510985	1.542	.385
47	7314.41	.864179	.684	.171	47	3212.98	.506908	1.556	.389
48	7162.03	.855036	.698	.174	48	3183.23	.502868	1.571	.393
49	7015.87	.846082	.713	.178	49	3154.03	.498866	1.585	.396
50	6875.55	3.837308	.727	.182	50	3125.36	3.494900	1.600	.400
51	6740.74	3.828708	.742	.185	51	3097.20	3.490970	1.614	.404
52	6611.12	.820275	.756	.189	52	3069.55	.487075	1.629	.407
53	6486.38	.812002	.771	.193	53	3042.39	.483215	1.643	.411
54	6366.26	.803885	.785	.196	54	3015.71	.479389	1.658	.414
55	6250.51	.795916	.800	.200	55	2989.48	.475596	1.673	.418
56	6138.90	.788091	.814	.204	56	2963.71	.471838	1.687	.422
57	6031.20	.780404	.829	.207	57	2938.39	.468109	1.702	.425
58	5927.22	.772851	.844	.211	58	2913.49	.464413	1.716	.429
59	5826.76	.765427	.858	.215	59	2889.01	.460749	1.731	[illegible]
60	5729.65	3.758128	.873	.218	60	2864.93	3.457115	1.745	[illegible]

Deg. D.	Radius. R.	Logarithm. log. R.	Tang. Off. t.	Mid. Ord. m.	Deg. D.	Radius. R.	Logarithm. log. R.	Tang. Off. t.
2° 0′	2864.93	3 457115	1.745	.436	3° 0′	1910.08	3.281051	2.618
1	2841.26	.453511	1.760	.440	1	1899.53	.278546	2.632
2	2817.97	.449937	1.774	.444	2	1889.09	.276253	2.647
3	2795.06	.446392	1.789	.447	3	1878.77	.273874	2.661
4	2772.53	.442876	1.803	.451	4	1868.56	.271508	2.676
5	2750.35	.439358	1.818	.454	5	1858.47	.269155	2.690
6	2728.52	.435928	1.832	.458	6	1848.48	.266814	2.705
7	2707.04	.432495	1.847	.462	7	1838.59	.264486	2.719
8	2685.89	.429089	1.862	.465	8	1828.82	.262170	2.734
9	2665.08	.425710	1.876	.469	9	1819.14	.259867	2.749
10	2644.58	3 422356	1.891	.473	10	1809.57	3.257576	2.763
11	2624.39	3.419029	1.905	.476	11	1800.10	3.255296	2.778
12	2604.51	.415727	1.920	.480	12	1790.73	.253029	2.792
13	2584.93	.412449	1.934	.484	13	1781.45	.250774	2.807
14	2565.65	.409197	1.949	.487	14	1772.27	.248530	2 821
15	2546.64	.405968	1.963	.491	15	1763.18	.246297	2.836
16	2527.92	.402763	1.978	.494	16	1754.19	.244077	2.850
17	2509.47	.399582	1.992	.498	17	1745.26	.241867	2.865
18	2491.29	.396424	2.007	.502	18	1736.48	.239669	2.879
19	2473.37	.393289	2.022	.505	19	1727 75	.237481	2.894
20	2455.70	3 390176	2.036	.509	20	1719.12	3.235305	2.908
21	2438.29	3.387085	2.051	.513	21	1710.56	3.233140	2.923
22	2421.12	.384016	2.065	.516	22	1702.10	.230985	2.938
23	2404.19	.380969	2.080	.520	23	1693.72	.228841	2.952
24	2387.50	.377943	2.094	.524	24	1685.42	.226707	2.967
25	2371.04	.374938	2.109	.527	25	1677.20	.224584	2.981
26	2354.80	.371954	2.123	.531	26	1669.06	.222472	2.996
27	2338.78	.368990	2.138	.534	27	1661.00	.220369	3.010
28	2322.98	.366046	2.152	.538	28	1653.01	.218277	3.025
29	2307.39	.363122	2.167	.542	29	1645.11	.216195	3.039
30	2292.01	3.360217	2.181	.545	30	1637.28	3.214122	3.054
31	2276.84	3.357332	2.196	.549	31	1629.52	3.212060	3.068
32	2261.86	.354466	2.211	.553	32	1621.84	.210007	3.083
33	2247.08	.351618	2.225	.556	33	1614.22	.207964	3.097
34	2232.49	.348789	2.240	.560	34	1606.68	.205930	3.112
35	2218.09	.345979	2.254	.564	35	1599.21	.203906	3.127
36	2203.87	.343187	2.269	.567	36	1591.81	.201892	3.141
37	2189.84	.340412	2.283	.571	37	1584.48	.199886	3.156
38	2175.98	.337655	2.298	.574	38	1577.21	.197890	3.170
39	2162.30	.334916	2.312	.578	39	1570.01	.195903	3.185
40	2148.79	3.332193	2.327	.582	40	1562.88	3.193925	3.199
41	2135.44	3.329488	2.341	.585	41	1555.81	3.191956	3.214
42	2122.26	.326799	2.356	.589	42	1548.80	.189996	3.228
43	2109.24	.324127	2 371	.593	43	1541.86	.188045	3.243
44	2096.39	.321471	2.385	.596	44	1534.98	.186103	3.257
45	2083.68	.318832	2.400	.600	45	1528.16	.184169	3.272
46	2071.13	.316208	2.414	.604	46	1521.40	.182244	3.28
47	2058.73	.313600	2.429	.607	47	1514.70	.180327	3.301
48	2046.48	.311008	2.443	.611	48	1508.06	.178419	3.316
49	2034.37	.308431	2.458	.614	49	1501.48	.176519	3.33
50	2022.41	3.305869	2.472	.618	50	1494.95	3.174627	3.34
51	2010.59	3.303323	2.487	.622	51	1488.48	3.172744	3.35
52	1998.90	.300791	2.501	.625	52	1482.07	.170868	3.37
53	1987.35	.298274	2.516	.629	53	1475.71	.169001	3.38
54	1975.93	.295771	2.530	.633	54	1469.41	.167142	3.40
55	1964.64	.293283	2.545	.636	55	1463.16	.165291	3.417
56	1953.48	.290809	2.560	.640	56	1456.96	.163447	3.43
57	1942.44	.288349	2.574	.644	57	1450.81	.161612	3.44
58	1931.53	.285902	2.589	.647	58	1444.72	.159784	3.4
59	1920.75	.283470	2.603	.651	59	1438.68	.157963	3.
60	1910.08	3.281051	2.618	.654	60	1432.69	3.156151	3

TABLE IV.—RADII, LOGARITHMS, OFFSETS, ETC.

Deg.	Radius.	Logarithm.	Tang. Off.	Mid. Ord.	Deg.	Radius.	Logarithm.	Tang. Off.	Mid. Ord.
D.	R.	log. R.	t.	m.	D.	R.	log. R.	t.	m.
4° 0′	1432.69	3.156151	3.490	.872	5° 0′	1146.28	3.059290	4.362	1.091
1	1426.74	.154346	3.505	.876	1	1142.47	.057846	4.376	1.094
2	1420.85	.152548	3.519	.880	2	1138.69	.056407	4.391	1.098
3	1415.01	.150758	3.534	.883	3	1134.94	.054972	4.405	1.102
4	1409.21	.148975	3.548	.887	4	1131.21	.053542	4.420	1.105
5	1403.46	.147200	3.563	.891	5	1127.50	.052116	4.435	1.109
6	1397.76	.145431	3.577	.894	6	1123.82	.050696	4.449	1.112
7	1392.10	.143670	3.592	.898	7	1120.16	.049280	4.464	1.116
8	1386.49	.141916	3.606	.902	8	1116.52	.047868	4.478	1.120
9	1380.92	.140170	3.621	.905	9	1112.91	.046462	4.493	1.123
10	1375.40	3.138430	3.635	.909	10	1109.33	3.045059	4.507	1.127
11	1369.92	3.136697	3.650	.912	11	1105.76	3.043662	4.522	1.131
12	1364.49	.134971	3.664	.916	12	1102.22	.042268	4.536	1.134
13	1359.10	.133251	3.679	.920	13	1098.70	.040880	4.551	1.138
14	1353.75	.131539	3.693	.923	14	1095.20	.039495	4.565	1.142
15	1348.45	.129833	3.708	.927	15	1091.73	.038115	4.580	1.146
16	1343.15	.128134	3.723	.931	16	1088.28	.036740	4.594	1.149
17	1337.65	.126442	3.736	.934	17	1084.85	.035368	4.609	1.153
18	1332.77	.124756	3.752	.938	18	1081.44	.034002	4.623	1.157
19	1327.63	.123077	3.766	.942	19	1078.05	.032639	4.638	1.160
20	1322.53	3.121404	3.781	.945	20	1074.68	3.031281	4.653	1.164
21	1317.46	3.119738	3.795	.949	21	1071.34	3.029927	4.667	1.168
22	1312.43	.118078	3.810	.952	22	1068.01	.028577	4.682	1.171
23	1307.45	.116424	3.824	.956	23	1064.71	.027231	4.696	1.175
24	1302.50	.114777	3.839	.960	24	1061.43	.025890	4.711	1.179
25	1297.58	.113136	3.853	.963	25	1058.16	.024552	4.725	1.182
26	1292.71	.111501	3.868	.967	26	1054.92	.023219	4.740	1.186
27	1287.87	.109872	3.882	.971	27	1051.70	.021890	4.754	1.190
28	1283.07	.108249	3.897	.974	28	1048.48	.020565	4.769	1.193
29	1278.30	.106632	3.911	.978	29	1045.31	.019244	4.783	1.197
30	1273.57	3.105022	3.926	.982	30	1042.14	3.017927	4.798	1.200
31	1268.87	3.103417	3.941	.985	31	1039.00	3.016614	4.812	1.204
32	1264.21	.101818	3.955	.989	32	1035.87	.015305	4.827	1.208
33	1259.58	.100225	3.970	.993	33	1032.76	.013999	4.841	1.211
34	1254.98	.098638	3.984	.996	34	1029.67	.012698	4.856	1.215
35	1250.42	.097057	3.999	1.000	35	1026.60	.011401	4.870	1.218
36	1245.89	.095481	4.013	1.003	36	1023.55	.010107	4.885	1.222
37	1241.40	.093912	4.028	1.007	37	1020.51	.008818	4.900	1.226
38	1236.94	.092347	4.042	1.011	38	1017.49	.007532	4.914	1.229
39	1232.51	.090789	4.057	1.014	39	1014.50	.006250	4.929	1.233
40	1228.11	3.089236	4.071	1.018	40	1011.51	3.004972	4.943	1.237
41	1223.74	3.087689	4.086	1.022	41	1008.55	3.003698	4.958	1.240
42	1219.40	.086147	4.100	1.025	42	1005.60	.002427	4.972	1.244
43	1215.30	.084610	4.115	1.029	43	1002.67	3.001160	4.987	1.247
44	1210.82	.083079	4.129	1.032	44	999.762	2.999897	5.001	1.251
45	1206.57	.081553	4.144	1.036	45	996.867	.998637	5.016	1.255
46	1202.36	.080033	4.159	1.040	46	993.988	.997381	5.030	1.258
47	1198.17	.078518	4.173	1.043	47	991.126	.996129	5.045	1.262
48	1194.01	.077008	4.188	1.047	48	988.280	.994880	5.059	1.266
49	1189.88	.075504	4.202	1.051	49	985.451	.993635	5.074	1.269
50	1185.78	3.074005	4.217	1.054	50	982.638	2.992393	5.088	1.273
51	1181.71	3.072511	4.231	1.058	51	979.840	2.991155	5.103	1.277
52	1177.66	.071022	4.246	1.062	52	977.060	.989921	5.117	1.280
53	1173.65	.069538	4.260	1.065	53	974.294	.988690	5.132	1.284
54	1169.66	.068059	4.275	1.069	54	971.544	.987463	5.146	1.288
55	1165.70	.066585	4.289	1.073	55	968.810	.986238	5.161	1.291
56	1161.76	.065116	4.304	1.076	56	966.091	.985018	5.175	1.295
57	1157.85	.063653	4.318	1.080	57	963.387	.983801	5.190	1.29
58	1153.97	.062194	4.333	1.083	58	960.698	.982587	5.205	1.3
59	1150.11	.060740	4.347	1.088	59	958.025	.981377	5.219	1.
60	1146.28	3.059290	4.362	1.091	60	955.366	2.980170	5.234	

TABLE IV.—RADII, LOGARITHMS, OFFSETS, ETC.

Deg. D.	Radius. R.	Logarithm. log. R.	Tang. Off. t.	Mid. Ord. m.	Deg. D.	Radius. R.	Logarithm. log. R.	Tang. Off. t.	Mid. Ord. m.
2° 0′	2864.93	3 457115	1.745	.436	3° 0′	1910.08	3.281051	2.618	.654
1	2841.26	.453511	1.760	.440	1	1899.53	.278646	2.632	.658
2	2817.97	.449937	1.774	.444	2	1889.09	.276253	2.647	.662
3	2795.06	.446392	1.789	.447	3	1878.77	.273874	2.661	.665
4	2772.53	.442876	1.803	.451	4	1868.56	.271508	2.676	.669
5	2750.35	.439388	1.818	.454	5	1858.47	.269155	2.690	.673
6	2728.52	.435928	1.832	.458	6	1848.48	.266814	2.705	.676
7	2707.04	.432495	1.847	.462	7	1838.59	.264486	2.719	.680
8	2685.89	.429089	1.862	.465	8	1828.82	.262170	2.734	.684
9	2665.08	.425710	1.876	.469	9	1819.14	.259867	2.749	.687
10	2644.58	3 422356	1.891	.473	10	1809.57	3.257576	2.763	.691
11	2624.39	3.419029	1.905	.476	11	1800.10	3.255296	2.778	.694
12	2604.51	.415727	1.920	.480	12	1790.73	.253029	2.792	.698
13	2584.93	.412449	1.934	.484	13	1781.45	.250774	2.807	.702
14	2565.65	.409197	1.949	.487	14	1772.27	.248530	2 821	.705
15	2546.64	.405968	1.963	.491	15	1763.18	.246297	2.836	.709
16	2527.92	.402763	1.978	.494	16	1754.19	.244077	2.850	.713
17	2509.47	.399582	1.992	.498	17	1745.26	.241867	2.865	.716
18	2491.29	.396424	2.007	.502	18	1736.48	.239669	2.879	.720
19	2473.37	.393289	2.022	.505	19	1727 75	.237481	2.894	.723
20	2455.70	3 390176	2.036	.509	20	1719.12	3.235305	2.908	.727
21	2438.29	3.387085	2.051	.513	21	1710.56	3.233140	2.923	.731
22	2421.12	.384016	2.065	.516	22	1702.10	.230985	2.938	.734
23	2404.19	.380969	2.080	.520	23	1693.72	.228841	2.952	.738
24	2387.50	.377943	2.094	.524	24	1685.42	.226707	2.967	.742
25	2371.04	.374938	2.109	.527	25	1677.20	.224584	2.981	.745
26	2354.80	.371954	2.123	.531	26	1669.06	.222472	2.996	.749
27	2338.78	.368990	2.138	.534	27	1661.00	.220369	3.010	.753
28	2322.98	.366046	2.152	.538	28	1653.01	.218277	3.025	.756
29	2307.39	.363122	2.167	.542	29	1645.11	.216195	3.039	.760
30	2292.01	3.360217	2.181	.545	30	1637.28	3.214122	3.054	.763
31	2276.84	3.357332	2.196	.549	31	1629.52	3.212060	3.068	.767
32	2261.86	.354466	2.211	.553	32	1621.84	.210007	3.083	.771
33	2247.08	.351618	2.225	.556	33	1614.22	.207964	3.097	.774
34	2232.49	.348789	2.240	.560	34	1606.68	.205930	3.112	.778
35	2218.09	.345979	2.254	.564	35	1599.21	.203906	3.127	.782
36	2203.87	.343187	2.269	.567	36	1591.81	.201892	3.141	.785
37	2189.84	.340412	2.283	.571	37	1584.48	.199886	3.156	.789
38	2175.98	.337655	2.298	.574	38	1577.21	.197890	3.170	.793
39	2162.30	.334916	2.312	.578	39	1570.01	.195903	3.185	.796
40	2148.79	3.332193	2.327	.582	40	1562.88	3.193925	3.199	.800
41	2135.44	3.329488	2.341	.585	41	1555.81	3.191956	3.214	.803
42	2122.26	.326799	2.356	.589	42	1548.80	.189996	3.228	.807
43	2109.24	.324127	2 371	.593	43	1541.86	.188045	3.243	.811
44	2096.39	.321471	2.385	.596	44	1534.98	.186103	3.257	.814
45	2083.68	.318832	2.400	.600	45	1528.16	.184169	3.272	.818
46	2071.13	.316208	2.414	.604	46	1521.40	.182244	3.286	.822
47	2058.73	.313600	2.429	.607	47	1514.70	.180327	3.301	.825
48	2046.48	.311008	2.443	.611	48	1508.06	.178419	3.316	.829
49	2034.37	.308431	2.458	.614	49	1501.48	.176519	3.330	.832
50	2022.41	3.305869	2.472	.618	50	1494.95	3.174627	3.345	.836
51	2010.59	3.303323	2.487	.622	51	1488.48	3.172744	3.359	.840
52	1998.90	.300791	2.501	.625	52	1482.07	.170868	3.374	.843
53	1987.35	.298274	2.516	.629	53	1475.71	.169001	3.388	.847
54	1975.93	.295771	2.530	.633	54	1469.41	.167142	3.403	.851
55	1964.64	.293283	2.545	.636	55	1463.16	.165291	3.417	.854
56	1953.48	.290809	2.560	.640	56	1456.96	.163447	3.432	.858
57	1942.44	.288349	2.574	.644	57	1450.81	.161612	3.446	.862
58	1931.53	.285902	2.589	.647	58	1444.72	.159784	3.461	.86
59	1920.75	.283470	2.603	.651	59	1438.68	.157963	3.475	
60	1910.08	3.281051	2.618	.654	60	1432.69	3.156151	3.490	

TABLE IV.—RADII, LOGARITHMS, OFFSETS, ETC.

Deg. D.	Radius. R.	Logarithm. log. R.	Tang. Off. t.	Mid. Ord. m.	Deg. D.	Radius. R.	Logarithm. log. R.	Tang. Off. t.	Mid. Ord. m.
4° 0′	1432.69	3.156151	3.490	.872	5° 0′	1146.28	3.059290	4.362	1.091
1	1426.74	.154346	3.505	.876	1	1142.47	.057846	4.376	1.094
2	1420.85	.152548	3.519	.880	2	1138.69	.056407	4.391	1.098
3	1415.01	.150758	3.534	.883	3	1134.94	.054972	4.405	1.102
4	1409.21	.148975	3.548	.887	4	1131.21	.053542	4.420	1.105
5	1403.46	.147200	3.563	.891	5	1127.50	.052116	4.435	1.109
6	1397.76	.145431	3.577	.894	6	1123.82	.050696	4.449	1.112
7	1392.10	.143670	3.592	.898	7	1120.16	.049280	4.464	1.116
8	1386.49	.141916	3.606	.902	8	1116.52	.047868	4.478	1.120
9	1380.92	.140170	3.621	.905	9	1112.91	.046462	4.493	1.123
10	1375.40	3.138430	3.635	.909	10	1109.33	3.045059	4.507	1.127
11	1369.92	3.136697	3.650	.912	11	1105.76	3.043662	4.522	1.131
12	1364.49	.134971	3.664	.916	12	1102.22	.042268	4.536	1.134
13	1359.10	.133251	3.679	.920	13	1098.70	.040880	4.551	1.138
14	1353.75	.131539	3.693	.923	14	1095.20	.039495	4.565	1.142
15	1348.45	.129833	3.708	.927	15	1091.73	.038115	4.580	1.146
16	1343.15	.128134	3.723	.931	16	1088.28	.036740	4.594	1.149
17	1337.65	.126442	3.736	.934	17	1084.85	.035368	4.609	1.153
18	1332.77	.124756	3.752	.938	18	1081.44	.034002	4.623	1.157
19	1327.63	.123077	3.766	.942	19	1078.05	.032639	4.638	1.160
20	1322.53	3.121404	3.781	.945	20	1074.68	3.031281	4.653	1.164
21	1317.46	3.119738	3.795	.949	21	1071.34	3.029927	4.667	1.168
22	1312.43	.118078	3.810	.952	22	1068.01	.028577	4.682	1.171
23	1307.45	.116424	3.824	.956	23	1064.71	.027231	4.696	1.175
24	1302.50	.114777	3.839	.960	24	1061.43	.025890	4.711	1.179
25	1297.58	.113136	3.853	.963	25	1058.16	.024552	4.725	1.182
26	1292.71	.111501	3.868	.967	26	1054.92	.023219	4.740	1.186
27	1287.87	.109872	3.882	.971	27	1051.70	.021890	4.754	1.190
28	1283.07	.108249	3.897	.974	28	1048.48	.020565	4.769	1.193
29	1278.30	.106632	3.911	.978	29	1045.31	.019244	4.783	1.197
30	1273.57	3.105022	3.926	.982	30	1042.14	3.017927	4.798	1.200
31	1268.87	3.103417	3.941	.985	31	1039.00	3.016614	4.812	1.204
32	1264.21	.101818	3.955	.989	32	1035.87	.015305	4.827	1.208
33	1259.58	.100225	3.970	.993	33	1032.76	.013999	4.841	1.211
34	1254.98	.098638	3.984	.996	34	1029.67	.012698	4.856	1.215
35	1250.42	.097057	3.999	1.000	35	1026.60	.011401	4.870	1.218
36	1245.89	.095481	4.013	1.003	36	1023.55	.010107	4.885	1.222
37	1241.40	.093912	4.028	1.007	37	1020.51	.008818	4.900	1.226
38	1236.94	.092347	4.042	1.011	38	1017.49	.007532	4.914	1.229
39	1232.51	.090789	4.057	1.014	39	1014.50	.006250	4.929	1.233
40	1228.11	3.089236	4.071	1.018	40	1011.51	3.004972	4.943	1.237
41	1223.74	3.087689	4.086	1.022	41	1008.55	3.003698	4.958	1.240
42	1219.40	.086147	4.100	1.025	42	1005.60	.002427	4.972	1.244
43	1215.30	.084610	4.115	1.029	43	1002.67	3.001160	4.987	1.247
44	1210.82	.083079	4.129	1.032	44	999.762	2.999897	5.001	1.251
45	1206.57	.081553	4.144	1.036	45	996.867	.998637	5.016	1.255
46	1202.36	.080033	4.159	1.040	46	993.988	.997381	5.030	1.258
47	1198.17	.078518	4.173	1.043	47	991.126	.996129	5.045	1.262
48	1194.01	.077008	4.188	1.047	48	988.280	.994880	5.059	1.266
49	1189.88	.075504	4.202	1.051	49	985.451	.993635	5.074	1.269
50	1185.78	3.074005	4.217	1.054	50	982.638	2.992393	5.088	1.273
51	1181.71	3.072511	4.231	1.058	51	979.840	2.991155	5.103	1.277
52	1177.66	.071022	4.246	1.062	52	977.060	.989921	5.117	1.280
53	1173.65	.069538	4.260	1.065	53	974.294	.988690	5.132	1.284
54	1169.66	.068059	4.275	1.069	54	971.544	.987463	5.146	1.288
55	1165.70	.066585	4.289	1.073	55	968.810	.986238	5.161	1.291
56	1161.76	.065116	4.304	1.076	56	966.091	.985018	5.175	1.29
57	1157.85	.063653	4.318	1.080	57	963.387	.983801	5.190	1.
58	1153.97	.062194	4.333	1.083	58	960.698	.982587	5.205	1
59	1150.11	.060740	4.347	1.088	59	958.025	.981377	5.219	
60	1146.28	3.059290	4.362	1.091	60	955.366	2.980170	5.234	

Deg. D.	Radius. R.	Logarithm. log. R.	Tang. Off. t.	Mid. Ord. m.	Deg. D.	Radius. R.	Logarithm. log. R.	Tang. Off. t.	Mid. Ord. m.
6° 0′	955.366	2.980170	5.234	1.309	7° 0′	819.020	2.913295	6.105	1.528
1	952.722	.978966	5.248	1.313	1	817.077	.912263	6.119	1.531
2	950.093	.977766	5.263	1.317	2	815.144	.911234	6.134	1.535
3	947.478	.976569	5.277	1.320	3	813.238	.910208	6.148	1.539
4	944.877	.975375	5.292	1.324	4	811.303	.909183	6.163	1.543
5	942.291	.974185	5.306	1.327	5	809.397	.908162	6.177	1.546
6	939.719	.972998	5.321	1.331	6	807.499	.907142	6.192	1.550
7	937.161	.971814	5.335	1.335	7	805.611	.906125	6.206	1.553
8	934.616	.970633	5.350	1.338	8	803.731	.905111	6.221	1.557
9	932.086	.969456	5.364	1.342	9	801.860	.904098	6.236	1.561
10	929.569	2.968282	5.379	1.346	10	799.997	2.903089	6.250	1.564
11	927.066	2.967111	5.393	1.349	11	798.144	2.902081	6.265	1.568
12	924.576	.965943	5.408	1.353	12	796.299	.901076	6.279	1.572
13	922.100	.964778	5.422	1.356	13	794.462	.900073	6.294	1.575
14	919.637	.963616	5.437	1.360	14	792.634	.899073	6.308	1.579
15	917.187	.962458	5.451	1.364	15	790.814	.898074	6.323	1.582
16	914.750	.961303	5.466	1.368	16	789.003	.897078	6.337	1.586
17	912.326	.960150	5.480	1.371	17	787.210	.896085	6.352	1.590
18	909.915	.959001	5.495	1.375	18	785.405	.895094	6.366	1.593
19	907.517	.957855	5.510	1.378	19	783.618	.894105	6.381	1.597
20	905.131	2.956711	5.524	1.382	20	781.840	2.893118	6.395	1.600
21	902.758	2.955571	5.539	1.386	21	780.069	2.892133	6.410	1.604
22	900.397	.954434	5.553	1.389	22	778.307	.891151	6.424	1.608
23	898.048	.953300	5.568	1.393	23	776.552	.890171	6.439	1.611
24	895.712	.952168	5.582	1.397	24	774.806	.889193	6.453	1.615
25	893.388	.951040	5.597	1.400	25	773.067	.888217	6.468	1.619
26	891.076	.949915	5.611	1.404	26	771.336	.887244	6.482	1.623
27	888.776	.948792	5.626	1.407	27	769.613	.886272	6.497	1.626
28	886.488	.947673	5.640	1.411	28	767.897	.885303	6.511	1.630
29	884.211	.946556	5.655	1.415	29	766.190	.884336	6.526	1.633
30	881.946	2.945442	5.669	1.418	30	764.489	2.883371	6.540	1.637
31	879.693	2.944331	5.684	1.422	31	762.797	2.882409	6.555	1.641
32	877.451	.943223	5.698	1.426	32	761.112	.881448	6.569	1.644
33	875.221	.942118	5.713	1.429	33	759.434	.880490	6.584	1.648
34	873.002	.941015	5.727	1.433	34	757.764	.879534	6.598	1.651
35	870.795	.939916	5.742	1.437	35	756.101	.878580	6.613	1.655
36	868.598	.938819	5.756	1.440	36	754.445	.877627	6.627	1.659
37	866.412	.937725	5.771	1.444	37	752.796	.876678	6.642	1.662
38	864.238	.936633	5.785	1.447	38	751.155	.875730	6.656	1.666
39	862.075	.935545	5.800	1.451	39	749.521	.874784	6.671	1.670
40	859.922	2.934459	5.814	1.455	40	747.894	2.873840	6.685	1.673
41	857.780	2.933376	5.829	1.458	41	746.274	2.872898	6.700	1.677
42	855.648	.932295	5.844	1.462	42	744.661	.871959	6.714	1.680
43	853.527	.931218	5.858	1.466	43	743.055	.871021	6.729	1.684
44	851.417	.930142	5.873	1.469	44	741.456	.870086	6.743	1.688
45	849.317	.929070	5.887	1.473	45	739.864	.869152	6.758	1.691
46	847.228	.928000	5.902	1.476	46	738.279	.868221	6.773	1.695
47	845.148	.926933	5.916	1.480	47	736.701	.867291	6.787	1.699
48	843.080	.925869	5.931	1.484	48	735.129	.866363	6.802	1.702
49	841.021	.924807	5.945	1.487	49	733.564	.865438	6.816	1.706
50	838.972	2.923747	5.960	1.491	50	732.005	2.864514	6.831	1.710
51	836.933	2.922691	5.974	1.495	51	730.454	2.863593	6.845	1.713
52	834.904	.921637	5.989	1.498	52	728.909	.862673	6.860	1.717
53	832.885	.920585	6.003	1.502	53	727.370	.861755	6.874	1.720
54	830.876	.919536	6.018	1.505	54	725.838	.860840	6.889	1.724
55	828.876	.918489	6.032	1.510	55	724.312	.859926	6.903	1.728
56	826.886	.917446	6.047	1.513	56	722.793	.859014	6.918	1.731
57	824.905	.916404	6.061	1.517	57	721.280	.858104	6.932	1.735
58	822.934	.915365	6.076	1.520	58	719.774	.857196	6.947	1.73
59	820.973	.914329	6.090	1.524	59	718.273	.856290	6.961	1.7
60	819.020	2.913295	6.105	1.528	60	716.779	2.855385	6.976	1.

Deg. D.	Radius. R.	Logarithm. log. R.	Tan. Off. t.	Mid. Ord. m.	Deg. D.	Radius. R.	Logarithm. log. R.	Tan. Off. t.	Mid. Ord. m.
14° 0′	410.275	2.613075	12.187	3.058	16° 0′	359.265	2.555415	13.917	3.496
2	409.306	.612048	12.216	3.065	2	358.523	.554517	13.946	3.504
4	408.341	.611023	12.245	3.073	4	357.784	.553621	13.975	3.511
6	407.380	.610000	12.274	3.080	6	357.048	.552727	14.004	3.518
8	406.424	.608980	12.302	3.087	8	356.315	.551834	14.033	3.526
10	405.473	.607962	12.331	3.095	10	355.585	.550944	14.061	3.533
12	404.526	.606946	12.360	3.102	12	354.859	.550055	14.090	3.540
14	403.583	.605933	12.389	3.109	14	354.135	.549169	14.119	3.547
16	402.645	.604923	12.418	3.117	16	353.414	.548284	14.148	3.555
18	401.712	.603914	12.447	3.124	18	352.696	.547401	14.177	3.562
20	400.782	2.602908	12.476	3.131	20	351.981	2.546519	14.205	3.569
22	399.857	.601905	12.504	3.138	22	351.269	.545640	14.234	3.577
24	398.937	.600904	12.533	3.146	24	350.560	.544762	14.263	3.584
26	398.020	.599905	12.562	3.153	26	349.854	.543887	14.292	3.591
28	397.108	.598908	12.591	3.160	28	349.150	.543013	14.320	3.599
30	396.200	.597914	12.620	3.168	30	348.450	.542140	14.349	3.606
32	395.296	.596922	12.649	3.175	32	347.752	.541270	14.378	3.613
34	394.396	.595933	12.678	3.182	34	347.057	.540401	14.407	3.621
36	393.501	.594945	12.706	3.190	36	346.365	.539535	14.436	3.628
38	392.609	.593960	12.735	3.197	38	345.676	.538670	14.464	3.635
40	391.722	2.592978	12.764	3.204	40	344.990	2.537806	14.493	3.643
42	390.838	.591997	12.793	3.211	42	344.306	.536945	14.522	3.650
44	389.959	.591019	12.822	3.219	44	343.625	.536085	14.551	3.657
46	389.084	.590043	12.851	3.226	46	342.947	.535227	14.580	3.664
48	388.212	.589069	12.880	3.233	48	342.271	.534370	14.608	3.672
50	387.345	.588097	12.908	3.241	50	341.598	.533516	14.637	3.679
52	386.481	.587128	12.937	3.248	52	340.928	.532663	14.666	3.686
54	385.621	.586161	12.966	3.255	54	340.260	.531811	14.695	3.694
56	384.765	.585196	12.995	3.263	56	339.595	.530962	14.723	3.701
58	383.913	.584233	13.024	3.270	58	338.933	.530114	14.752	3.708
15° 0	383.065	2.583272	13.053	3.277	17° 0	338.273	2.529268	14.781	3.716
2	382.220	.582314	13.081	3.284	2	337.616	.528424	14.810	3.723
4	381.380	.581358	13.110	3.292	4	336.962	.527581	14.838	3.730
6	380.543	.580403	13.139	3.299	6	336.310	.526740	14.867	3.738
8	379.709	.579451	13.168	3.306	8	335.660	.525900	14.896	3.745
10	378.880	.578501	13.197	3.314	10	335.013	.525062	14.925	3.752
12	378.054	.577553	13.226	3.321	12	334.369	.524226	14.954	3.760
14	377.231	.576608	13.254	3.328	14	333.727	.523392	14.982	3.767
16	376.412	.575664	13.283	3.336	16	333.088	.522559	15.011	3.774
18	375.597	.574722	13.312	3.343	18	332.451	.521728	15.040	3.781
20	374.786	2.573783	13.341	3.350	20	331.816	2.520898	15.069	3.789
22	373.977	.572845	13.370	3.358	22	331.184	.520070	15.097	3.796
24	373.173	.571910	13.399	3.365	24	330.555	.519244	15.126	3.803
26	372.372	.570977	13.427	3.372	26	329.928	.518419	15.155	3.811
28	371.574	.570045	13.456	3.379	28	329.303	.517596	15.184	3.818
30	370.780	.569116	13.485	3.387	30	328.689	.516774	15.212	3.825
32	369.989	.568189	13.514	3.394	32	328.061	.515954	15.241	3.833
34	369.202	.567264	13.543	3.401	34	327.443	.515136	15.270	3.840
36	368.418	.566340	13.572	3.409	36	326.828	.514319	15.299	3.847
38	367.637	.565419	13.600	3.416	38	326.215	.513504	15.327	3.855
40	366.859	2.564500	13.629	3.423	40	325.604	2.512690	15.356	3.862
42	366.085	.563582	13.658	3.431	42	324.996	.511878	15.385	3.869
44	365.315	.562667	13.687	3.438	44	324.390	.511067	15.414	3.877
46	364.517	.561754	13.716	3.445	46	323.786	.510258	15.442	3.884
48	363.783	.560843	13.744	3.452	48	323.184	.509451	15.471	3.891
50	363.022	.559933	13.773	3.460	50	322.585	.508645	15.500	3.899
52	362.264	.559026	13.802	3.467	52	321.989	.507840	15.529	3.906
54	361.510	.558120	13.831	3.474	54	321.394	.507037	15.557	3.913
56	360.758	.557216	13.860	3.482	56	320.801	.506236	15.586	3.920
58	360.010	.556315	13.889	3.489	58	320.211	.505436	15.615	3.928
60	359.265	2.555415	13.917	3.496	60	319.623	2.504638	15.643	3.93

Deg. D.	Radius. R.	Logarithm. log. R.	Tang. Off. t.	Mid. Ord. m.
10° 0′	573.686	2.758674	8.716	2.183
2	571.784	.757232	8.745	2.190
4	569.896	.755796	8.774	2.198
6	568.020	.754364	8.803	2.205
8	566.156	.752937	8.831	2.212
10	564.305	.751514	8.860	2.219
12	562.466	.750096	8.889	2.227
14	560.638	.748683	8.918	2.234
16	558.823	.747274	8.947	2.241
18	557.019	2.745870	8.976	2.234
20	555.227	2.744471	9.005	2.256
22	553.447	.743076	9.034	2.263
24	551.678	.741686	9.063	2.270
26	549.920	.740300	9.092	2.278
28	548.174	.738918	9.121	2.285
30	546.438	.737541	9.150	2.293
32	544.714	.736169	9.179	2.300
34	543.001	.734800	9.208	2.307
36	541.298	.733436	9.237	2.314
38	539.606	2.732077	9.266	2.321
40	537.924	2.730721	9.295	2.329
42	536.253	.729370	9.324	2.336
44	534.593	.728023	9.353	2.343
46	532.943	.726681	9.382	2.351
48	531.303	.725342	9.411	2.358
50	529.673	.724008	9.440	2.365
52	528.053	.722677	9.469	2.372
54	526.443	.721351	9.498	2.380
56	524.843	.720029	9.527	2.387
58	523.252	2.718711	9.556	2.394
11° 0′	521.671	2.717397	9.585	2.402
2	520.100	.716087	9.614	2.409
4	518.539	.714781	9.642	2.416
6	516.986	.713479	9.671	2.423
8	515.443	.712181	9.700	2.431
10	513.909	.710887	9.729	2.438
12	512.385	.709596	9.758	2.445
14	510.869	.708310	9.787	2.453
16	509.363	.707027	9.816	2.460
18	507.865	2.705748	9.845	2.467
20	506.376	2.704473	9.874	2.475
22	504.896	.703202	9.903	2.482
24	503.425	.701934	9.932	2.489
26	501.962	.700671	9.961	2.496
28	500.507	.699410	9.990	2.504
30	499.061	.698154	10.019	2.511
32	497.624	.696901	10.048	2.518
34	496.195	.695652	10.077	2.526
36	494.774	.694407	10.106	2.533
38	493.361	2.693165	10.135	2.540
40	491.956	2.691926	10.164	2.547
42	490.559	.690692	10.192	2.555
44	489.171	.689460	10.221	2.562
46	487.790	.688233	10.250	2.569
48	486.417	.687008	10.279	2.577
50	485.051	.685788	10.308	2.584
52	483.694	.684570	10.337	2.591
54	482.344	.683357	10.366	2.598
56	481.001	.682146	10.395	2.606
58	479.666	.680939	10.424	2.613
60	478.339	2.679735	10.453	2.620

Deg. D.	Radius. R.	Logarithm. log. R.	Tang. Off. t.
12° 0′	478.339	2.679735	10.453
2	477.018	.678535	10.482
4	475.705	.677338	10.511
6	474.400	.676145	10.540
8	473.102	.674954	10.569
10	471.810	.673767	10.597
12	470.526	.672584	10.626
14	469.249	.671403	10.655
16	467.978	.670226	10.684
18	466.715	2.669052	10.713
20	465.459	2.667881	10.742
22	464.209	.666713	10.771
24	462.966	.665549	10.800
26	461.729	.664387	10.829
28	460.500	.663229	10.858
30	459.276	.662074	10.887
32	458.060	.660922	10.916
34	456.850	.659773	10.945
36	455.646	.658628	10.973
38	454.449	2.657485	11.002
40	453.259	2.656345	11.031
42	452.073	.655208	11.060
44	450.894	.654075	11.089
46	449.722	.652944	11.118
48	448.556	.651816	11.147
50	447.395	.650691	11.176
52	446.241	.649570	11.205
54	445.093	.648451	11.234
56	443.951	.647335	11.263
58	442.814	2.646221	11.291
13° 0′	441.684	2.645111	11.320
2	440.559	.644004	11.349
4	439.440	.642899	11.378
6	438.326	.641798	11.407
8	437.219	.640699	11.436
10	436.117	.639603	11.465
12	435.020	.638510	11.494
14	433.929	.637419	11.523
16	432.844	.636331	11.552
18	431.764	2.635246	11.580
20	430.690	2.634164	11.609
22	429.620	.633085	11.638
24	428.557	.632008	11.667
26	427.498	.630934	11.696
28	426.445	.629863	11.725
30	425.396	.628794	11.754
32	424.354	.627728	11.783
34	423.316	.626665	11.812
36	422.283	.625604	11.840
38	421.256	2.624546	11.869
40	420.233	2.623490	11.898
42	419.215	.622437	11.927
44	418.203	.621387	11.956
46	417.195	.620339	11.985
48	416.192	.619294	12.014
50	415.194	.618251	12.043
52	414.201	.617211	12.071
54	413.212	.616173	12.100
56	412.229	.615138	12.12
58	411.250	.614106	12.1
60	410.275	2.613075	12.

Deg.	Radius.	Logarithm.	Tan. Off.	Mid. Ord.	Deg.	Radius.	Logarithm.	Tan. Off.	Mid. Ord.
D.	R.	log. R.	t.	m.	D.	R.	log. R.	t.	m.
14° 0′	410.275	2.613075	12.187	3.058	16° 0′	359.265	2.555415	13.917	3.496
2	409.306	.612048	12.216	3.065	2	358.523	.554517	13.946	3.504
4	408.341	.611023	12.245	3.073	4	357.784	.553621	13.975	3.511
6	407.380	.610000	12.274	3.080	6	357.048	.552727	14.004	3.518
8	406.424	.608980	12.302	3.087	8	356.315	.551834	14.033	3.526
10	405.473	.607962	12.331	3.095	10	355.585	.550944	14.061	3.533
12	404.526	.606946	12.360	3.102	12	354.859	.550055	14.090	3.540
14	403.583	.605933	12.389	3.109	14	354.135	.549169	14.119	3.547
16	402.645	.604923	12.418	3.117	16	353.414	.548284	14.148	3.555
18	401.712	.603914	12.447	3.124	18	352.696	.547401	14.177	3.562
20	400.782	2.602908	12.476	3.131	20	351.981	2.546519	14.205	3.569
22	399.857	.601905	12.504	3.138	22	351.269	.545640	14.234	3.577
24	398.937	.600904	12.533	3.146	24	350.560	.544762	14.263	3.584
26	398.020	.599905	12.562	3.153	26	349.854	.543887	14.292	3.591
28	397.108	.598908	12.591	3.160	28	349.150	.543013	14.320	3.599
30	396.200	.597914	12.620	3.168	30	348.450	.542140	14.349	3.606
32	395.296	.596922	12.649	3.175	32	347.752	.541270	14.378	3.613
34	394.396	.595933	12.678	3.182	34	347.057	.540401	14.407	3.621
36	393.501	.594945	12.706	3.190	36	346.365	.539535	14.436	3.628
38	392.609	.593960	12.735	3.197	38	345.676	.538670	14.464	3.635
40	391.722	2.592978	12.764	3.204	40	344.990	2.537806	14.493	3.643
42	390.838	.591997	12.793	3.211	42	344.306	.536945	14.522	3.650
44	389.959	.591019	12.822	3.219	44	343.625	.536085	14.551	3.657
46	389.084	.590043	12.851	3.226	46	342.947	.535227	14.580	3.664
48	388.212	.589069	12.880	3.233	48	342.271	.534370	14.608	3.672
50	387.345	.588097	12.908	3.241	50	341.598	.533516	14.637	3.679
52	386.481	.587128	12.937	3.248	52	340.928	.532663	14.666	3.686
54	385.621	.586161	12.966	3.255	54	340.260	.531811	14.695	3.694
56	384.765	.585196	12.995	3.263	56	339.595	.530962	14.723	3.701
58	383.913	.584233	13.024	3.270	58	338.933	.530114	14.752	3.708
15° 0	383.065	2.583272	13.053	3.277	17° 0	338.273	2.529268	14.781	3.716
2	382.220	.582314	13.081	3.284	2	337.616	.528424	14.810	3.723
4	381.380	.581358	13.110	3.292	4	336.962	.527581	14.838	3.730
6	380.543	.580403	13.139	3.299	6	336.310	.526740	14.867	3.738
8	379.709	.579451	13.168	3.306	8	335.660	.525900	14.896	3.745
10	378.880	.578501	13.197	3.314	10	335.013	.525062	14.925	3.752
12	378.054	.577553	13.226	3.321	12	334.369	.524226	14.954	3.760
14	377.231	.576608	13.254	3.328	14	333.727	.523392	14.982	3.767
16	376.412	.575664	13.283	3.336	16	333.088	.522559	15.011	3.774
18	375.597	.574722	13.312	3.343	18	332.451	.521728	15.040	3.781
20	374.786	2.573783	13.341	3.350	20	331.816	2.520898	15.069	3.789
22	373.977	.572845	13.370	3.358	22	331.184	.520070	15.097	3.796
24	373.173	.571910	13.399	3.365	24	330.555	.519244	15.126	3.803
26	372.372	.570977	13.427	3.372	26	329.928	.518419	15.155	3.811
28	371.574	.570045	13.456	3.379	28	329.303	.517596	15.184	3.818
30	370.780	.569116	13.485	3.387	30	328.689	.516774	15.212	3.825
32	369.989	.568189	13.514	3.394	32	328.061	.515954	15.241	3.833
34	369.202	.567264	13.543	3.401	34	327.443	.515136	15.270	3.840
36	368.418	.566340	13.572	3.409	36	326.828	.514319	15.299	3.847
38	367.637	.565419	13.600	3.416	38	326.215	.513504	15.327	3.855
40	366.859	2.564500	13.629	3.423	40	325.604	2.512690	15.356	3.862
42	366.085	.563582	13.658	3.431	42	324.996	.511878	15.385	3.869
44	365.315	.562667	13.687	3.438	44	324.390	.511067	15.414	3.877
46	364.547	.561754	13.716	3.445	46	323.788	.510258	15.442	3.884
48	363.783	.560843	13.744	3.452	48	323.184	.509451	15.471	3.891
50	363.022	.559933	13.773	3.460	50	322.585	.508645	15.500	3.899
52	362.264	.559026	13.802	3.467	52	321.989	.507840	15.529	3.906
54	361.510	.558120	13.831	3.474	54	321.394	.507037	15.557	3.913
56	360.758	.557216	13.860	3.482	56	320.801	.506236	15.586	3.920
58	360.010	.556315	13.889	3.489	58	320.211	.505436	15.615	3.928
60	359.265	2.555415	13.917	3.496	60	319.623	2.504638	15.643	3.935

Deg. D.	Radius. R.	Logarithm. log. R.	Tang. Off. t.	Mid. Ord. m.
18° 0′	319.623	2.504638	15.643	3.935
2	319.037	.503841	15.672	3.942
4	318.453	.503045	15.701	3.950
6	317.871	.502251	15.730	3.957
8	317.292	.501459	15.758	3.964
10	316.715	.500668	15.787	3.972
12	316.139	.499879	15.816	3.979
14	315.566	.499091	15.845	3.986
16	314.993	.498304	15.873	3.994
18	314.426	.497519	15.902	4.001
20	313.860	2.496736	15.931	4.008
22	313.295	.495953	15.959	4.016
24	312.732	.495173	15.988	4.023
26	312.172	.494393	16.017	4.030
28	311.613	.493616	16.046	4.038
30	311.056	.492839	16.074	4.045
32	310.502	.492064	16.103	4.052
34	309.949	.491291	16.132	4.060
36	309.399	.490518	16.160	4.067
38	308.850	.489748	16.189	4.074
40	308.303	2.488978	16.218	4.081
42	307.759	.488210	16.246	4.089
44	307.216	.487444	16.275	4.096
46	306.675	.486679	16.304	4.103
48	306.136	.485915	16.333	4.111
50	305.599	.485152	16.361	4.118
52	305.064	.484391	16.390	4.125
54	304.531	.483632	16.419	4.133
56	304.000	.482873	16.447	4.140
58	303.470	.482116	16.476	4.147
19° 0′	302.943	2.481361	16.505	4.155
2	302.417	.480607	16.533	4.162
4	301.893	.479854	16.562	4.169
6	301.371	.479102	16.591	4.177
8	300.851	.478352	16.620	4.184
10	300.333	.477603	16.648	4.191
12	299.816	.476855	16.677	4.199
14	299.302	.476109	16.706	4.206
16	298.789	.475364	16.734	4.213
18	298.278	.474621	16.763	4.221
20	297.768	2.473878	16.792	4.228
22	297.260	.473137	16.820	4.235
24	296.755	.472398	16.849	4.243
26	296.250	.471659	16.878	4.250
28	295.748	.470922	16.906	4.257
30	295.247	.470186	16.935	4.265
32	294.748	.469452	16.964	4.272
34	294.251	.468718	16.992	4.279
36	293.756	.467986	17.021	4.287
38	293.262	.467256	17.050	4.294
40	292.770	2.466526	17.078	4.301
42	292.279	.465798	17.107	4.308
44	291.790	.465071	17.136	4.316
46	291.303	.464345	17.164	4.323
48	290.818	.463621	17.193	4.330
50	290.334	.462897	17.222	4.338
52	289.851	.462175	17.250	4.345
54	289.371	.461455	17.279	4.352
56	288.892	.460735	17.308	4.360
58	288.414	.460017	17.336	4.367
60	287.939	2.459300	17.365	4.374

Deg. D.	Radius. R.	Logarithm. log. R.	Tang. Off. t.
20° 0′	287.939	2.459300	17.365
10	285.583	.455733	17.508
20	283.267	.452195	17.651
30	280.988	.448688	17.794
40	278.746	.445209	17.937
50	276.541	.441759	18.081
21° 0′	274.370	2.438337	18.224
10	272.234	.434943	18.367
20	271.032	.431576	18.509
30	268.062	.428235	18.652
40	266.024	.424921	18.795
50	264.018	.421633	18.938
22° 0′	262.042	2.418371	19.081
10	260.098	.415134	19.224
20	258.180	.411922	19.366
30	256.292	.408734	19.509
40	254.431	.405571	19.652
50	252.599	.402431	19.794
23° 0′	250.793	2.399315	19.937
10	249.013	.396222	20.079
20	247.258	.393151	20.222
30	245.473	.390103	20.364
40	243.825	.387077	20.507
50	242.144	.384074	20.649
24° 0′	240.487	2.381091	20.791
10	238.853	.378130	20.933
20	237.241	.375190	21.076
30	235.652	.372270	21.218
40	234.084	.369371	21.360
50	232.537	.366492	21.502
25° 0′	231.011	2.363633	21.644
10	229.506	.360794	21.786
20	228.020	.357974	21.928
30	226.555	.355173	22.070
40	225.108	.352391	22.212
50	223.680	.349627	22.353
26° 0′	222.271	2.346882	22.495
10	220.879	.344155	22.637
20	219.506	.341446	22.778
30	218.150	.338755	22.920
40	216.811	.336081	23.062
50	215.489	.333424	23.203
27° 0′	214.183	2.330785	23.345
10	212.893	.328162	23.486
20	211.620	.325556	23.627
30	210.362	.322967	23.769
40	209.119	.320393	23.910
50	207.891	.317836	24.051
28° 0′	206.678	2.315295	24.192
10	205.480	.312769	24.333
20	204.296	.310259	24.474
30	203.125	.307764	24.615
40	201.969	.305285	24.756
50	200.826	.302820	24.897
29° 0′	199.696	2.300370	25.038
10	198.580	.297935	25.179
20	197.476	.295515	25.320
30	196.385	.293108	25.460
40	195.306	.290716	25.601
50	194.240	.288338	25.74
30° 0′	193.185	2.285974	25.8

Deg.	Radius.	Logarithm.	Tang. Off.	Mid. Ord.	Deg.	Radius.	Logarithm.	Tang. Off.	Mid. Ord.
D.	R.	log. R.	t.	m.	D.	R.	log. R.	t.	m.
30° 20′	191.111	2.281286	26.163	6.657	**38°**30′	151.657	2.180863	32.969	8.479
40	189.083	.276652	26.443	6.731	**39°** 0′	149.787	.175475	33.381	8.592
31° 0′	187.099	.272071	26.724	6.805	30	147.965	.170160	33.792	8.704
20	185.158	.267541	27.004	6.879	**40°** 0′	146.190	.164918	34.202	8.816
40	183.258	.263062	27.284	6.958	30	144.460	.159747	34.612	8.929
32° 0′	181.398	.258632	27.564	7.027	**41°** 0′	142.773	.154645	35.021	9.041
20	179.577	.254250	27.843	7.101	30	141.127	.149610	35.429	9.154
40	177.794	.249916	28.123	7.175	**42°** 0′	139.521	.144641	35.837	9.267
33° 0′	176.047	.245628	28.402	7.250	30	137.955	.139736	36.244	9.380
20	174.336	.241386	28.680	7.324	**43°** 0′	136.425	.134895	36.650	9.493
40	172.659	.237188	28.959	7.398	30	134.932	.130114	37.056	9.606
34° 0′	171.015	2.233035	29.237	7.473	**44°** 0′	133.473	2.125395	37.461	9.719
20	169.404	.228924	29.515	7.547	30	132.049	.120734	37.865	9.832
40	167.825	.224855	29.793	7.621	**45°** 0′	130.656	.116130	38.268	9.946
35° 0′	166.275	.220828	30.071	7.696	30	129.296	.111584	38.671	10.059
20	164.756	.216842	30.348	7.770	**46°** 0′	127.965	.107092	39.073	10.173
40	163.266	.212805	30.625	7.845	30	126.664	.102655	39.474	10.286
36° 0′	161.803	.208988	30.902	7.919	**47°** 0′	125.392	.098270	39.875	10.400
20	160.368	.205119	31.178	7.994	30	124.148	.093938	40.275	10.516
40	158.960	.201288	31.454	8 068	**48°** 0′	122.930	.089657	40.674	10.628
37° 0′	157.577	.197494	31.730	8.143	30	121.738	.085425	41.072	10.742
20	156.220	.193736	32 006	8.218	**49°** 0′	120.571	.081243	41.469	10.856
40	154.887	.190014	32.282	8.292	30	119.429	.077109	41.866	10.970
38° 0′	153.578	2.186328	32.557	8.367	**50°** 0′	118.310	2.073022	42.262	11.085

TABLE V.—CORRECTIONS FOR TANGENTS AND EXTERNALS.

Ang Δ	For Tangents, add						Ang Δ	For Externals, add					
	5° Cur.	10° Cur.	15° Cur.	20° Cur.	25° Cur.	30° Cur.		5° Cur.	10° Cur.	15° Cur.	20° Cur.	25° Cur.	30° Cur.
10°	.03	.06	.09	.13	.16	.19	10°	001	.003	.004	.006	.007	.008
20	.06	.13	.19	.26	.32	.39	20	.006	.011	.017	.022	.028	.034
30	.10	.19	.29	.39	.49	.59	30	.013	.025	.038	.051	.065	.078
40	.13	.26	.40	.53	.67	80	40	.023	.046	.070	.093	.117	.141
50	.17	.34	.51	.68	.85	1 02	50	.037	.075	.116	.151	.189	.227
60	.21	.42	.63	.84	1.05	1.27	60	.056	.112	.168	.225	.283	.340
70	.25	.51	.76	1.02	1.28	1.54	70	.080	.159	.240	.321	.403	.485
80	.30	.61	.91	1.22	1.53	1.84	80	.110	.220	.332	.445	.558	.671
90	.36	.72	1.09	1.45	1.83	2.20	90	.149	.299	.450	.603	.756	.910
100	.43	.86	1.30	1.74	2 18	2.62	100	.200	.401	.604	.809	1.015	1.221
110	.51	1.03	1.56	2 08	2.61	3.14	110	.268	.536	.806	1.082	1.355	1.633
120	.62	1.25	1.93	2.52	3 16	3.81	120	.360	.721	1.086	1.456	1.825	2.197

TABLE VI.—TANGENTS AND EXTERNALS TO A 1° CURV

Angle. Δ	Tangent. T.	External. E.	Angle. Δ	Tangent. T.	External. E.	Angle. Δ	Tangent. T.
1°	50.00	.218	11	551.70	26.500	21°	1061.9
10′	58.34	.297	10′	560.11	27.313	10′	1070.6
20	66.67	.388	20	568.53	28.137	20	1079.2
30	75.01	.491	30	576.95	28.974	30	1087.8
40	83.34	.606	40	585.36	29.824	40	1096.4
50	91.68	.733	50	593.79	30.686	50	1105.1
2	100.01	.873	12	602.21	31.561	22	1113.7
10	108.35	1.024	10	610.64	32.447	10	1122.4
20	116.68	1.188	20	619.07	33.347	20	1131.0
30	125.02	1.364	30	627.50	34.259	30	1139.7
40	133.36	1.552	40	635.93	35.183	40	1148.4
50	141.70	1.752	50	644.37	36.120	50	1157.0
3	150.04	1.964	13	652.81	37.070	23	1165.7
10	158.38	2.188	10	661.25	38.031	10	1174.4
20	166.72	2.425	20	669.70	39.006	20	1183.1
30	175.06	2.674	30	678.15	39.993	30	1191.8
40	183.40	2.934	40	686.60	40.992	40	1200.5
50	191.74	3.207	50	695.06	42.004	50	1209.2
4	200.08	3.492	14	703.51	43.029	24	1217.9
10	208.43	3.790	10	711.97	44.066	10	1226.6
20	216.77	4.099	20	720.44	45.116	20	1235.3
30	225.12	4.421	30	728.90	46.178	30	1244.0
40	233.47	4.755	40	737.37	47.253	40	1252.8
50	241.81	5.100	50	745.85	48.341	50	1261.5
5	250.16	5.459	15	754.32	49.441	25	1270.2
10	258.51	5.829	10	762.80	50.554	10	1279.0
20	266.86	6.211	20	771.29	51.679	20	1287.7
30	275.21	6.606	30	779.77	52.818	30	1296.5
40	283.57	7.013	40	788.26	53.969	40	1305.3
50	291.92	7.432	50	796.75	55.132	50	1314.0
6	300.28	7.863	16	805.25	56.309	26	1322.8
10	308.64	8.307	10	813.75	57.498	10	1331.6
20	316.99	8.762	20	822.25	58.699	20	1340.4
30	325.35	9.230	30	830.76	59.914	30	1349.2
40	333.71	9.710	40	839.27	61.141	40	1358.0
50	342.08	10.202	50	847.78	62.381	50	1366.8
7	350.44	10.707	17	856.30	63.634	27	1375.6
10	358.81	11.224	10	864.82	64.900	10	1384.4
20	367.17	11.753	20	873.35	66.178	20	1393.2
30	375.54	12.294	30	881.88	67.470	30	1402.0
40	383.91	12.847	40	890.41	68.774	40	1410.9
50	392.28	13.413	50	898.95	70.091	50	1419.7
8	400.66	13.991	18	907.49	71.421	28	1428.6
10	409.03	14.582	10	916.03	72.764	10	1437.4
20	417.41	15.184	20	924.58	74.119	20	1446.3
30	425.79	15.799	30	933.13	75.488	30	1455.1
40	434.17	16.426	40	941.69	76.869	40	1464.0
50	442.55	17.065	50	950.25	78.264	50	1472.9
9	450.93	17.717	19	958.81	79.671	29	1481.8
10	459.32	18.381	10	967.38	81.092	10	1490.7
20	467.71	19.058	20	975.96	82.525	20	1499.6
30	476.10	19.746	30	984.53	83.972	30	1508.5
40	484.49	20.447	40	993.12	85.431	40	1517.4
50	492.88	21.161	50	1001.7	86.904	50	1526.3
10	501.28	21.887	20	1010.3	88.389	30	1535.3
10	509.68	22.624	10	1018.9	89.888	10	1544.2
20	518.08	23.375	20	1027.5	91.399	20	1553.1
30	526.48	24.138	30	1036.1	92.924	30	1562.1
40	534.89	24.913	40	1044.7	94.462	40	1571.0
50	543.29	25.700	50	1053.3	96.013	50	1580.

Angle. △	Tan-gent. T.	Exter-nal. E.	Angle. △	Tan-gent. T.	Exter-nal. E.	Angle. △	Tan-gent. T.	Exter-nal. E.
31°	1589.0	216.25	**41°**	2142.2	387.38	**51°**	2732.9	618.39
10	1598.0	218.66	10′	2151.7	390.71	10′	2743.1	622.81
20	1606.9	221.08	20	2161.2	394.06	20	2753.4	627.24
30	1615.9	223.51	30	2170.8	397.43	30	2763.7	631.69
40	1624.9	225.96	40	2180.3	400.82	40	2773.9	636.17
50	1633.9	228.42	50	2189.9	404.22	50	2784.2	640.66
32	1643.0	230.90	**42**	2199.4	407.64	**52**	2794.5	645.17
10	1652.0	233.39	10	2209.0	411.07	10	2804.9	649.70
20	1661.0	235.90	20	2218.6	414.52	20	2815.2	654.25
30	1670.0	238.43	30	2228.1	417.99	30	2825.6	658.83
40	1679.1	240.96	40	2237.7	421.48	40	2835.9	663.42
50	1688.1	243.52	50	2247.3	424.98	50	2846.3	668.03
33	1697.2	246.08	**43**	2257.0	428.50	**53**	2856.7	672.66
10	1706.3	248.66	10	2266.6	432.04	10	2867.1	677.32
20	1715.3	251.26	20	2276.2	435.59	20	2877.5	681.99
30	1724.4	253.87	30	2285.9	439.16	30	2888.0	686.68
40	1733.5	256.50	40	2295.6	422.75	40	2898.4	691.40
50	1742.6	259.14	50	2305.2	446.35	50	2908.9	696.13
34	1751.7	261.80	**44**	2314.9	449.98	**54**	2919.4	700.89
10	1760.8	264.47	10	2324.6	453.62	10	2929.9	705.66
20	1770.0	267.16	20	2334.3	457.27	20	2940.4	710.46
30	1779.1	269.86	30	2344.1	460.95	30	2951.0	715.28
40	1788.2	272.58	40	2353.8	464.64	40	2961.5	720.11
50	1797.4	275.31	50	2363.5	468.35	50	2972.1	724.97
35	1806.6	278.05	**45**	2373.3	472.08	**55**	2982.7	729.85
10	1815.7	280.82	10	2383.1	475.82	10	2993.3	734.76
20	1824.9	283.60	20	2392.8	479.59	20	3003.9	739.68
30	1834.1	286.39	30	2402.6	483.37	30	3014.5	744.62
40	1843.3	289.20	40	2412.4	487.17	40	3025.2	749.59
50	1852.5	292.02	50	2422.3	490.98	50	3035.8	754.57
36	1861.7	294.86	**46**	2432.1	494.82	**56**	3046.5	759.58
10	1870.9	297.72	10	2441.9	498.67	10	3057.2	764.61
20	1880.1	300.59	20	2451.8	502.54	20	3067.9	769.66
30	1889.4	303.47	30	2461.7	506.42	30	3078.7	774.73
40	1898.6	306.37	40	2471.5	510.33	40	3089.4	779.83
50	1907.9	309.29	50	2481.4	514.25	50	3100.2	784.94
37	1917.1	312.22	**47**	2491.3	518.20	**57**	3110.9	790.08
10	1926.4	315.17	10	2501.2	522.16	10	3121.7	795.24
20	1935.7	318.13	20	2511.2	526.13	20	3132.6	800.42
30	1945.0	321.11	30	2521.1	530.13	30	3143.4	805.62
40	1954.3	324.11	40	2531.1	534.15	40	3154.2	810.85
50	1963.6	327.12	50	2541.0	538.18	50	3165.1	816.10
38	1972.9	330.15	**48**	2551.0	542.23	**58**	3176.0	821.37
10	1982.2	333.19	10	2561.0	546.30	10	3186.9	826.66
20	1991.5	336.25	20	2571.0	550.39	20	3197.8	831.98
30	2000.9	339.32	30	2581.0	554.50	30	3208.8	837.31
40	2010.2	342.41	40	2591.1	558.63	40	3219.7	842.67
50	2019.6	345.52	50	2601.1	562.77	50	3230.7	848.06
39	2029.0	348.64	**49**	2611.2	566.94	**59**	3241.7	853.46
10	2038.4	351.78	10	2621.2	571.12	10	3252.7	858.89
20	2047.8	354.94	20	2631.3	575.32	20	3263.7	864.34
30	2057.2	358.11	30	2641.4	579.54	30	3274.8	869.82
40	2066.6	361.29	40	2651.5	583.78	40	3285.8	875.32
50	2076.0	364.50	50	2661.6	588.04	50	3296.9	880.84
40	2085.4	367.72	**50**	2671.8	592.32	**60**	3308.0	886.38
10	2094.9	370.95	10	2681.9	596.62	10	3319.1	891.95
20	*2104.3*	*374.20*	20	2692.1	600.93	20	3330.3	897.[illegible]4
30	*2113.8*	*377.47*	30	2702.3	605.27	30	3341.4	903.15
40	*2123.3*	*380.76*	40	2712.5	609.62	40	3352.6	908.7
50	*2132.7*	*384.06*	50	2722.7	614.00	50	3363.8	914

TABLE VI.—TANGENTS AND EXTERNALS TO A 1° CURVE.

Angle. Δ	Tan-gent. T.	Exter-nal. E.	Angle. Δ	Tan-gent. T.	Exter-nal. E.	Angle. Δ	Tan-gent. T.	Exter-nal. E.
61°	3375.0	920.14	**71°**	4086.9	1308.2	**81°**	4893.6	1805.3
10′	3386.3	925.85	10′	4099.5	1315.6	10′	4908.0	1814.7
20	3397.5	931.58	20	4112.1	1322.9	20	4922.5	1824.1
30	3408.8	937.34	30	4124.8	1330.3	30	4937.0	1833.6
40	3420.1	943.12	40	4137.4	1337.7	40	4951.5	1843.1
50	3431.4	948.92	50	4150.1	1345.1	50	4966.1	1852.6
62	3442.7	954.75	**72**	4162.8	1352.6	**82**	4980.7	1862.2
10	3454.1	960.60	10	4175.6	1360.1	10	4995.4	1871.8
20	3465.4	966.48	20	4188.5	1367.6	20	5010.0	1881.5
30	3476.8	972.38	30	4201.2	1375.2	30	5024.8	1891.2
40	3488.3	978.31	40	4214.0	1382.8	40	5039.5	1900.9
50	3499.7	984.27	50	4226.8	1390.4	50	5054.3	1910.7
63	3511.1	990.24	**73**	4239.7	1398.0	**83**	5069.2	1920.5
10	3522.6	996.24	10	4252.6	1405.7	10	5084.0	1930.4
20	3534.1	1002.3	20	4265.6	1413.5	20	5099.0	1940.3
30	3545.6	1008.3	30	4278.5	1421.2	30	5113.9	1950.3
40	3557.2	1014.4	40	4291.5	1429.0	40	5128.9	1960.2
50	3568.7	1020.5	50	4304.6	1436.8	50	5143.9	1970.3
64	3580.3	1026.6	**74**	4317.6	1444.6	**84**	5159.0	1980.4
10	3591.9	1032.8	10	4330.7	1452.5	10	5174.1	1990.5
20	3603.5	1039.0	20	4343.8	1460.4	20	5189.3	2000.6
30	3615.1	1045.2	30	4356.9	1468.4	30	5204.4	2010.8
40	3626.8	1051.4	40	4370.1	1476.4	40	5219.7	2021.1
50	3638.5	1057.7	50	4383.3	1484.4	50	5234.9	2031.4
65	3650.2	1063.9	**75**	4396.5	1492.4	**85**	5250.3	2041.7
10	3661.9	1070.2	10	4409.8	1500.5	10	5265.6	2052.1
20	3673.7	1076.6	20	4423.1	1508.6	20	5281.0	2062.5
30	3685.4	1082.9	30	4436.4	1516.7	30	5296.4	2073.0
40	3697.2	1089.3	40	4449.7	1524.9	40	5311.9	2083.5
50	3709.0	1095.7	50	4463.1	1533.1	50	5327.4	2094.1
66	3720.9	1102.2	**76**	4476.5	1541.4	**86**	5343.0	2104.7
10	3732.7	1108.6	10	4489.9	1549.7	10	5358.6	2115.3
20	3744.6	1115.1	20	4503.4	1558.0	20	5374.2	2126.0
30	3756.5	1121.7	30	4516.9	1566.3	30	5389.9	2136.7
40	3768.5	1128.2	40	4530.4	1574.7	40	5405.6	2147.5
50	3780.4	1134.8	50	4544.0	1583.1	50	5421.4	2158.4
67	3792.4	1141.4	**77**	4557.6	1591.6	**87**	5437.2	2169.2
10	3804.4	1148.0	10	4571.2	1600.1	10	5453.1	2180.2
20	3816.4	1154.7	20	4584.8	1608.6	20	5469.0	2191.1
30	3828.4	1161.3	30	4598.5	1617.1	30	5484.9	2202.2
40	3840.5	1168.1	40	4612.2	1625.7	40	5500.9	2213.2
50	3852.6	1174.8	50	4626.0	1634.4	50	5517.0	2224.3
68	3864.7	1181.6	**78**	4639.8	1643.0	**88**	5533.1	2235.5
10	3876.8	1188.4	10	4653.6	1651.7	10	5549.2	2246.7
20	3889.0	1195.2	20	4667.4	1660.5	20	5565.4	2258.0
30	3901.2	1202.0	30	4681.3	1669.2	30	5581.6	2269.3
40	3913.4	1208.9	40	4695.2	1678.1	40	5597.8	2280.6
50	3925.6	1215.8	50	4709.2	1686.9	50	5614.2	2292.0
69	3937.9	1222.7	**79**	4723.2	1695.8	**89**	5630.5	2303.5
10	3950.2	1229.7	10	4737.2	1704.7	10	5646.9	2315.0
20	3962.5	1236.7	20	4751.2	1713.7	20	5663.4	2326.6
30	3974.8	1243.7	30	4765.3	1722.7	30	5679.9	2338.2
40	3987.2	1250.8	40	4779.4	1731.7	40	5696.4	2349.8
50	3999.5	1257.9	50	4793.6	1740.8	50	5713.0	2361.5
70	*4011.9*	*1265.0*	**80**	4807.7	1749.9	**90**	5729.7	2373.3
10	*4024.4*	*1272.1*	10	4822.0	1759.0	10	5746.3	2385.1
20	*4036.8*	*1279.3*	20	4836.2	1768.2	20	5763.1	2397.0
30	*4049.3*	*1286.5*	30	4850.5	1777.4	30	5779.9	2408.9
40	*4061.8*	*1293.6*	40	4864.8	1786.7	40	5796.7	2420.9
50	*4074.4*	*1300.9*	50	4879.2	1796.0	50	5813.6	2432.9

TABLE VI.—TANGENTS AND EXTERNALS TO A 1° CURVE.

Angle. Δ	Tan-gent. T.	Ex-ternal. E.	Angle. Δ	Tan-gent. T.	Ex-ternal. E.	Angle. Δ	Tan-gent. T.	Ex-ternal E.
91°	5830.5	2444.9	**101°**	6950.6	3278.1	**111°**	8336.7	4386.1
10′	5847.5	2457.1	10′	6971.3	3294.1	10′	8362.7	4407.6
20	5864.6	2469.3	20	6992.0	3310.1	20	8388.9	4429.2
30	5881.7	2481.5	30	7012.7	3326.1	30	8415.1	4450.9
40	5898.8	2493.8	40	7033.6	3342.3	40	8441.5	4472.7
50	5916.0	2506.1	50	7054.5	3358.5	50	8468.0	4494.6
92	5933.2	2518.5	**102**	7075.5	3374.9	**112**	8494.6	4516.6
10	5950.5	2531.0	10	7096.6	3391.2	10	8521.3	4538.8
20	5967.9	2543.5	20	7117.8	3407.7	20	8548.1	4561.1
30	5985.3	2556.0	30	7139.0	3424.3	30	8575.0	4583.4
40	6002.7	2568.6	40	7160.3	3440.9	40	8602.1	4606.0
50	6020.2	2581.3	50	7181.7	3457.6	50	8629.3	4628.6
93	6037.8	2594.0	**103**	7203.2	3474.4	**113**	8656.6	4651.3
10	6055.4	2606.8	10	7224.7	3491.3	10	8684.0	4674.2
20	6073.1	2619.7	20	7246.3	3508.2	20	8711.5	4697.2
30	6090.8	2632.6	30	7268.0	3525.2	30	8739.2	4720.3
40	6108.6	2645.5	40	7289.8	3542.4	40	8767.0	4743.6
50	6126.4	2658.5	50	7311.7	3559.6	50	8794.9	4766.9
94	6144.3	2671.6	**104**	7333.6	3576.8	**114**	8822.9	4790.4
10	6162.2	2684.7	10	7355.6	3594.2	10	8851.0	4814.1
20	6180.2	2697.9	20	7377.8	3611.7	20	8879.3	4837.8
30	6198.3	2711.2	30	7399.9	3629.2	30	8907.7	4861.7
40	6216.4	2724.5	40	7422.2	3646.8	40	8936.3	4885.7
50	6234.6	2737.9	50	7444.6	3664.5	50	8965 0	4909.9
95	6252.8	2751.3	**105**	7467.0	3682.3	**115**	8993.8	4934.1
10	6271.1	2764.8	10	7489.6	3700.2	10	9022.7	4958.6
20	6289.4	2778.3	20	7512.2	3718.2	20	9051.7	4983.1
30	6307.9	2792.0	30	7534.9	3736.2	30	9080.9	5007.8
40	6326.3	2805.6	40	7557.7	3754.4	40	9110.3	5032.6
50	6344.8	2819.4	50	7580.5	3772.6	50	9139.8	5057.6
96	6363.4	2833.2	**106**	7603.5	3791.0	**116**	9169.4	5082.7
10	6382.1	2847.0	10	7626.6	3809.4	10	9199.1	5107.9
20	6400.8	2861.0	20	7649.7	3827.9	20	9229.0	5133.3
30	6419.5	2875.0	30	7672.9	3846.5	30	9259.0	5158.8
40	6438.4	2889.0	40	7696.3	3865.2	40	9289.2	5184.5
50	6457.3	2903.1	50	7719.7	3884.0	50	9319.5	5210.3
97	6476.2	2917.3	**107**	7743.2	3902.9	**117**	9349.9	5236.2
10	6495.2	2931.6	10	7766.8	3921.9	10	9380.5	5262.3
20	6514.3	2945.9	20	7790.5	3940.9	20	9411.3	5288.6
30	6533.4	2960.3	30	7814.3	3960.1	30	9442.2	5315.0
40	6552.6	2974.7	40	7838.1	3979.4	40	9473.2	5341.5
50	6571.9	29[illegible].2	50	7862.1	3998.7	50	9504.4	5368.2
98	6591.2	3003.8	**108**	7886.2	4018.2	**118**	9535.7	5395.1
10	6610.6	3018.4	10	7910.4	4037.8	10	9567.2	5422.1
20	6630.1	3033.1	20	7934.6	4057.4	20	9598.9	5449.2
30	6649.6	3047.9	30	7959.0	4077.2	30	9630.7	5476.5
40	6669.2	3062.8	40	7983.5	4097.1	40	9662.6	5504.0
50	6688.8	3077.7	50	8008.0	4117.0	50	9694.7	5531.7
99	6708.6	3092.7	**109**	8032.7	4137.1	**119**	9727.0	5559.4
10	6728.4	3107.7	10	8057.4	4157.3	10	9759.4	5587.4
20	6748.2	3122.9	20	8082.3	4177.5	20	9792.0	5615.5
30	6768.1	3138.1	30	8107.3	4197.9	30	9824.8	5643.8
40	6788.1	3153.3	40	8132.3	4218.4	40	9857.7	5672.3
50	6808.2	3168.7	50	8157.5	4239.0	50	9890.8	5700.9
100	6828.3	3184.1	**110**	8182.8	4259.7	**120**	9924.0	5729.7
10	6848.5	3199.6	10	8208.2	4280.5	10	9957.5	5758.6
20	6868.8	3215.1	20	8233.7	4301.4	20	9991.0	5787.7
30	6889.2	3230.8	30	8259.3	4322.4	30	10025.0	5817.0
40	6909.6	3246.5	40	8285.0	4343.6	40	10059.0	5846.5
50	6930.1	3262.3	50	8310.8	4364.8	50	10093.0	5876.

TABLE VII.—LONG CHORDS.

Degree of Curve.	Actual Arc, One Station.	LONG CHORDS.				
		2 Stations.	**3** Stations.	**4** Stations.	**5** Stations.	**6** Stations.
0° 10′	100.000	200.000	299.999	399.998	499.996	599.993
20	.000	199.999	299.997	399.992	499.983	599.970
30	.000	199.998	299.992	399.981	499.962	599.933
40	.001	199.997	299.986	399.966	499.932	599.882
50	.001	199.995	299.979	399.947	499.894	599.815
1	100.001	199.992	299.970	399.924	499.848	599.733
10	.002	199.990	299.959	399.896	499.793	599.637
20	.002	199.986	299.946	399.865	499.729	599.526
30	.003	199.983	299.932	399.829	499.657	599.401
40	.003	199.979	299.915	399.789	499.577	599.260
50	.004	199.974	299.898	399.744	499.488	599.105
2	100.005	199.970	299.878	399.695	499.391	598.934
10	.006	199.964	299.857	399.643	499.285	598.750
20	.007	199.959	299.834	399.586	499.171	598.550
30	.008	199.952	299.810	399.524	499.049	598.336
40	.009	199.946	299.783	399.459	498.918	598.106
50	.010	199.939	299.756	399.389	498.778	597.862
3	100.011	199.931	299.726	399.315	498.630	597.604
10	.013	199.924	299.695	399.237	498.474	597.331
20	.014	199.915	299.662	399.154	498.309	597.043
30	.015	199.907	299.627	399.068	498.136	596.740
40	·017	199.898	299.591	398.977	497.955	596.423
50	.019	199.888	299.553	398.882	497.765	596.091
4	100.020	199.878	299.513	398.782	497.566	595.744
10	.022	199.868	299.471	398.679	497.360	595.383
20	.024	199.857	299.428	398.571	497.145	595.007
30	.026	199.846	299.383	398.459	496.921	594.617
40	.028	199.834	299.337	398.343	496.689	594.212
50	.030	199.822	299.289	398.223	496.449	593.792
5	100.032	199.810	299.239	398.099	496.201	593.358
10	.034	199.797	299.187	397.970	495.944	592.909
20	.036	199.783	299.134	397.837	495.678	592.446
30	.038	199.770	299.079	397.700	495.405	591.968
40	.041	199.756	299.023	397.559	495.123	591.476
50	.043	199.741	298.964	397.413	494.832	590.970
6	100.046	199.726	298.904	397.264	494.534	590.449
10	.048	199.710	298.843	397.110	494.227	589.913
20	.051	199.695	298.779	396.952	493.912	589.364
30	.054	199.678	298.714	396.790	493.588	588.800
40	.056	199.662	298.648	396.623	493.257	588.221
50	.059	199.644	298.579	396.453	492.917	587.628
7	100.062	199.627	298.509	396.278	492.568	587.021
10	.065	199.609	298.438	396.099	492.212	586.400
20	.068	199.591	298.364	395.916	491.847	585.765
30	.071	199.572	298.289	395.729	491.474	585.115
40	.075	199.553	298.212	395.538	491.093	584.451
50	.078	199.533	298.134	395.342	490.704	583.773
8	100.081	199.513	298.054	395.142	490.306	583.081
10	.085	199.492	297.972	394.938	489.900	582.375
20	.088	199.471	297.888	394.731	489.486	581.654
30	.092	199.450	297.803	394.518	489.064	580.920
40	.095	199.428	297.716	394.302	488.634	580.172
50	.099	199.406	297.628	394.082	488.196	579.409
9	100.103	199.383	297.538	393.857	487.749	578.633
10	.107	199.360	297.446	393.629	487.294	577.843
20	.111	199.337	297.352	393.396	486.822	577.039
30	.115	199.313	297.257	393.159	486.361	576.222
40	.119	199.289	297.160	392.918	485.882	575.390
50	.123	199.264	297.062	392.673	485.395	574.545
10	100.127	199.239	296.962	392.424	484.900	573.686

TABLE VII.—LONG CHORDS.

Degree of Curve.	Long Chords.					
	7 Stations.	8 Stations.	9 Stations.	10 Stations.	11 Stations.	12 Stations.
0° 10′	699.988	799.982	899.974	999.965	1099.95	1199.94
20	699.953	799.929	899.899	999.860	1099.81	1199.76
30	699.893	799.840	899.772	999.686	1099.58	1199.46
40	699.810	799 716	899.594	999.442	1099.25	1199.03
50	699.704	799.556	899.365	999.128	1098.84	1198.49
1	699.574	799.360	899.086	998.744	1098.33	1197.82
10	699.420	799.130	898.757	998.290	1097.72	1197.04
20	699.242	798.863	898.376	997.768	1097.02	1196.13
30	699.041	798.562	897.945	997.175	1096.23	1195.11
40	698.816	798.224	897.464	996.513	1095.35	1193.96
50	698.567	797.852	896.931	995.782	1094.38	1192.69
2	698.295	797.444	896.349	994.981	1093.31	1191.31
10	698.000	797 000	895.716	994.112	1092.15	1189.80
20	697.680	796.522	895.033	993.173	1090.90	1188.18
30	697.338	796.008	894.299	992.165	1089.56	1186.43
40	696.971	795.459	893.515	991.088	1088.12	1184.57
50	696.581	794.874	892.681	989.943	1086.60	1182.59
3	696.168	794.255	891.798	988.729	1084.98	1180.49
10	695.731	793.600	890.864	987.447	1083.28	1178.28
20	695.271	792.911	889.880	986.096	1081.48	1175.94
30	694.787	792.186	888.846	984.677	1079.59	1173.49
40	694.280	791.427	887.763	983.190	1077.61	1170.93
50	693.750	790.632	886.630	981.636	1075.54	1168.25
4	693.196	789.803	885.448	980.014	1073.38	1165.45
10	692.619	788.939	884.217	978.325	1071.14	1162.54
20	692.018	788.040	882.936	976.569	1068.81	1159.51
30	691.395	787.108	881.606	974.746	1066.38	1156.37
40	690.748	786.140	880.228	972.856	1063.87	1153.12
50	690.079	785.138	878.800	970.900	1061.27	1149.76
5	689.386	784.101	877.324	968.877	1058.59	1146.28
10	688.670	783.030	875.800	966.788	1055.81	1142.69
20	687.930	781.925	874.227	964.634	1052.95	1138.99
30	687.169	780.786	872.605	962.415	1050.01	1135.18
40	686.384	779.612	870.936	960.130	1046.97	1131.26
50	685.576	778.406	869.219	957.780	1043.86	1127.24
6	684.745	777.165	867.454	955.366	1040.66	1123.10
10	683.892	775.890	865.642	952.888	1037.37	1118.86
20	683.016	774.582	863.782	950.345	1034.01	1114.51
30	682.117	773.240	861.875	947.739	1030.55	1110.05
40	681.195	771.864	859.922	945.069	1027.02	1105.49
50	680.251	770.455	857.921	942.337	1023.40	1100.83
7	679.285	769.014	855.874	939.542	1019.70	1096.06
10	678.296	767.539	853.780	936.684	1015.93	1091.19
20	677.284	766.030	851.640	933.764	1012.07	1086.22
30	676.250	764.490	849.455	930.783	1008.13	1081.15
40	675.194	762.916	847.224	927.741	1004.11	1075.98
50	674.116	761.309	844.947	924.638	1000.01	1070.71
8	673.015	759.670	842.625	921.474	995.834	1065.34
10	671.892	757.999	840.258	918.250	991.580	1059.88
20	670.748	756.295	837.845	914.966	987.250	1054.32
30	669.581	754.560	835.389	911.623	982.844	1048.66
40	668.393	752.792	832.888	908.221	978.362	1042.91
50	667.182	750.993	830.342	904.761	973.806	1037.06
9	665.950	749.161	827.754	901.242	969.175	1031.13
10	664.697	747.299	825.121	897.667	964.471	1025.11
20	663.421	745.404	822.445	894.033	959.694	1018.99
30	662.124	743.479	819.726	890.343	954.844	1012.79
40	660.806	741.522	816.965	886.597	949.924	1006.49
50	659.466	739.535	814.160	882.795	944.933	1000.12
0	658.105	737.516	811.314	878.938	939.871	993.65

TABLE VII.—LONG CHORDS.

Degree of Curve.	Actual Arc, One Station.	Long Chords.				
		2 Stations.	3 Stations.	4 Stations.	5 Stations.	6 Stations.
10° 10′	100.131	199.213	296.860	392.171	484.397	572.813
20	.136	199.187	296.756	391.914	483.886	571.926
30	.140	199.161	296.651	391.652	483.367	571.027
40	.145	199.134	296.544	391.387	482.840	570.113
50	.149	199.107	296.436	391.117	482.305	569.186
11	100.154	199.079	296.325	390.843	481.762	568.245
10	.158	199.051	296.214	390.565	481.211	567.292
20	.163	199.023	296.100	390.284	480.653	566.324
30	.168	198.994	295.985	389.998	480.086	565.343
40	.173	198.964	295.868	389.708	479.511	564.349
50	.178	198.935	295.750	389.414	478.929	563.341
12	100.183	198.904	295.629	389.116	478.338	562.321
10	.188	198.874	295.508	388.814	477.740	561.287
20	.193	198.843	295.384	388.508	477.135	560.240
30	.199	198.811	295.259	388.197	476.521	559.180
40	.204	198.779	295.132	387.883	475.899	558.107
50	.209	198.747	295.004	387.565	475.270	557.020
13	100.215	198.714	294.874	387.243	474.633	555.921
10	.220	198.681	294.742	386.916	473.988	554.809
20	.226	198.648	294.609	386.586	473.336	553.684
30	.232	198.614	294.474	386.252	472.675	552.546
40	.237	198.579	294.337	385.914	472.007	551.395
50	.243	198.544	294.199	385.572	471.332	550.232
14	100.249	198.509	294.059	385.225	470.649	549.056
10	.255	198.474	293.918	384.875	469.958	547.867
20	.261	198.437	293.774	384.521	469.260	546.666
30	.267	198.401	293.629	384.163	468.554	545.452
40	.274	198.364	293.483	383.801	467.840	544.226
50	.280	198.327	293.335	383.435	467.119	542.987
15	100.286	198.289	293.185	383.065	466.390	541.736
10	.292	198.251	293.034	382.691	465.654	540.472
20	.299	198.212	292.881	382.313	464.911	539.196
30	.306	198.173	292.726	381.931	464.160	537.908
40	.312	198.134	292.570	381.546	463.401	536.608
50	.319	198.094	292.412	381.156	462.635	535.296
16	100.326	198.054	292.252	380.763	461.862	533.972
10	.333	198.013	292.091	380.365	461.081	532.635
20	.339	197.972	291.928	379.964	460.293	531.287
30	.346	197.930	291.764	379.559	459.498	529.927
40	.353	197.888	291.598	379.150	458.695	528.555
50	.361	197.846	291.430	378.737	457.886	527.171
17	100.368	197.803	291.261	378.320	457.069	525.776
10	.375	197.760	291.090	377.900	456.244	524.369
20	.382	197.716	290.918	377.475	455.413	522.950
30	.390	197.672	290.743	377.047	454.574	521.519
40	.397	197.628	290.568	376.615	453.728	520.078
50	.405	197.583	290.390	376.179	452.875	518.625
18	100.412	197.538	290.211	375.739	452.015	517.160
10	.420	197.492	290.031	375.295	451.147	515.685
20	.428	197.446	289.849	374.848	450.373	514.198
30	.436	197.399	289.665	374.397	449.392	512.699
40	.444	197.352	289.479	373.942	448.504	511.190
50	.452	197.305	289.292	373.483	447.608	509.670
19	100.460	197.256	289.104	373.021	446.706	508.139
10	.468	197.209	288.913	372.554	445.797	506.597
20	.476	197.160	288.722	372.084	444.881	505.043
30	.484	197.111	288.528	371.610	443.957	503.479
40	.493	197.062	288.333	371.133	443.028	501.905
50	.501	197.012	288.137	370.652	442.091	500.320
20	100.510	196.962	287.939	370.167	441.147	498.72

TABLE VII.—LONG CHORDS.

egree of Curve.	Long Chords.					
	7 Stations.	8 Stations.	9 Stations.	10 Stations.	11 Stations.	12 Stations.
10° 10′	656.723	735.467	808.426	875.025	934.741	987.105
20	655.320	733.387	805.495	871.058	929.542	980.473
30	653.895	731.277	802.524	867.038	924.276	973.760
40	652.450	729.137	799.512	862.963	918.943	966.967
50	650.983	726.967	796.458	858.836	913.544	960.093
11	649.496	724.767	793.364	854.656	908.080	953.141
10	647.989	722.537	790.230	850.425	902.550	946.112
20	646.460	720.278	787.056	846.140	896.957	939.007
30	644.911	717.990	783.843	841.808	891.303	931.828
40	643.342	715.672	780.590	837.424	885.586	924.575
50	641.752	713.325	777.298	832.990	879.807	917.250
12	640.142	710.950	773.968	828.507	873.968	909.854
10	638.512	708.546	770.600	823.974	868.070	902.389
20	636.862	706.113	767.193	819.394	862.113	894.855
30	635.191	703.653	763.749	814.766	856.099	887.254
40	633.501	701.164	760.268	810.092	850.028	879.588
50	631.792	698.647	756.749	805.370	843.900	871.857
13	630.062	696.103	753.194	800.602	837.718	864.063
10	628.313	693.531	749.603	795.790	831.482	856.208
20	626.544	690.932	745.976	790.932	825.192	848.293
30	624.756	688.306	742.313	786.030	818.850	840.318
40	622.949	685.653	738.616	781.085	812.457	832.286
50	621.123	682.974	734.883	776.096	806.013	824.198
14	619.278	680.268	731.116	771.066	799.520	816.056
10	617.413	677.535	727.315	765.993	792.979	807.860
20	615.530	674.777	723.480	760.879	786.389	799.612
30	613.628	671.993	719.612	755.725	779.753	791.313
40	611.708	669.183	715.711	750.531	773.072	782.966
50	609.769	666.348	711.777	745.297	766.345	774.571
15	607.812	663.488	707.811	740.024	759.575	766.130
10	605.836	660.603	703.814	734.714	752.763	
20	603.842	657.693	699.785	729.366	745.908	
30	601.831	654.758	695.725	723.982	739.014	
40	599.801	651.799	691.634	718.561	732.078	
50	597.753	648.817	687.513	713.105	725.104	
16	595.688	645.810	683.362	707.614	718.092	
10	593.605	642.780	679.182	702.088	711.043	
20	591.505	639.727	674.973	696.529	703.959	
30	589.388	636.650	670.735	690.938		
40	587.253	633.550	666.469	685.314		
50	585.101	630.428	662.175	679.659		
17	582.933	627.283	657.854	673.972		
10	580.747	624.117	653.506	668.256		
20	578.545	620.928	649.131	662.510		
30	576.326	617.717	644.730	656.735		
40	574.091	614.485	640.304	650.933		
50	571.839	611.232	635.852	645.103		
18	569.571	607.958	631.375	639.245		
10	567.287	604.664	626.874			
20	564.988	601.349	622.349			
30	562.673	598.013	617.801			
40	560.342	594.658	613.229			
50	557.996	591.283	608.635			
19	555.634	587.888	604.018			
10	553.257	584.475	599.379			
20	550.864	581.042	594.720			
30	548.457	577.591	590.039			
40	546.035	574.121	585.339			
50	543.599	570.634	580.618			
	541.147	567.128	575.877			

TABLE VII.—LONG CHORDS.

Degree of Curve.	Actual Arc, One Station.	Long Chords.				
		2 Stations.	3 Stations.	4 Stations.	5 Stations.	6 Stations.
21°	100.562	196.651	286.716	367.179	435.345	488.931
22	100.617	196.325	285.437	364.060	429.305	478.775
23	100.675	195.985	284.101	360.810	423.083	468.270
24	100.735	195.630	282.709	357.433	416.535	457.433
25	100.798	195.259	281.262	353.930	409.819	446.280
26	100.863	194.874	279.759	350.303	402.891	434.827
27	100.931	194.474	278.201	346.555	395.758	423.092
28	101.002	194.059	276.589	342.688	388.428	411.092
29	101.075	193.630	274.924	338.704	380.908	398.846
30	101.152	193.185	273.205	334.607	373.205	386.370

TABLE VIII.—MIDDLE ORDINATES.

TABLE VIII.—MIDDLE ORDINATES.

Degree of Curve.	1 Station.	2 Stations.	3 Stations.	4 Stations.	5 Stations.	6 Stations.
0° 10′	.036	.145	.327	.582	.909	1.309
20	.073	.291	.654	1.164	1.818	2.618
30	.109	.436	.982	1.745	2.727	3.926
40	.145	.582	1.309	2.327	3.636	5.235
50	.182	.727	1.636	2.909	4.545	6.544
1	.218	.873	1.963	3.490	5.453	7.852
10	.255	1.018	2.291	4.072	6.362	9.160
20	.291	1.164	2.618	4.654	7.270	10.468
30	.327	1.309	2.945	5.235	8.179	11.775
40	.364	1.454	3.272	5.816	9.087	13.082
50	.400	1.600	3.599	6.398	9.994	14.389
2	.436	1.745	3.926	6.979	10.902	15.694
10	.473	1.891	4.253	7.560	11.809	17.000
20	.509	2.036	4.580	8.141	12.716	18.304
30	.545	2.181	4.907	8.722	13.623	19.608
40	.582	2.327	5.234	9.303	14.529	20.912
50	.618	2.472	5.561	9.883	15.435	22.214
3	.654	2.618	5.888	10.464	16.341	23.516
10	.691	2.763	6.215	11.044	17.246	24.817
20	.727	2.908	6.542	11.624	18.151	26.117
30	.763	3.054	6.868	12.204	19.055	27.416
40	.800	3.199	7.195	12.784	19.959	28.714
50	.836	3.345	7.522	13.363	20.863	30.012
4	.872	3.490	7.848	13.943	21.766	31.308
10	.909	3.635	8.175	14.522	22.668	32.603
20	.945	3.781	8.501	15.101	23.570	33.896
30	.982	3.926	8.828	15.680	24.471	35.189
40	1.018	4.071	9.154	16.258	25.372	36.480
50	1.054	4.217	9.480	16.837	26.272	37.770
5	1.091	4.362	9.807	17.415	27.171	39.059
10	1.127	4.507	10.133	17.992	28.070	40.346
20	1.164	4.653	10.459	18.570	28.968	41.631
30	1.200	4.798	10.785	19.147	29.866	42.916
40	1.237	4.943	11.111	19.724	30.762	44.198
50	1.273	5.088	11.436	20.301	31.658	45.479
6	1.309	5.234	11.762	20.877	32.553	46.759
10	1.346	5.379	12.088	21.453	33.448	48.037
20	1.382	5.524	12.413	22.029	34.341	49.313
30	1.418	5.669	12.739	22.604	35.234	50.587
40	1.455	5.814	13.064	23.179	36.126	51.860
50	1.491	5.960	13.389	23.754	37.017	53.130
7	1.528	6.105	13.715	24.328	37.907	54.399
10	1.564	6.250	14.040	24.902	38.796	55.666
20	1.600	6.395	14.365	25.476	39.684	56.931
30	1.637	6.540	14.689	26.049	40.571	58.193
40	1.673	6.685	15.014	26.622	41.458	59.454
50	1.710	6.831	15.339	27.195	42.343	60.712
8	1.746	6.976	15.663	27.767	43.227	61.969
10	1.782	7.121	15.988	28.338	44.110	63.223
20	1.819	7.266	16.312	28.910	44.992	64.475
30	1.855	7.411	16.636	29.481	45.873	65.724
40	1.892	7.556	16.960	30.051	46.753	66.972
50	1.928	7.701	17.284	30.621	47.632	68.216
9	1.965	7.846	17.608	31.190	48.510	69.459
10	2.001	7.991	17.932	31.759	49.386	70.699
20	2.037	8.136	18.255	32.328	50.261	71.936
30	2.074	8.281	18.578	32.896	51.135	73.171
40	2.110	8.426	18.902	33.464	52.008	74.408
50	2.147	8.571	19.225	34.031	52.880	75.632
10	2.183	8.716	19.548	34.597	53.750	76.859

TABLE VIII.—MIDDLE ORDINATES.

Degree of Curve.	7 Stations.	8 Stations.	9 Stations.	10 Stations.	11 Stations.	12 Stations.
0° 10′	1.782	2.327	2.945	3.636	4.400	5.236
20	3.563	4.654	5.890	7.272	8.799	10.471
30	5.345	6.981	8.835	10.907	13.197	15.704
40	7.126	9.307	11.778	14.540	17.593	20.936
50	8.907	11.632	14.721	18.173	21.987	26.164
1	10.687	13.957	17.663	21.803	26.378	31.388
10	12.467	16.281	20.603	25.431	30.766	36.607
20	14.246	18.604	23.541	29.057	35.150	41.821
30	16.024	20.925	26.477	32.679	39.530	47.028
40	17.802	23.246	29.411	36.298	43.905	52.229
50	19.579	25.564	32.343	39.914	48.274	57.422
2	21.355	27.881	35.272	43.525	52.637	62.606
10	23.130	30.197	38.198	47.131	56.993	67.780
20	24.903	32.510	41.121	50.733	61.343	72.945
30	26.676	34.821	44.040	54.330	65.684	78.098
40	28.447	37.130	46.956	57.921	70.018	83.240
50	30.216	39.436	49.868	61.506	74.342	88.370
3	31.984	41.740	52.776	65.084	78.657	93.486
10	33.751	44.041	55.679	68.656	82.963	98.588
20	35.516	46.339	58.577	72.221	87.258	103.675
30	37.279	48.634	61.471	75.778	91.542	108.747
40	39.040	50.926	64.360	79.328	95.814	113.803
50	40.800	53.215	67.243	82.869	100.075	118.841
4	42.557	55.500	70.121	86.402	104.323	123.862
10	44.312	57.781	72.992	89.925	108.558	128.864
20	46.065	60.059	75.858	93.440	112.779	133.847
30	47.816	62.333	78.717	96.945	116.986	138.810
40	49.564	64.602	81.570	100.439	121.178	143.753
50	51.310	66.868	84.416	103.924	125.356	148.674
5	53.053	69.129	87.255	107.397	129.517	153.572
10	54.794	71.386	90.087	110.860	133.663	158.448
20	56.532	73.638	92.911	114.311	137.791	163.300
30	58.267	75.885	95.728	117.751	141.903	168.128
40	59.999	78.127	98.536	121.178	145.997	172.931
50	61.729	80.364	101.337	124.593	150.072	177.708
6	63.455	82.596	104.129	127.995	154.129	182.459
10	65.178	84.822	106.912	131.384	158.166	187.182
20	66.898	87.043	109.686	134.759	162.184	191.878
30	68.615	89.258	112.452	138.120	166.182	196.545
40	70.328	91.468	115.298	141.468	170.159	201.183
50	72.037	93.671	117.954	144.800	174.114	205.792
7	73.744	95.868	120.691	148.118	178.048	210.370
10	75.446	98.059	123.417	151.421	181.960	214.916
20	77.145	100.244	126.134	154.708	185.850	219.431
30	78.840	102.422	128.840	157.979	189.716	223.914
40	80.531	104.594	131.535	161.234	193.559	228.363
50	82.218	106.758	134.219	· 164.473	197.377	232.779
8	83.901	108.916	136.893	167.695	201.171	237.160
10	85.580	111.067	139.555	170.899	204.941	241.507
20	87.254	113.210	142.205	174.086	208.685	245.818
30	88.924	115.346	144.844	177.255	212.403	250.093
40	90.590	117.475	147.470	180.407	216.095	254.331
50	92.252	119.596	150.085	183.539	219.760	258.531
9	93.909	121.709	152.687	186.653	223.398	262.694
10	95.561	123.814	155.277	189.748	227.008	266.818
20	97.208	125.911	157.854	192.824	230.591	270.904
30	98.851	128.000	160.417	195.880	234.145	274.949
40	100.489	130.081	162.968	198.916	237.670	278.955
50	102.122	132.153	165.505	201.932	241.167	282.919
10	103.750	134.217	168.029	204.928	244.633	286.84

TABLE VIII.—MIDDLE ORDINATES.

Degree of Curve.	1 Station.	2 Stations.	3 Stations.	4 Stations.	5 Stations.	6 Stations.
10° 10′	2.219	8.860	19.870	35.164	54.619	78.083
20	2.256	9.005	20.193	35.729	55.486	79.305
30	2.293	9.150	20.516	36.294	56.353	80.523
40	2.329	9.295	20.838	36.859	57.218	81.739
50	2.365	9.440	21.160	37.423	58.081	82.951
11	2.402	9.585	21.483	37.986	58.943	84.161
10	2.438	9.729	21.804	38.549	59.804	85.368
20	2.475	9.874	22.126	39.111	60.663	86.571
30	2.511	10.019	22.448	39.673	61.521	87.772
40	2.547	10.164	22.769	40.234	62.377	88.969
50	2.584	10.308	23.090	40.795	63.232	90.164
12	2.620	10.453	23.412	41.355	64.085	91.355
10	2.657	10.597	23.732	41.914	64.937	92.542
20	2.693	10.742	24.053	42.473	65.787	93.727
30	2.730	10.887	24.374	43.031	66.636	94.908
40	2.766	11.031	24.694	43.588	67.482	96.086
50	2.803	11.176	25.014	44.145	68.328	97.260
13	2.839	11.320	25.334	44.701	69.171	98.431
10	2.876	11.465	25.654	45.256	70.012	99.598
20	2.912	11.609	25.974	45.811	70.854	100.762
30	2.949	11.754	26.293	46.365	71.692	101.922
40	2.985	11.898	26.612	46.919	72.529	103.079
50	3.022	12.043	26.931	47.472	73.364	104.232
14	3.058	12.187	27.250	48.024	74.197	105.381
10	3.095	12.331	27.569	48.575	75.029	106.527
20	3.131	12.476	27.887	49.126	75.859	107.669
30	3.168	12.620	28.206	49.676	76.687	108.807
40	3.204	12.764	28.524	50.225	77.513	109.941
50	3.241	12.908	28.841	50.773	78.337	111.071
15	3.277	13.053	29.159	51.321	79.159	112.197
10	3.314	13.197	29.476	51.868	79.979	113.319
20	3.350	13.341	29.778	52.414	80.798	114.428
30	3.387	13.485	30.111	52.959	81.614	115.522
40	3.423	13.629	30.427	53.504	82.429	116.662
50	3.460	13.773	30.744	54.048	83.241	117.768
16	3.496	13.917	31.060	54.591	84.052	118.870
10	3.533	14.061	31.370	55.133	84.861	119.967
20	3.569	14.205	31.692	55.675	85.667	121.061
30	3.606	14.349	32.008	56.215	86.471	122.150
40	3.643	14.493	32.323	56.755	87.274	123.225
50	3.679	14.637	32.638	57.294	88.074	124.215
17	3.716	14.781	32.953	57.832	88.872	125.391
10	3.752	14.925	33.267	58.369	89.668	126.463
20	3.789	15.069	33.582	58.906	90.462	127.530
30	3.825	15.212	33.896	59.441	91.254	128.598
40	3.862	15.356	34.210	59.976	92.043	129.651
50	3.899	15.500	34.523	60.510	92.830	130.704
18	3.935	15.643	34.837	61.042	93.616	131.753
10	3.972	15.787	35.150	61.574	94.398	132.797
20	4.008	15.931	35.463	62.106	95.179	133.837
30	4.045	16.074	35.775	62.636	95.957	134.872
40	4.081	16.218	36.088	63.165	96.733	135.902
50	4.118	16.361	36.400	63.693	97.506	136.928
19	4.155	16.505	36.712	64.221	98.278	137.948
10	4.191	16.648	37.023	64.747	99.047	138.964
20	4.228	16.792	37.334	65.273	99.813	139.975
30	4.265	16.935	37.645	65.797	100.577	140.981
40	4.301	17.078	37.956	66.321	101.339	141.982
50	4.338	17.222	38.266	66.843	102.098	142.978
20	4.374	17.365	38.576	67.365	102.855	143.969

TABLE IX.—LINEAR DEFLECTION TABLE.

Deflection.	100.	200.	300.	400.	500.	600.	700.	800.	900.	1000.
30′	0.87	1.75	2.62	3.49	4.36	5.24	6.11	6.98	7.85	8.73
1°	1.75	3.49	5.24	6.98	8.73	10.47	12.22	13.96	15.71	17,45
30	2.62	5.24	7.85	10.47	13.09	15.71	18.33	20.94	23.56	26.18
2	3.49	6.98	10.47	13.96	17.45	20.94	24.43	27.92	31.41	34.90
30	4.36	8.72	13.09	17.45	21.81	26.18	30.54	34.90	39.27	43.63
3	5.24	10.47	15.71	20.94	26.18	31.41	36.65	41.88	47.12	52.35
30	6.11	12.22	18.32	24.43	30.54	36.65	42.75	48.86	54.97	61.08
4	6.98	13.96	20.94	27.92	34.90	41.88	48.86	55.84	66.82	69.80
30	7.85	15.70	23.56	31.41	39.26	47.11	54.96	62.82	70.67	78.52
5	8.73	17.45	26.17	34.89	43.62	52.34	61.07	69.79	78.51	87.24
30	9.60	19.19	28.79	38.38	47.98	57.57	67.17	76.76	86.36	95.96
6	10.47	20.93	31.40	41.87	52.34	62.80	73.27	83.74	94.20	104.67
30	11.34	22.68	34.02	45.35	56.67	68.03	79.37	90.71	102.05	113.39
7	12.21	24.42	36.63	48.84	61.05	73.26	85.47	97.68	109.89	122.10
30	13.08	26.16	39.24	52.32	65.40	78.48	91.56	104.64	117.73	130.81
8	13.95	27.90	41.85	55.80	69.76	83.71	97.66	111.61	125.56	139.51
30	14.82	29.64	44.47	59.29	74.11	88.93	103.75	118.57	133.40	148.22
9	15.69	31.38	47.08	62.77	78.46	94.15	109.84	125.53	141.23	156.92
30	16.56	33.12	49.68	66.25	82.81	99.37	115.93	132.49	149.05	165.62
10	17.43	34.86	52.29	69.72	87.16	104.59	122.02	139.45	156.88	174.31
30	18.30	36.60	54.90	73.20	91.50	109.80	128.10	146.40	164.70	183.00
11	19.17	38.34	57.51	76.68	95.85	115.01	134.18	153.35	172.52	191.69
30	20.04	40.08	60.11	80.15	100.19	120.23	140.26	160.30	180.34	200.38
12	20.91	41.81	62.72	83.62	104.53	125.43	146.34	167.25	188.15	209.06
30	21.77	43.55	65.32	87.09	108.87	130.64	152.41	174.19	195.96	217.73
13	22.64	45.28	67.92	90.56	113.20	135.84	158.48	181.13	203.77	226.41
30	23.51	47.01	70.52	94.03	117.54	141.04	164.55	188.06	211.57	235.07
14	24.37	48.75	73.12	97.50	121.87	146.24	170.62	194.99	219.36	243.74
30	25.24	50.48	75.72	100.96	126.20	151.44	176.68	201.92	227.16	252.40
15	26.11	52.21	78.32	104.42	130.53	156.63	182.74	208.84	234.95	261.05
30	26.97	53.94	80.91	107.88	134.85	161.82	188.79	215.76	242.73	269.70
16	27.83	55.67	83.50	111.34	139.17	167.01	194.84	222.68	250.51	278.35
30	28.70	57.40	86.10	114.79	143.49	172.19	200.89	229.59	258.29	286.99
17	29.56	59.12	88.69	118.25	147.81	177.37	206.93	236.50	266.06	295.62
30	30.42	60.85	91.27	121.70	152.12	182.55	212.97	243.40	273.82	304.25
18	31.29	62.57	93.86	125.15	156.43	187.72	219.01	250.30	281.58	312.87
30	32.15	64.30	96.45	128.59	160.74	192.89	225.04	257.19	289.34	321.49
19	33.01	66.02	99.03	132.04	165.05	198.06	231.07	264.08	297.08	330.09
30	33.87	67.74	101.61	135.48	169.35	203.22	237.09	270.96	304.83	338.70
20	34.73	69.46	104.19	138.92	173.65	208.38	243.11	277.84	312.57	347.30
30	35.59	71.18	106.77	142.35	177.94	213.53	249.12	284.71	320.30	355.89
21	36.45	72.89	109.34	145.79	182.24	218.68	255.13	291.58	328.02	364.47
30	37.30	74.61	111.91	149.22	186.52	223.83	261.13	298.44	335.74	373.05
22	38.16	76.32	114.49	152.65	190.81	228.97	267.13	305.29	343.46	381.62
30	39.02	78.04	117.05	156.07	195.09	234.11	273.13	312.14	351.16	390.18
23	39.87	79.75	119.62	159.49	199.37	239.24	279.12	318.99	358.86	398.74
30	40.73	81.46	122.19	162.91	203.64	244.37	285.10	325.83	366.56	407.28
24	41.58	83.16	124.75	166.33	207.91	249.49	291.08	332.66	374.24	415.82
30	42.41	84.87	127.31	169.74	212.18	254.61	297.05	339.48	381.92	424.36
25	43.29	86.58	129.86	173.15	216.44	259.73	303.02	364.30	389.59	432.88
30	44.14	88.28	132.42	176.56	220.70	264.84	308.98	353.12	397.26	441.39
26	44.99	89.98	134.97	179.96	224.95	269.94	314.93	359.92	404.91	449.90
30	45.84	91.68	137.52	183.36	229.20	275.04	320.88	366.72	412.56	458.40
27	46.69	93.38	140.07	186.76	233.45	280.14	326.82	373.51	420.20	466.89
30	47.54	95.07	142.61	190.15	237.69	285.22	332.76	380.30	427.83	475.37
28	*48.38*	*96.77*	*145.15*	193.54	241.92	290.31	338.69	387.08	435.46	483.84
30	*49.23*	*98.46*	*147.69*	196.92	246.15	295.38	344.62	393.85	443.08	492.3
29	*50.08*	*100.15*	*150.23*	200.30	250.38	300.46	350.53	400.16	450.68	500
30	*50.92*	*101.84*	*152.76*	203.68	254.60	305.52	356.44	407.36	458.28	50
30	*51.76*	*103.53*	*155.29*	207.06	258.82	310.59	362.35	414.11	465.87	5

TABLE X.—COEFFICIENTS FOR VALVOID ARCS.

I.—RATIO OF $u = \frac{i}{\Delta}$.

L	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°	110°	120°
300	.3518	.3516	.3514	.3510	.3506	.3500	.3493	.3485	.3476	.3466	.3455	.3444
400	.3437	.3436	.3433	.3430	.3426	.3421	.3415	.3408	.3399	.3390	.3380	.3368
500	.3400	.3398	.3396	.3393	.3389	.3383	.3379	.3372	.3364	.3356	.3345	.3335
600	.3379	.3378	.3376	.3373	.3369	.3365	.3359	.3353	.3345	.3337	.3327	.3317
700	.3367	.3366	.3364	.3361	.3357	.3353	.3347	.3341	.3334	.3326	.3316	.3306
800	.3359	.3358	.3356	.3353	.3349	.3345	.3340	.3333	.3326	.3318	.3309	.3299
900	.3353	.3352	.3350	.3348	.3344	.3340	.3334	.3328	.3321	.3313	.3304	.3294
1000	.3350	.3348	.3346	.3344	.3340	.3336	.3331	.3324	.3317	.3310	.3301	.3291
1200	.3345	.3343	.3341	.3339	.3336	.3331	.3326	.3320	.3313	.3305	.3296	.3286
1500	.3340	.3339	.3337	.3335	.3331	.3327	.3322	.3316	.3309	.3301	.3292	.3283
2000	.3337	.3336	.3333	.3331	.3328	.3324	.3319	.3313	.3306	.3298	.3289	.3280

II.—RATIO OF $v = \frac{r}{L}$.

L	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°	110°	120°
300	.7706	.7683	.7643	.7588	.7518	.7432	.7332	.7218	.7090	.6949	.6795	.6630
400	.7611	.7588	.7549	.7495	.7425	.7341	.7243	.7130	.7004	.6865	.6714	.6551
500	.7568	.7545	.7506	.7452	.7384	.7300	.7202	.7091	.6966	.6828	.6678	.6516
600	.7545	.7522	.7483	.7430	.7361	.7273	.7181	.7070	.6946	.6808	.6659	.6498
700	.7531	.7508	.7469	.7416	.7348	.7265	.7168	.7057	.6933	.6797	.6648	.6487
800	.7522	.7499	.7461	.7407	.7339	.7257	.7160	.7049	.6926	.6789	.6640	.6480
900	.7516	.7492	.7454	.7401	.7333	.7251	.7154	.7044	.6920	.6784	.6635	.6475
1000	.7512	.7489	.7450	.7397	.7329	.7247	.7150	.7040	.6917	.6780	.6632	.6472
1200	.7505	.7483	.7444	.7391	.7324	.7241	.7145	.7035	.6912	.6775	.6627	.6468
1500	.7501	.7478	.7440	.7387	.7319	.7237	.7141	.7031	.6908	.6772	.6624	.6464
2000	.7497	.7474	.7436	.7383	.7316	.7234	.7137	.7028	.6904	.6769	.6621	.6461

III.—RATIO OF $l_1 = \frac{l}{\Delta' - \Delta}$ = LENGTH OF VALVOID ARC CORRESPONDING TO A CHANGE OF ONE DEGREE IN THE ANGLE Δ.

L	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°	110°	120°
300	2.62	2.61	2.60	2.59	2.57	2.55	2.52	2.49	2.46	2.42	2.38	2.34
400	3.49	3.48	3.46	3.44	3.42	3.38	3.35	3.30	3.25	3.20	3.14	3.08
500	4.36	4.35	4.33	4.30	4.26	4.22	4.17	4.11	4.05	3.98	3.90	3.81
600	5.23	5.22	5.19	5.16	5.11	5.06	4.99	4.92	4.84	4.75	4.65	4.55
700	6.10	6.09	6.06	6.02	5.96	5.90	5.83	5.74	5.65	5.54	5.43	5.31
800	6.97	6.95	6.92	6.87	6.82	6.74	6.66	6.56	6.45	6.33	6.20	6.06
900	7.85	7.82	7.79	7.73	7.67	7.59	7.49	7.38	7.26	7.13	6.98	6.82
1000	8.72	8.69	8.65	8.59	8.52	8.43	8.32	8.20	8.07	7.92	7.75	7.58
1100	9.59	9.56	9.52	9.45	9.37	9.27	9.16	9.02	8.87	8.71	8.53	8.34
1200	10.46	10.43	10.38	10.31	10.22	10.11	9.99	9.84	9.68	9.50	9.31	9.09
1300	11.33	11.30	11.25	11.17	11.07	10.96	10.82	10.66	10.49	10.29	10.08	9.85
1400	12.21	12.17	12.11	12.03	11.93	11.80	11.65	11.48	11.29	11.08	10.86	10.61
1500	13.08	13.04	12.98	12.89	12.78	12.64	12.48	12.30	12.10	11.88	11.63	11.37
1600	13.95	13.91	13.84	13.75	13.63	13.49	13.32	13.12	12.91	12.67	12.41	12.13
1700	14.82	14.78	14.71	14.61	14.48	14.33	14.15	13.94	13.71	13.46	13.18	12.88
1800	15.69	15.65	15.57	15.47	15.33	15.17	14.98	14.76	14.52	14.25	13.96	13.64
1900	16.57	16.52	16.44	16.33	16.19	16.01	15.81	15.58	15.33	15.04	14.73	14.40
2000	17.44	17.39	17.30	17.19	17.04	16.86	16.65	16.40	16.13	15.83	15.51	15.1

TABLE XI.—TURNOUTS AND SWITCHES FROM A STRAIGHT TRACK. §§ 180, 181, 182.

GAUGE, 4 FEET 8½ INCHES = 4.708. THROW, 5 INCHES = 0.417.

No. *n*.	Angle *F*.	Dist. *BF*.	Chord *af*.	Switch *AD*.	Radius *r*.	Log'thm. *log. r*.	Degree of Curve.
4	14° 15′ 00″	37.664	37.373	11.209	150.656	2.177986	38° 45′ 57″
4½	13 40 49	42.372	42.118	12.610	190.674	2.280292	30 24 09
5	11 25 16	47.080	46.846	14.012	235.400	2.371806	24 31 36
5½	10 23 20	51.788	51.575	15.413	284.834	2.454592	20 13 13
6	9 31 39	56.496	56.301	16.814	338.976	2.530169	16 57 52
6½	8 47 51	61.204	61.024	18.215	397.826	2.599693	14 26 25
7	8 10 16	65.912	65.744	19.616	461.384	2.664063	12 26 34
7½	7 37 41	70.620	70.464	21.017	529.650	2.723989	10 50 02
8	7 09 10	75.328	75.181	22.418	602.624	2.780046	9 31 07
8½	6 43 59	80.036	79.898	23.820	680.306	2.832704	8 25 47
9	6 21 35	84.744	84.613	25.221	762.696	2.882352	7 31 04
9½	6 01 32	89.452	89.328	26.622	849.794	2.929314	6 44 46
10	5 43 29	94.160	94.043	28.023	941.600	2.973866	6 05 16
10½	5 27 09	98.868	98.756	29.424	1038.114	3.016245	5 31 17
11	5 12 18	103.576	103.469	30.825	1139.336	3.056652	5 01 50
11½	4 58 45	108.284	108.182	32.227	1245.266	3.095262	4 36 08
12	4 46 19	112.992	112.894	33.628	1355.904	3.132229	4 13 36

GAUGE, 3 FEET. THROW, 4 INCHES = 0.333.

No. *n*.	Angle *F*.	Dist. *BF*.	Chord *af*.	Switch *AD*.	Radius *r*.	Log'thm. *log. r*.	Degree of Curve.
4	14° 15′ 00″	24	23.815	8	96.0	1.982271	62° 46′ 34″
4½	12 40 49	27	26.835	9	121.5	2.084576	48 36 04
5	11 25 16	30	29.851	10	150.0	2.176091	38 56 33
5½	10 23 20	33	32.865	11	181.5	2.258877	31 58 55
6	9 31 39	36	35.876	12	216.0	2.334454	26 46 07
6½	8 47 51	39	38.885	13	253.5	2.403978	22 45 04
7	8 10 16	42	41.893	14	294.0	2.468347	19 35 01
7½	7 37 41	45	44.900	15	337.5	2.528274	17 02 21
8	7 09 10	48	47.906	16	384.0	2.584331	14 57 48
8½	6 43 59	51	50.912	17	433.5	2.636989	13 14 47
9	6 21 35	54	53.917	18	486.0	2.686636	11 48 37
9½	6 01 32	57	56.921	19	541.5	2.733598	10 35 46
10	5 43 29	60	59.925	20	600.0	2.778151	9 33 38
10½	5 27 09	63	62.929	21	661.5	2.820580	8 40 12
11	5 12 18	66	65.932	22	726.0	2.860037	7 53 54
11½	4 58 45	69	68.935	23	793.5	2.899547	7 13 32
12	4 46 19	72	71.938	24	864.0	2.936514	6 58 06

ANGLE AND DISTANCE OF MIDDLE FROG, *F*″

No. *n*.	No. *n*″.	Angle *F*″.	Gauge 4, 8½. Dist. *aF*″.	Gauge 3. Dist. *aF*″.	No. *n*.	No. *n*″.	Angle *F*″.	Gauge 4, 8½. Dist. *aF*″.	Gauge 3. Dist. *aF*″.
4	2.817	20° 07′ 36″	26.736	17.037	8	5.651	10° 06′ 44″	53.317	33.974
4½	3.172	17 54 52	30.054	19.151	8½	6.005	9 31 08	56.643	36.094
5	3.527	16 08 19	33 374	21.266	9	6.359	8 59 30	59.969	38.213
5½	3.881	14 40 58	36.695	23.383	9½	6.713	8 31 10	63.296	40.333
6	4.235	13 27 57	40.018	25.500	10	7.067	8 05 40	66.623	42.453
6½	4.589	12 26 07	43.342	27.618	10½	7.420	7 42 35	69.950	44.573
7	4.943	11 33 04	46.666	29.736	11	7.774	7 21 36	73.277	46.69
7½	5.297	10 47 02	49.991	31.855	11½	8.128	7 02 26	76.605	48.8
8	5.651	10 06 44	53.317	33.974	12	8.482	6 44 51	79.932	50.

TABLE XII.—MIDDLE ORDINATES FOR CURVING RAILS. § 199.

D	Length of Rail-Chord.												D
	32	30	28	26	24	22	20	18	16	14	12	10	
1°	.022	.020	.017	.015	.013	.011	.009	.007	.006	.004	.003	.002	1°
2	.045	.039	.034	.030	.025	.021	.017	.014	.011	.009	.006	.004	2
3	.067	.059	.051	.044	.038	.032	.026	.021	.017	.013	.009	.007	3
4	.089	.079	.068	.059	.050	.042	.035	.028	.022	.017	.013	.009	4
5	.112	.098	.086	.074	.063	.053	.044	.035	.028	.021	.016	.011	5
6	.134	.118	.103	.088	.075	.063	.052	.042	.034	.026	.019	.013	6
7	.156	.137	.120	.103	.088	.074	.061	.049	.039	.030	.022	.015	7
8	.179	.157	.137	.118	.100	.084	.070	.057	.045	.034	.025	.017	8
9	.201	.177	.154	.133	.113	.095	.078	.064	.050	.038	.028	.020	9
10	.223	.196	.171	.147	.126	.105	.087	.071	.056	.043	.031	.022	10
11	.245	.216	.188	.162	.138	.116	.096	.078	.061	.047	.035	.024	11
12	.268	.235	.205	.177	.151	.127	.105	.085	.067	.051	.038	.026	12
14	.312	.274	.238	.206	.175	.147	.122	.099	.078	.060	.044	.030	14
16	.356	.313	.273	.235	.200	.168	.139	.113	.089	.068	.050	.035	16
18	.400	.352	.307	.264	.225	.189	.156	.127	.100	.077	.056	.039	18
20	.445	.391	.340	.293	.250	.210	.174	.141	.111	.085	.063	.043	20
24	.531	.467	.407	.351	.299	.251	.207	.168	.133	.102	.075	.052	24
28	.618	.543	.473	.408	.347	.292	.241	.195	.154	.118	.087	.060	28
32	.705	.619	.539	.465	.396	.333	.275	.223	.176	.135	.099	.069	32
36	.791	.696	.606	.522	.445	.373	.309	.250	.197	.151	.111	.077	36
40	.878	.772	.672	.579	.493	.414	.342	.277	.219	.168	.123	.086	40
45	.983	.863	.752	.648	.552	.463	.383	.305	.245	.188	.137	.096	45
50	1.087	.955	.831	.716	.610	.512	.423	.343	.271	.207	.152	.106	50

TABLE XIII.—DIFFERENCE IN ELEVATION OF RAILS ON CURVES. § 201.

D	Velocity in Miles per Hour.										D
	10	15	20	25	30	35	40	45	50	60	
1	.006	.013	.023	.036	.051	.070	.091	.116	.143	.206	1
2	.011	.026	.046	.071	.103	.140	.183	.231	.285	.410	2
3	.017	.039	.069	.107	.154	.210	.274	.346	.427	.612	3
4	.023	.051	.091	.143	.206	.280	.365	.461	.568	.811	4
5	.029	.064	.114	.179	.257	.349	.455	.574	.707	1.006	5
6	.034	.077	.137	.214	.308	.418	.545	.687	.844	1.196	6
7	.040	.090	.160	.250	.359	.487	.634	.798	.979		
8	.046	.103	.183	.285	.410	.556	.723	.908	1.112		
9	.051	.116	.206	.320	.460	.624	.811	1.017			
10	.057	.129	.228	.356	.511	.692	.898	1.124			
11	.063	.142	.251	.391	.561	.760	.984				
12	.069	.154	.274	.427	.611	.826	1.069				
14	.080	.180	.319	.497	.711	.959					
16	.091	.206	.365	.567	.809	1.088					
18	.102	.231	.410	.637	.906						
20	.114	.256	.455	.707	1.002						
25	.141	.318	.563	.775							
30	.168	.380	.672	.844							
35	.195	.441	.778								
40	.222	.501	.881								
50	.276	.618									

TABLE XIV.—GRADES AND GRADE ANGLES.

TABLE XIV.—GRADES AND GRADE ANGLES.

Feet per Station.	Feet per Mile.	Inclination.	Feet per Station.	Feet per Mile.	Inclination.	Feet per Station.	Feet per Mile.	Inclination.
		° ′ ″			° ′ ″			° ′ ″
.01	.528	21	.51	26.928	17 32	1.01	53.328	34 43
.02	1.056	41	.52	27.456	17 53	1.02	53.856	35 04
.03	1.584	1 02	.53	27.984	18 13	1.03	54.384	35 24
.04	2.112	1 23	.54	28.512	18 34	1.04	54.912	35 45
.05	2.640	1 43	.55	29.040	18 54	1.05	55.440	36 05
.06	3.168	2 04	.56	29.568	19 15	1.06	55.968	36 26
.07	3.696	2 24	.57	30.096	19 36	1.07	56.496	36 47
.08	4.224	2 45	.58	30.624	19 56	1.08	57.024	37 08
.09	4.752	3 06	.59	31.152	20 17	1.09	57.552	37 28
.10	5.280	3 26	.60	31.680	20 38	1.10	58.080	37 49
.11	5.808	3 47	.61	32.208	20 58	1.11	58.608	38 09
.12	6.336	4 08	.62	32.736	21 19	1.12	59.136	38 30
.13	6.864	4 28	.63	33.264	21 39	1.13	59.664	38 51
.14	7.392	4 49	.64	33.792	22 00	1.14	60.192	39 11
.15	7.920	5 09	.65	34.320	22 21	1.15	60.720	39 32
.16	8.448	5 30	.66	34.848	22 41	1.16	61.248	39 53
.17	8.976	5 51	.67	35.376	23 02	1.17	61.776	40 13
.18	9.504	6 11	.68	35.904	23 23	1.18	62.304	40 34
.19	10.032	6 32	.69	36.432	23 43	1.19	62.832	40 54
.20	10.560	6 53	.70	36.960	24 04	1.20	63.360	41 15
.21	11.088	7 13	.71	37.488	24 24	1.21	63.888	41 35
.22	11.616	7 34	.72	38.016	24 45	1.22	64.416	41 56
.23	12.144	7 54	.73	38.544	25 06	1.23	64.944	42 17
.24	12.672	8 15	.74	39.072	25 26	1.24	65.472	42 38
.25	13.200	8 36	.75	39.600	25 47	1.25	66.000	42 58
.26	13.728	8 56	.76	40.128	26 08	1.26	66.528	43 19
.27	14.256	9 17	.77	40.656	26 28	1.27	67.056	43 39
.28	14.784	9 38	.78	41.184	26 49	1.28	67.584	44 00
.29	15.312	9 58	.79	41.712	27 09	1.29	68.112	44 21
.30	15.840	10 19	.80	42.240	27 30	1.30	68.640	44 41
.31	16.368	10 39	.81	42.768	27 51	1.31	69.168	45 02
.32	16.896	11 00	.82	43.296	28 11	1.32	69.696	45 23
.33	17.424	11 21	.83	43.824	28 32	1.33	70.224	45 43
.34	17.952	11 41	.84	44.352	28 53	1.34	70.752	46 04
.35	18.480	12 02	.85	44.880	29 13	1.35	71.280	46 24
.36	19.008	12 23	.86	45.408	29 34	1.36	71.808	46 45
.37	19.536	12 43	.87	45.936	29 54	1.37	72.336	47 06
.38	20.064	13 04	.88	46.464	30 15	1.38	72.864	47 26
.39	20.592	13 24	.89	46.992	30 36	1.39	73.392	47 47
.40	21.120	13 45	.90	47.520	30 57	1.40	73.920	48 08
.41	21.648	14 06	.91	48.048	31 17	1.41	74.448	48 28
.42	22.176	14 26	.92	48.576	31 38	1.42	74.976	48 49
.43	22.704	14 47	.93	49.104	31 58	1.43	75.504	49 09
.44	23.232	15 08	.94	49.632	32 19	1.44	76.032	49 30
.45	23.760	15 28	.95	50.160	32 39	1.45	76.560	49 51
.46	24.288	15 49	.96	50.688	33 00	1.46	77.088	50 11
.47	24.816	16 09	.97	51.216	33 21	1.47	77.616	50 32
.48	25.344	16 30	.98	51.744	33 41	1.48	78.144	50 52
.49	25.872	16 51	.99	52.272	34 02	1.49	78.672	51 13
.50	26.400	17 11	1.00	52.800	34 23	1.50	79.200	51 3

TABLE XIV.—GRADES AND GRADE ANGLES.

Feet per Mile.	Inclination.	Feet per Station.	Feet per Mile.	Inclination.	Feet per Station.	Feet per Mile.	Inclination.
	° ′ ″			° ′ ″			° ′ ″
79.728	51 54	2.05	108.240	1 10 28	5.10	269.280	2 55 10
80.256	52 15	2.10	110.880	1 12 11	5.20	274.560	2 58 36
80.784	52 36	2.15	113.520	1 13 54	5.30	279.840	3 02 09
81.312	52 56	2.20	116.160	1 15 37	5.40	285.120	3 05 27
81.840	53 17	2.25	118.800	1 17 20	5.50	290.400	3 08 53
82.368	53 37	2.30	121.440	1 19 03	5.60	295.680	3 12 19
82.896	53 58	2.35	124.080	1 20 46	5.70	300.960	3 15 44
83.424	54 19	2.40	126.720	1 22 29	5.80	306.240	3 19 10
83.952	54.39	2.45	129.360	1 24 12	5.90	311.520	3 22 36
84.480	55 00	2.50	132.000	1 25 56	6.00	316.800	3 26 01
85.008	55 21	2.55	134.640	1 27 39	6.10	322.080	3 29 27
85.536	55 41	2.60	137.280	1 29 22	6.20	327.360	3 32 52
86.064	56 02	2.65	139.920	1 31 05	6.30	332.640	3 36 18
86.592	56 22	2.70	142.560	1 32 48	6.40	337.920	3 39 43
87.120	56 43	2.75	145.200	1 34 31	6.50	343.200	3 43 08
87.648	57 04	2.80	147.840	1 36 14	6.60	348.480	3 46 34
88.176	57 24	2.85	150.480	1 37 57	6.70	353.760	3 49 59
88.704	57 45	2.90	153.120	1 39 40	6.80	359.040	3 53 24
89.232	58 06	2.95	155.760	1 41 23	6.90	364.320	3 56 50
89.760	58 26	3.00	158.400	1 43 06	7.00	369.600	4 00 15
90.288	58 47	3.05	161.040	1 44 49	7.10	374.880	4 03 40
90.816	59 07	3.10	163.680	1 46 32	7.20	380.160	4 07 06
91.344	59 28	3.15	166.320	1 48 15	7.30	385.440	4 10 31
91.872	59 49	3.20	168.960	1 49 58	7.40	390.720	4 13 56
92.400	1 00 09	3.25	171.600	1 51 41	7.50	396.000	4 17 21
92.928	1 00 30	3.30	174.240	1 53 24	7.60	401.280	4 20 46
93.456	1 00 51	3.35	176.880	1 55 07	7.70	406.560	4 24 11
93.984	1 01 11	3.40	179.520	1 56 50	7.80	411.840	4 27 36
94.512	1 01 32	3.45	182.160	1 58 33	7.90	417.120	4 31 01
95.040	1 01 52	3.50	184.800	2 00 16	8.00	422.400	4 34 26
95.568	1 02 13	3.55	187.440	2 01 59	8.10	427.680	4 37 51
96.096	1 02 34	3.60	190.080	2 03 42	8.20	432.960	4 41 16
96.624	1 02 54	3.65	192.720	2 05 25	8.30	428.240	4 44 41
97.152	1 03 15	3.70	195.360	2 07 08	8.40	443.520	4 48 06
97.680	1 03 35	3.75	198.000	2 08 51	8.50	448.800	4 51 30
98.208	1 03 56	3.80	200.640	2 10 34	8.60	454.080	4 54 55
98.736	1 04 17	3.85	203.280	2 12 17	8.70	459.360	4 58 20
99.264	1 04 37	3.90	205.920	2 14 00	8.80	464.640	5 01 44
99.792	1 04 58	3.95	208.560	2 15 43	8.90	469.920	5 05 10
100.320	1 05 19	4.00	211.200	2 17 26	9.00	475.200	5 08 34
100.848	1 05 39	4.10	216.480	2 20 52	9.10	480.480	5 11 59
101.376	1 06 00	4.20	221.760	2 24 18	9.20	485.760	5 15 23
101.904	1 06 20	4.30	227.040	2 27 44	9 30	491.040	5 18 48
102.432	1 06 41	4.40	232.320	2 31 10	9.40	496.320	5 22 12
102.960	1 07 02	4.50	237.600	2 34 36	9 50	501.600	5 25 37
103.488	1 07 22	4.60	242.880	2 38 01	9.60	506.880	5 29 01
104.016	1 07 43	4.70	248.160	2 41 27	9.70	512.160	5 32 25
104.544	*1 08 04*	4.80	253.440	2 44 53	9.80	517.440	5 35 50
105.072	*1 08 24*	4.90	258.720	2 48 19	9.90	522.720	5 39 14
105.600	*1 08 45*	5.00	264.000	2 51 45	10.00	528.000	5 42 38

TABLE XV.—FOR OBTAINING BAROMETRIC HEIGHTS IN FEET.

Barom-eter. Inches = *h*.	0.00	0.02	0.04	0.06	0.08	Diff. per .002 in.
19°.0	16832	16860	16888	16915	16943	2.8
.1	16970	16997	17025	17052	17080	2.8
.2	17107	17134	17162	17189	17216	2.7
.3	17243	17270	17298	17325	17352	2.7
.4	17379	17406	17433	17460	17487	2.7
.5	17514	17540	17567	17594	17621	2.7
.6	17648	17674	17701	17728	17755	2.7
.7	17781	17808	17834	17861	17887	2.7
.8	17914	17940	17967	17993	18020	2.7
.9	18046	18072	18099	18125	18151	2.6
20°.0	18178	18204	18230	18256	18282	2.6
.1	18308	18334	18360	18386	18413	2.6
.2	18438	18464	18490	18516	18542	2.6
.3	18568	18594	18620	18645	18671	2.6
.4	18697	18723	18748	18774	18799	2.6
.5	18825	18851	18876	18902	18927	2.6
.6	18953	18978	19004	19029	19054	2.5
.7	19080	19105	19130	19156	19181	2.5
.8	19206	19231	19256	19282	19307	2.5
.9	19332	19357	19382	19407	19432	2.5
21°.0	19457	19482	19507	19532	19557	2.5
.1	19582	19606	19631	19656	19681	2.5
.2	19706	19730	19755	19780	19804	2.5
.3	19829	19854	19878	19903	19927	2.5
.4	19952	19976	20001	20025	20050	2.5
.5	20074	20098	20123	20147	20172	2.5
.6	20196	20220	20244	20269	20293	2.4
.7	20317	20341	20365	20389	20413	2.4
.8	20438	20462	20186	20510	20534	2.4
.9	20558	20581	20605	20629	20653	2.4
22°.0	20677	20701	20725	20748	20772	2.4
.1	20796	20820	20843	20867	20891	2.4
.2	20914	20938	20962	20985	21009	2.4
.3	21032	21056	21079	21103	21126	2.4
.4	21150	21173	21196	21220	21243	2.3
.5	21266	21290	21313	21336	21359	2.3
.6	21383	21406	21429	21452	21475	2.3
.7	21498	21522	21545	21568	21591	2.3
.8	21614	21637	21660	21683	21706	2.3
.9	21728	21751	21774	21797	21820	2.3
23°.0	21843	21866	21888	21911	21934	2.3
.1	21957	21979	22002	22025	22047	2.3
.2	22070	22092	22115	22138	22160	2.3
.3	22183	22205	22228	22250	22272	2.2
.4	22295	22317	22340	22362	22384	2.2
.5	22407	22429	22451	22474	22496	2.2
.6	22518	22540	22562	22585	22607	2.2
.7	22629	22651	22673	22695	22717	2.2
.8	22739	22761	22783	22805	22827	2.2
.9	22849	22871	22893	22915	22937	2.2
24°.0	22959	22981	23003	23024	23046	2.2
.1	23068	23090	23111	23133	23155	2.2
.2	23176	23198	23220	23241	23263	2.2
.3	23285	23306	23328	23349	23371	2.2
.4	23392	23414	23435	23457	23178	2.2
.5	23500	23521	23542	23564	23585	2.1
.6	23606	23628	23649	23670	23692	2.1
.7	23713	23734	23755	23776	23798	2.1
.8	23819	23840	23861	23882	23903	2.1
.9	23924	23945	23966	23987	24008	2.1

TABLE XV.—FOR OBTAINING BAROMETRIC HEIGHTS IN FEET.

Barometer. Inches $= h$.	0.00	0.02	0.04	0.06	0.08	Diff. per .002 in.
25°.0	24029	24050	24071	24092	24113	2.1
.1	24134	24155	24176	24197	24217	2.1
.2	24238	24259	24280	24301	24321	2.1
.3	24342	24363	24384	24404	24425	2.1
.4	24446	24466	24487	24508	24528	2.1
.5	24549	24569	24590	24610	24631	2.1
.6	24651	24672	24692	24713	24733	2.0
.7	24754	24774	24794	24815	24835	2.0
.8	24855	24876	24896	24916	24937	2.0
.9	24957	24977	24997	25018	25038	2.0
26°.0	25058	25078	25098	25118	25138	2.0
.1	25159	25179	25199	25219	25239	2.0
.2	25259	25279	25299	25319	25339	2.0
.3	25359	25379	25399	25419	25438	2.0
.4	25458	25478	25498	25518	25538	2.0
.5	25557	25577	25597	25617	25637	2.0
.6	25656	25676	25696	25715	25735	2.0
.7	25755	25774	25794	25813	25833	2.0
.8	25853	25872	25892	25911	25931	2.0
.9	25950	25970	25989	26009	26028	2.0
27°.0	26048	26067	26086	26106	26125	1.9
.1	26145	26164	26183	26203	26222	1.9
.2	26241	26260	26280	26299	26318	1.9
.3	26337	26357	26376	26395	26414	1.9
.4	26433	26452	26472	26491	26510	1.9
.5	26529	26548	26567	26586	26605	1.9
.6	26624	26643	26662	26681	26700	1.9
.7	26719	26738	26757	26776	26795	1.9
.8	26813	26832	26851	26870	26889	1.9
.9	26908	26926	26945	26964	26983	1.9
28°.0	27001	27020	27039	27058	27076	1.9
.1	27095	27114	27132	27151	27169	1.9
.2	27188	27207	27225	27244	27262	1.9
.3	27281	27299	27318	27336	27355	1.8
.4	27373	27392	27410	27429	27447	1.8
.5	27466	27484	27502	27521	27539	1.8
.6	27557	27576	27594	27612	27631	1.8
.7	27649	27667	27685	27704	27722	1.8
.8	27740	27758	27777	27795	27813	1.8
.9	27831	27849	27867	27885	27904	1.8
29°.0	27922	27940	27958	27976	27994	1.8
.1	28012	28030	28048	28066	28084	1.8
.2	28102	28120	28138	28156	28174	1.8
.3	28192	28209	28227	28245	28263	1.8
.4	28281	28299	28317	28334	28352	1.8
.5	28370	28388	28405	28423	28441	1.8
.6	28459	28476	28494	28512	28529	1.8
.7	28547	28565	28582	28600	28618	1.8
.8	28635	28653	28670	28688	28706	1.8
.9	28723	28741	28758	28776	28793	1.8
30°.0	28811	28828	28846	28863	28881	1.8
.1	28898	28915	28933	28950	28968	1.8
.2	28985	29002	29020	29037	29054	1.7
.3	29072	29089	29106	29124	29141	1.7
.4	29158	29175	29192	29210	29227	1.7
.5	29244	29261	29278	29296	29313	1.7
.6	29330	29347	29364	29381	29398	1.7
.7	29416	29433	29450	29467	29484	1.
.8	29501	29518	29535	29552	29569	1
.9	29586	29603	29620	29637	29654	

TABLE XV.—FOR OBTAINING BAROMETRIC HEIGHTS IN FEET.

Barometer. Inches = *h*.	0.00	0.02	0.04	0.06	0.08	Diff. per .002 in.
19°.0	16832	16860	16888	16915	16943	2.8
.1	16970	16997	17025	17052	17080	2.8
.2	17107	17134	17162	17189	17216	2.7
.3	17243	17270	17298	17325	17352	2.7
.4	17379	17406	17433	17460	17487	2.7
.5	17514	17540	17567	17594	17621	2.7
.6	17648	17674	17701	17728	17755	2.7
.7	17781	17808	17834	17861	17887	2.7
.8	17914	17940	17967	17993	18020	2.7
.9	18046	18072	18099	18125	18151	2.6
20°.0	18178	18204	18230	18256	18282	2.6
.1	18308	18334	18360	18386	18413	2.6
.2	18438	18464	18490	18516	18542	2.6
.3	18568	18594	18620	18645	18671	2.6
.4	18697	18723	18748	18774	18799	2.6
.5	18825	18851	18876	18902	18927	2.6
.6	18953	18978	19004	19029	19054	2.5
.7	19080	19105	19130	19156	19181	2.5
.8	19206	19231	19256	19282	19307	2.5
.9	19332	19357	19382	19407	19432	2.5
21°.0	19457	19482	19507	19532	19557	2.5
.1	19582	19606	19631	19656	19681	2.5
.2	19706	19730	19755	19780	19804	2.5
.3	19829	19854	19878	19903	19927	2.5
.4	19952	19976	20001	20025	20050	2.5
.5	20074	20098	20123	20147	20172	2.5
.6	20196	20220	20244	20269	20293	2.4
.7	20317	20341	20365	20389	20413	2.4
.8	20438	20462	20486	20510	20534	2.4
.9	20558	20581	20605	20629	20653	2.4
22°.0	20677	20701	20725	20748	20772	2.4
.1	20796	20820	20843	20867	20891	2.4
.2	20914	20938	20962	20985	21009	2.4
.3	21032	21056	21079	21103	21126	2.4
.4	21150	21173	21196	21220	21243	2.3
.5	21266	21290	21313	21336	21359	2.3
.6	21383	21406	21429	21452	21475	2.3
.7	21498	21522	21545	21568	21591	2.3
.8	21614	21637	21660	21683	21706	2.3
.9	21728	21751	21774	21797	21820	2.3
23°.0	21843	21866	21888	21911	21934	2.3
.1	21957	21979	22002	22025	22047	2.3
.2	22070	22092	22115	22138	22160	2.3
.3	22183	22205	22228	22250	22272	2.2
.4	22295	22317	22340	22362	22384	2.2
.5	22407	22429	22451	22474	22496	2.2
.6	22518	22540	22562	22585	22607	2.2
.7	22629	22651	22673	22695	22717	2.2
.8	22739	22761	22783	22805	22827	2.2
.9	22849	22871	22893	22915	22937	2.2
24°.0	22959	22981	23003	23024	23046	2.2
.1	23068	23090	23111	23133	23155	2.2
.2	23176	23198	23220	23241	23263	2.2
.3	23285	23306	23328	23349	23371	2.2
.4	23392	23414	23435	23457	23478	2.2
.5	23500	23521	23542	23564	23585	2.1
.6	23606	23628	23649	23670	23692	2.1
.7	23713	23734	23755	23776	23798	2.1
.8	23819	23840	23861	23882	23903	2.1
.9	23924	23945	23966	23987	24008	2.1

TABLE XVIII.—COEFFICIENT FOR REDUCING INCLINED STADIA MEASUREMENTS TO THE HORIZONTAL. § 224.

α	0′	10′	20′	30′	40′	50′
0°	1.000000	.999992	.999967	.999924	.999865	.999789
1	.999696	.999586	.999459	.999315	.999154	.998977
2	.998782	.998571	.998343	.998098	.997836	.997557
3	.997261	.996949	.996619	.996273	.995910	.995531
4	.995134	.994721	.994291	.993844	.993381	.992901
5	.992404	.991891	.991360	.990814	.990250	.989670
6	.989074	.988461	.987831	.987185	.986522	.985843
7	.985148	.984436	.983708	.982963	.982202	.981424
8	.980631	.979821	.978995	.978152	.977294	.976419
9	.975528	.974621	.973698	.972759	.971804	.970833
10	.969846	.968843	.967824	.966790	.965739	.964673
11°	.963591	.962494	.961380	.960252	.959107	.957948
12	.956772	.955581	.954375	.953153	.951916	.950664
13	.949396	.948113	.946815	.945502	.944174	.942831
14	.941473	.940100	.938711	.937309	.935891	.934459
15	.933011	.931550	.930073	.928582	.927077	.925557
16	.924022	.922474	.920911	.919334	.917742	.916137
17	.914517	.912883	.911236	.909574	.907899	.906209
18	.904507	.902790	.901060	.899316	.897558	.895787
19	.894003	.892206	.890395	.888571	.886733	.884883
20	.883020	.881143	.879254	.877352	.875437	.873510
21°	.871569	.869617	.867652	.865674	.863684	.861681
22	.859667	.857640	.855601	.853550	.851487	.849412
23	.847326	.845227	.843117	.840996	.838862	.836718
24	.834561	.832394	.830215	.828025	.825825	.823613
25	.821390	.819156	.816911	.814656	.812390	.810113
26	.807826	.805529	.803221	.800903	.798575	.796236
27	.793888	.791529	.789161	.786783	.784396	.781998
28	.779591	.777175	.774749	.772314	.769870	.767416
29	.764954	.762483	.760002	.757513	.755015	.752509
30	.749994	.747471	.744939	.742399	.739850	.737294
31°	.734729	.732157	.729577	.726989	.724393	.721790
32	.719179	.716561	.713935	.711302	.708662	.706015
33	.703361	.700700	.698033	.695358	.692677	.689990
34	.687296	.684595	.681889	.679176	.676457	.673733
35	.671002	.668266	.665524	.662776	.660023	.657264
36	.654500	.651731	.648957	.646177	.643393	.640604
37	.637810	.635011	.632208	.629401	.626588	.623772
38	.620952	.618127	.615299	.612466	.609630	.606790
39	.603946	.601099	.598248	.595395	.592537	.589677
40	.586814	.583948	.581079	.578207	.575332	.572455
41°	.569576	.566694	.563810	.560924	.558036	.555145
42	.552253	.549359	.546464	.543567	.540668	.537768
43	.534867	.531964	.529061	.526156	.523251	.520345
44	.517438	.514530	.511622	.508714	.505805	.502897
45	.499988	.497079	.494170	.491261	.488353	.485445

TABLE XVI.—COEFFICIENT OF CORRECTION FOR TEMPERATURE.

$t+t'$	$\frac{t+t'-64^\circ}{900}$	$t+t'$	$\frac{t+t'-64^\circ}{900}$	$t+t'$	$\frac{t+t'-64^\circ}{900}$	$t+t'$	$\frac{t+t'-64^\circ}{900}$
20°	.0489	65°	+ .0011	110°	+ .0511	155°	.1011
21	− .0478	66	.0022	111	.0522	156	.1022
22	.0467	67	.0033	112	.0533	157	.1033
23	.0456	68	.0044	113	.0544	158	.1044
24	.0444	69	.0056	114	.0556	159	.1056
25	.0433	70	.0067	115	.0567	160	.1067
26	.0422	71	.0078	116	.0578	161	.1078
27	.0411	72	.0089	117	.0589	162	.1089
28	.0400	73	.0100	118	.0600	163	.1100
29	.0389	74	.0111	119	.0611	164	.1111
30	.0378	75	.0122	120	+ .0622	165	.1122
31	− .0367	76	+ .0133	121	.0633	166	+ .1133
32	.0356	77	.0144	122	.0644	167	.1144
33	.0344	78	.0156	123	.0656	168	.1156
34	.0333	79	.0167	124	.0667	169	.1167
35	.0322	80	.0178	125	.0678	170	.1178
36	.0311	81	.0189	126	.0689	171	.1189
37	.0300	82	.0200	127	.0700	172	.1200
38	.0289	83	.0211	128	.0711	173	.1211
39	.0278	84	.0222	129	.0722	174	.1222
40	.0267	85	.0233	130	+ .0733	175	.1233
41	− .0256	86	+ .0244	131	.0744	176	+ .1244
42	.0244	87	.0256	132	.0756	177	.1256
43	.0233	88	.0267	133	.0767	178	.1267
44	.0222	89	.0278	134	.0778	179	.1278
45	.0211	90	.0289	135	.0789	180	.1289
46	.0200	91	.0300	136	.0800	181	.1300
47	.0189	92	.0311	137	.0811	182	.1311
48	.0178	93	.0322	138	.0822	183	.1322
49	.0167	94	.0333	139	.0833	184	.1333
50	− .0156	95	.0344	140	+ .0844	185	.1344
51	.0144	96	+ .0356	141	.0856	186	+ .1356
52	.0133	97	.0367	142	.0867	187	.1367
53	.0122	98	.0378	143	.0878	188	.1378
54	.0111	99	.0389	144	.0889	189	.1389
55	.0100	100	.0400	145	.0900	190	.1400
56	.0089	101	.0411	146	.0911	191	.1411
57	.0078	102	.0422	147	.0922	192	.1422
58	.0067	103	.0433	148	.0933	193	.1433
59	.0056	104	.0444	149	.0944	194	.1444
60	.0044	105	.0456	150	+ .0956	195	.1456
61	− .0033	106	+ .0467	151	.0967	196	+ .1467
62	.0022	107	.0478	152	.0978	197	.1478
63	.0011	108	.0489	153	.0989	198	.1489
64	.0000	109	.0500	154	.1000	199	.1500

TABLE XVII.—CORRECTION FOR EARTH'S CURVATURE AND REFRACTION. § 119.

L°	H°	L°	H°	L°	H°	L°	H°	L°	H°	Miles	H°
300	.002	1300	.035	2300	.108	3300	.223	4300	.379	1	.571
400	.003	1400	.040	2400	.118	3400	.237	4400	.397	2	2.285
500	.005	1500	.046	2500	.128	3500	.251	4500	.415	3	5.142
600	007	1600	.052	2600	.139	3600	.266	4600	.434	4	9.141
700	.010	1700	.059	2700	.149	3700	.281	4700	.453	5	14.282
800	.013	1800	.066	2800	.161	3800	.296	4800	.472	6	20.567
900	.017	1900	.074	2900	.172	3900	.312	4900	.492	7	27.994
1000	020	2000	082	3000	.184	4000	.328	5000	.512	8	36.563
1100	.025	2100	.090	3100	.197	4100	.345	5100	.533	9	46.275
1200	.030	2200	.099	3200	.210	4200	.362	5200	.554	10	57.130

TABLE XX.—LENGTHS OF CIRCULAR ARCS; RADIUS = 1.

Sec.	Length.	Min.	Length.	Deg.	Length.	Deg.	Length.
1	.0000048	1	.0002909	1	.0174533	61	1.0646508
2	.0000097	2	.0005818	2	.0349066	62	1.0821041
3	.0000145	3	.0008727	3	.0523599	63	1.0995574
4	.0000194	4	.0011636	4	.0698132	64	1.1170107
5	.0000242	5	.0014544	5	.0872665	65	1.1344640
6	.0000291	6	.0017453	6	.1047198	66	1.1519173
7	.0000339	7	.0020362	7	.1221730	67	1.1693706
8	.0000388	8	.0023271	8	.1396263	68	1.1868239
9	.0000436	9	.0026180	9	.1570796	69	1.2042772
10	.0000485	10	.0029089	10	.1745329	70	1.2217305
11	.0000533	11	.0031998	11	.1919862	71	1.2391838
12	.0000582	12	.0034907	12	.2094395	72	1.2566371
13	.0000630	13	.0037815	13	.2268928	73	1.2740904
14	.0000679	14	.0040724	14	.2443461	74	1.2915436
15	.0000727	15	.0043633	15	.2617994	75	1.3089969
16	.0000776	16	.0046542	16	.2792527	76	1.3264502
17	.0000824	17	.0049451	17	.2967060	77	1.3439035
18	.0000873	18	.0052360	18	.3141593	78	1.3613568
19	.0000921	19	.0055269	19	.3316126	79	1.3788101
20	.0000970	20	.0058178	20	.3490659	80	1.3962634
21	.0001018	21	.0061087	21	.3665191	81	1.4137167
22	.0001067	22	.0063995	22	.3839724	82	1.4311700
23	.0001115	23	.0066904	23	.4014257	83	1.4486233
24	.0001164	24	.0069813	24	.4188790	84	1.4660766
25	.0001212	25	.0072722	25	.4363323	85	1.4835299
26	.0001261	26	.0075631	26	.4537856	86	1.5009832
27	.0001309	27	.0078540	27	.4712389	87	1.5184364
28	.0001357	28	.0081449	28	.4886922	88	1.5358897
29	.0001406	29	.0084358	29	.5061455	89	1.5533430
30	.0001454	30	.0087266	30	.5235988	90	1.5707963
31	.0001503	31	.0090175	31	.5410521	91	1.5882496
32	.0001551	32	.0093084	32	.5585054	92	1.6057029
33	.0001600	33	.0095993	33	.5759587	93	1.6231562
34	.0001648	34	.0098902	34	.5934119	94	1.6406095
35	.0001697	35	.0101811	35	.6108652	95	1.6580628
36	.0001745	36	.0104720	36	.6283185	96	1.6755161
37	.0001794	37	.0107629	37	.6457718	97	1.6929694
38	.0001842	38	.0110538	38	.6632251	98	1.7104227
39	.0001891	39	.0113446	39	.6806784	99	1.7278760
40	.0001939	40	.0116355	40	.6981317	100	1.7453293
41	.0001988	41	.0119264	41	.7155850	101	1.7627825
42	.0002036	42	.0122173	42	.7330383	102	1.7802358
43	.0002085	43	.0125082	43	.7504916	103	1.7976891
44	.0002133	44	.0127991	44	.7679449	104	1.8151424
45	.0002182	45	.0130900	45	.7853982	105	1.8325957
46	.0002230	46	.0133809	46	.8028515	106	1.8500490
47	.0002279	47	.0136717	47	.8203047	107	1.8675023
48	.0002327	48	.0139626	48	.8377580	108	1.8849556
49	.0002376	49	.0142535	49	.8552113	109	1.9024089
50	.0002424	50	.0145444	50	.8726646	110	1.9198622
51	.0002473	51	.0148353	51	.8901179	111	1.9373155
52	.0002521	52	.0151262	52	.9075712	112	1.9547688
53	.0002570	53	.0154171	53	.9250245	113	1.9722221
54	.0002618	54	.0157080	54	.9424778	114	1.9896753
55	.0002666	55	.0159989	55	.9599311	115	2.0071286
56	.0002715	56	.0162897	56	.9773844	116	2.0245819
57	*.0002763*	*57*	.0165806	57	.9948377	117	2.0420352
58	*.0002812*	*58*	.0168715	58	1.0122910	118	2.059488
59	*.0002860*	*59*	.0171624	59	1.0297443	119	2.07694
60	*.0002909*	*60*	.0174533	60	1.0471976	120	2.0943

TABLE XIX.—LOGARITHM OF COEFFICIENT FOR REDUCING INCLINED STADIA MEASUREMENTS TO THE HORIZONTAL. § 224.

α	0′	10′	20′	30′	40′	50′
0°	0.000000	9.999996	9.999985	9.999967	9.999941	9.999908
1	9.999868	.999820	.999765	.999702	.999633	.999555
2	.999471	.999379	.999280	.999173	.999059	.998938
3	.998809	.998673	.998529	.998379	.998220	.998055
4	.997882	.997701	.997514	.997318	.997116	.996906
5	.996689	.996464	.996232	.995992	.995745	.995491
6°	9.995229	9.994959	9.994683	9.994399	9.994107	9.993808
7	.993501	.993187	.992866	.992537	.992201	.991857
8	.991506	.991147	.990780	.990406	.990025	.989636
9	.989240	.988836	.988424	.988005	.987579	.987144
10	.986703	.986253	.985797	.985332	.984860	.984380
11°	9.983893	9.983398	9.982895	9.982385	9.981867	9.981342
12	.980808	.980268	.979719	.979163	.978599	.978027
13	.977447	.976860	.976265	.975663	.975052	.974434
14	.973808	.973174	.972532	.971883	.971225	.970560
15	.969887	.969206	.968517	.967820	.967116	.966403
16°	9.965683	9.964954	9.964218	9.963473	9.962721	9.961960
17	.961192	.960415	.959631	.958838	.958037	.957229
18	.956412	.955587	.954753	.953912	.953063	.952205
19	.951339	.950465	.949583	.948692	.947793	.946886
20	.945970	.945047	.944114	.943174	.942225	.941268
21°	9.940302	9.939328	9.938345	9.937354	9.936355	9.935347
22	.934330	.933305	.932271	.931229	.930178	.929119
23	.928050	.926974	.925888	.924794	.923691	.922579
24	.921458	.920329	.919191	.918044	.916888	.915723
25	.914549	.913366	.912175	.910974	.909764	.908546
26°	9.907318	9.906081	9.904835	9.903580	9.902316	9.901042
27	.899759	.898467	.897166	.895855	.894535	.893206
28	.891867	.890519	.889161	.887794	.886417	.885031
29	.883635	.882230	.880815	.879390	.877956	.876512
30	.875058	.873594	.872121	.870637	.869144	.867641
31°	9.866127	9.864604	9.863071	9.861528	9.859974	9.858411
32	.856837	.855253	.853659	.852054	.850439	.848814
33	.847178	.845532	.843876	.842209	.840531	.838843
34	.837144	.835434	.833714	.831982	.830240	.828488
35	.826724	.824949	.823163	.821367	.819559	.817740
36°	9.815910	9.814068	9.812216	9.810352	9.808476	9.806589
37	.804691	.802781	.800860	.798927	.796982	.795026
38	.793058	.791078	.789086	.787082	.785066	.783038
39	.780998	.778946	.776882	.774805	.772716	.770614
40	.768500	.766374	.764235	.762083	.759919	.757742
41°	9.755552	9.753349	9.751133	9.748904	9.746662	9.744407
42	.742138	.739857	.737561	.735253	.732931	.730595
43	.728246	.725883	.723506	.721115	.718710	.716291
44	.713858	.711411	.708950	.706474	.703983	.701479
45	9.698959	9.696425	9.693876	9.691313	9.688734	9.686140

TABLE XX.—LENGTHS OF CIRCULAR ARCS; RADIUS = 1.

Sec.	Length.	Min.	Length.	Deg.	Length.	Deg.	Length.
1	.0000048	1	.0002909	1	.0174533	61	1.0646508
2	.0000097	2	.0005818	2	.0349066	62	1.0821041
3	.0000145	3	.0008727	3	.0523599	63	1.0995574
4	.0000194	4	.0011636	4	.0698132	64	1.1170107
5	.0000242	5	.0014544	5	.0872665	65	1.1344640
6	.0000291	6	.0017453	6	.1047198	66	1.1519173
7	.0000339	7	.0020362	7	.1221730	67	1.1693706
8	.0000388	8	.0023271	8	.1396263	68	1.1868239
9	.0000436	9	.0026180	9	.1570796	69	1.2042772
10	.0000485	10	.0029089	10	.1745329	70	1.2217305
11	.0000533	11	.0031998	11	.1919862	71	1.2391838
12	.0000582	12	.0034907	12	.2094395	72	1.2566371
13	.0000630	13	.0037815	13	.2268928	73	1.2740904
14	.0000679	14	.0040724	14	.2443461	74	1.2915436
15	.0000727	15	.0043633	15	.2617994	75	1.3089969
16	.0000776	16	.0046542	16	.2792527	76	1.3264502
17	.0000824	17	.0049451	17	.2967060	77	1.3439035
18	.0000873	18	.0052360	18	.3141593	78	1.3613568
19	.0000921	19	.0055269	19	.3316126	79	1.3788101
20	.0000970	20	.0058178	20	.3490659	80	1.3962634
21	.0001018	21	.0061087	21	.3665191	81	1.4137167
22	.0001067	22	.0063995	22	.3839724	82	1.4311700
23	.0001115	23	.0066904	23	.4014257	83	1.4486233
24	.0001164	24	.0069813	24	.4188790	84	1.4660766
25	.0001212	25	.0072722	25	.4363323	85	1.4835299
26	.0001261	26	.0075631	26	.4537856	86	1.5009832
27	.0001309	27	.0078540	27	.4712389	87	1.5184364
28	.0001357	28	.0081449	28	.4886922	88	1.5358897
29	.0001406	29	.0084358	29	.5061455	89	1.5533430
30	.0001454	30	.0087266	30	.5235988	90	1.5707963
31	.0001503	31	.0090175	31	.5410521	91	1.5882496
32	.0001551	32	.0093084	32	.5585054	92	1.6057029
33	.0001600	33	.0095993	33	.5759587	93	1.6231562
34	.0001648	34	.0098902	34	.5934119	94	1.6406095
35	.0001697	35	.0101811	35	.6108652	95	1.6580628
36	.0001745	36	.0104720	36	.6283185	96	1.6755161
37	.0001794	37	.0107629	37	.6457718	97	1.6929694
38	.0001842	38	.0110538	38	.6632251	98	1.7104227
39	.0001891	39	.0113446	39	.6806784	99	1.7278760
40	.0001939	40	.0116355	40	.6981317	100	1.7453293
41	.0001988	41	.0119264	41	.7155850	101	1.7627825
42	.0002036	42	.0122173	42	.7330383	102	1.7802358
43	.0002085	43	.0125082	43	.7504916	103	1.7976891
44	.0002133	44	.0127991	44	.7679449	104	1.8151424
45	.0002182	45	.0130900	45	.7853982	105	1.8325957
46	.0002230	46	.0133809	46	.8028515	106	1.8500490
47	.0002279	47	.0136717	47	.8203047	107	1.8675023
48	.0002327	48	.0139626	48	.8377580	108	1.8849556
49	.0002376	49	.0142535	49	.8552113	109	1.9024089
50	.0002424	50	.0145444	50	.8726646	110	1.9198622
51	.0002473	51	.0148353	51	.8901179	111	1.9373155
52	.0002521	52	.0151262	52	.9075712	112	1.9547688
53	.0002570	53	.0154171	53	.9250245	113	1.9722221
54	.0002618	54	.0157080	54	.9424778	114	1.9896753
55	.0002666	55	.0159989	55	.9599311	115	2.0071286
56	.0002715	56	.0162897	56	.9773844	116	2.0245819
57	.0002763	57	.0165806	57	.9948377	117	2.0420352
58	.0002812	58	.0168715	58	1.0122910	118	2.0594885
59	.0002860	59	.0171624	59	1.0297443	119	2.0769418
60	.0002909	60	.0174533	60	1.0471976	120	2.094395

TABLE XXI.—MINUTES IN DECIMALS OF A DEGREE.

′	0″	10″	15″	20″	30″	40″	45″	50″	′
0	.00000	.00278	.00417	.00556	.00833	.01111	.01250	.01389	0
1	.01667	.01944	.02083	.02222	.02500	.02778	.02917	.03055	1
2	.03333	.03611	.03750	.03889	.04167	.04444	.04583	.04722	2
3	.05000	.05278	.05417	.05556	.05833	.06111	.06250	.06389	3
4	.06667	.06944	.07083	.07222	.07500	.07778	.07917	.08056	4
5	.08333	.08611	.08750	.08889	.09167	.09444	.09583	.09722	5
6	.10000	.10278	.10417	.10556	.10833	.11111	.11250	.11389	6
7	.11667	.11944	.12083	.12222	.12500	.12778	.12917	.13056	7
8	.13333	.13611	.13750	.13889	.14167	.14444	.14583	.14722	8
9	.15000	.15278	.15417	.15556	.15833	.16111	.16250	.16389	9
10	.16667	.16944	.17083	.17222	.17500	.17778	.17917	.18056	10
11	.18333	.18611	.18750	.18889	.19167	.19444	.19583	.19722	11
12	.20000	.20278	.20417	.20556	.20833	.21111	.21250	.21389	12
13	.21667	.21944	.22083	.22222	.22500	.22778	.22917	.23056	13
14	.23333	.23611	.23750	.23889	.24167	.24444	.24583	.24722	14
15	.25000	.25278	.25417	.25556	.25833	.26111	.26250	.26389	15
16	.26667	.26944	.27083	.27222	.27500	.27778	.27917	.28056	16
17	.28333	.28611	.28750	.28889	.29167	.29444	.29583	.29722	17
18	.30000	.30278	.30417	.30556	.30833	.31111	.31250	.31389	18
19	.31667	.31944	.32083	.32222	.32500	.32778	.32917	.33056	19
20	.33333	.33611	.33750	.33889	.34167	.34444	.34583	.34722	20
21	.35000	.35278	.35417	.35556	.35833	.36111	.36250	.36389	21
22	.36667	.36944	.37083	.37222	.37500	.37778	.37917	.38056	22
23	.38333	.38611	.38750	.38889	.39167	.39444	.39583	.39722	23
24	.40000	.40278	.40417	.40556	.40833	.41111	.41250	.41389	24
25	.41667	.41944	.42083	.42222	.42500	.42778	.42917	.43056	25
26	.43333	.43611	.43750	.43889	.44167	.44444	.44583	.44722	26
27	.45000	.45278	.45417	.45556	.45833	.46111	.46250	.46389	27
28	.46667	.46944	.47083	.47222	.47500	.47778	.47917	.48056	28
29	.48333	.48611	.48750	.48889	.49167	.49444	.49583	.49722	29
30	.50000	.50278	.50417	.50556	.50833	.51111	.51250	.51389	30
31	.51667	.51944	.52083	.52222	.52500	.52778	.52917	.53056	31
32	.53333	.53611	.53750	.53889	.54167	.54444	.54583	.54722	32
33	.55000	.55278	.55417	.55556	.55833	.56111	.56250	.56389	33
34	.56667	.56944	.57083	.57222	.57500	.57778	.57917	.58056	34
35	.58333	.58611	.58750	.58889	.59167	.59444	.59583	.59722	35
36	.60000	.60278	.60417	.60556	.60833	.61111	.61250	.61389	36
37	.61667	.61944	.62083	.62222	.62500	.62778	.62917	.63056	37
38	.63333	.63611	.63750	.63889	.64167	.64444	.64583	.64722	38
39	.65000	.65278	.65417	.65556	.65833	.66111	.66250	.66389	39
40	.66667	.66944	.67083	.67222	.67500	.67778	.67917	.68056	40
41	.68333	.68611	.68750	.68889	.69167	.69444	.69583	.69722	41
42	.70000	.70278	.70417	.70556	.70833	.71111	.71250	.71389	42
43	.71667	.71944	.72083	.72222	.72500	.72778	.72917	.73056	43
44	.73333	.73611	.73750	.73889	.74167	.74444	.74583	.74722	44
45	.75000	.75278	.75417	.75556	.75833	.76111	.76250	.76389	45
46	.76667	.76944	.77083	.77222	.77500	.77778	.77917	.78056	46
47	.78333	.78611	.78750	.78889	.79167	.79444	.79583	.79722	47
48	.80000	.80278	.80417	.80556	.80833	.81111	.81250	.81389	48
49	.81667	.81944	.82083	.82222	.82500	.82778	.82917	.83056	49
50	.83333	.83611	.83750	.83889	.84167	.84444	.84583	.84722	50
51	.85000	.85278	.85417	.85556	.85833	86111	.86250	.86389	51
52	.86667	.86944	.87083	.87222	.87500	.87778	.87917	.88056	52
53	.88333	.88611	.88750	.88889	.89167	.89444	.89583	.89722	53
54	.90000	.90278	.90417	.90556	.90833	.91111	.91250	.91389	54
55	.91667	.91944	.92083	.92222	.92500	.92778	.92917	.93056	55
56	.93333	.93611	.93750	.93889	.94167	.94444	.94583	.94722	56
57	.95000	.95278	.95417	.95556	.95833	.96111	.96250	.96389	57
58	.96667	.96944	.97085	.97222	.97500	.97778	.97917	.98056	58
59	.98333	.98611	.98750	.98889	.99167	.99444	.99583	.99722	59
′	0″	10″	15″	20″	30″	40″	45″	50″	′

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
63	3969	250047	7.9372539	3.9790571	.015873016
64	4096	262144	8.0000000	4.0000000	.015625000
65	4225	274625	8.0622577	4.0207256	.015384615
66	4356	287496	8.1240384	4.0412401	.015151515
67	4489	300763	8.1853528	4.0615480	.014925373
68	4624	314432	8.2462113	4.0816551	.014705882
69	4761	328509	8.3066239	4.1015661	.014492754
70	4900	343000	8.3666003	4.1212853	.014285714
71	5041	357911	8.4261498	4.1408178	.014084507
72	5184	373248	8.4852814	4.1601676	.013888889
73	5329	389017	8.5440037	4.1793390	.013698630
74	5476	405224	8.6023253	4.1983364	.013513514
75	5625	421875	8.6602540	4.2171633	.013333333
76	5776	438976	8.7177979	4.2358236	.013157895
77	5929	456533	8.7749644	4.2543210	.012987013
78	6084	474552	8.8317609	4.2726586	.012820513
79	6241	493039	8.8881944	4.2908404	.012658228
80	6400	512000	8.9442719	4.3088695	.012500000
81	6561	531441	9.0000000	4.3267487	.012345679
82	6724	551368	9.0553851	4.3444815	.012195122
83	6889	571787	9.1104336	4.3620707	.012048193
84	7056	592704	9.1651514	4.3795191	.011904762
85	7225	614125	9.2195445	4.3968296	.011764706
86	7396	636056	9.2736185	4.4140049	.011627907
87	7569	658503	9.3273791	4.4310476	.011494253
88	7744	681472	9.3808315	4.4479602	.011363636
89	7921	704969	9.4339811	4.4647451	.011235955
90	8100	729000	9.4868330	4.4814047	.011111111
91	8281	753571	9.5393920	4.4979414	.010989011
92	8464	778688	9.5916630	4.5143574	.010869565
93	8649	804357	9.6436508	4.5306549	.010752688
94	8836	830584	9.6953597	4.5468359	.010638298
95	9025	857375	9.7467943	4.5629026	.010526316
96	9216	884736	9.7979590	4.5788570	.010416667
97	9409	912673	9.8488578	4.5947009	.010309278
98	9604	941192	9.8994949	4.6104363	.010204082
99	9801	970299	9.9498744	4.6260650	.010101010
100	10000	1000000	10.0000000	4.6415888	.010000000
101	10201	1030301	10.0498756	4.6570095	.009900990
102	10404	1061208	10.0995049	4.6723287	.009803922
103	10609	1092727	10.1488916	4.6875482	.009708738
104	10816	1124864	10.1980390	4.7026694	.009615385
105	11025	1157625	10.2469508	4.7176940	.009523810
106	11236	1191016	10.2956301	4.7326235	.009433962
107	11449	1225043	10.3440804	4.7474594	.009345794
108	11664	1259712	10.3923048	4.7622032	.009259259
109	11881	1295029	10.4403065	4.7768562	.009174312
110	12100	1331000	10.4880885	4.7914199	.009090909
111	12321	1367631	10.5356538	4.8058955	.009009009
112	12544	1404928	10.5830052	4.8202845	.008928571
113	12769	1442897	10.6301458	4.8345881	.008849558
114	12996	1481544	10.6770783	4.8488076	.008771930
115	13225	1520875	10.7238053	4.8629442	.008695652
116	13456	1560896	10.7703296	4.8769990	.008620690
117	13689	1601613	10.8166538	4.8909732	.008547009
118	13924	1643032	10.8627805	4.9048681	.008474576
119	14161	1685159	10.9087121	4.9186847	.008403361
120	14400	1728000	10.9544512	4.9324242	.00833333
121	*14641*	1771561	11.0000000	4.9460874	.0082644
122	*14884*	1815848	11.0453610	4.9596757	.0081967
123	*15129*	1860867	11.0905365	4.9731898	.00813
124	*15376*	1906624	11.1355287	4.9866310	.0080

TABLE XXIII.—SQUARES, CUBES, SQUARE ROOTS.

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
125	15625	1953125	11.1803399	5.0000000	.008000000
126	15876	2000376	11.2249722	5.0132979	.007936508
127	16129	2048383	11.2694277	5.0265257	.007874016
128	16384	2097152	11.3137085	5.0396842	.007812500
129	16641	2146689	11.3578167	5.0527743	.007751938
130	16900	2197000	11.4017543	5.0657970	.007692308
131	17161	2248091	11.4455231	5.0787531	.007633588
132	17424	2299968	11.4891253	5.0916434	.007575758
133	17689	2352637	11.5325626	5.1044687	.007518797
134	17956	2406104	11.5758369	5.1172299	.007462687
135	18225	2460375	11.6189500	5.1299278	.007407407
136	18496	2515456	11.6619038	5.1425632	.007352941
137	18769	2571353	11.7046999	5.1551367	.007299270
138	19044	2628072	11.7473401	5.1676493	.007246377
139	19321	2685619	11.7898261	5.1801015	.007194245
140	19600	2744000	11.8321596	5.1924941	.007142857
141	19881	2803221	11.8743421	5.2048279	.007092199
142	20164	2863288	11.9163753	5.2171034	.007042254
143	20449	2924207	11.9582607	5.2293215	.006993007
144	20736	2985984	12.0000000	5.2414828	.006944444
145	21025	3048625	12.0415946	5.2535879	.006896552
146	21316	3112136	12.0830460	5.2656374	.006849315
147	21609	3176523	12.1243557	5.2776321	.006802721
148	21904	3241792	12.1655251	5.2895725	.006756757
149	22201	3307949	12.2065556	5.3014592	.006711409
150	22500	3375000	12.2474487	5.3132928	.006666667
151	22801	3442951	12.2882057	5.3250740	.006622517
152	23104	3511808	12.3288280	5.3368033	.006578947
153	23409	3581577	12.3693169	5.3484812	.006535948
154	23716	3652264	12.4096736	5.3601084	.006493506
155	24025	3723875	12.4498996	5.3716854	.006451613
156	24336	3796416	12.4899960	5.3832126	.006410256
157	24649	3869893	12.5299641	5.3946907	.006369427
158	24964	3944312	12.5698051	5.4061202	.006329114
159	25281	4019679	12.6095202	5.4175015	.006289308
160	25600	4096000	12.6491106	5.4288352	.006250000
161	25921	4173281	12.6885775	5.4401218	.006211180
162	26244	4251528	12.7279221	5.4513618	.006172840
163	26569	4330747	12.7671453	5.4625556	.006134969
164	26896	4410944	12.8062485	5.4737037	.006097561
165	27225	4492125	12.8452326	5.4848066	.006060606
166	27556	4574296	12.8840987	5.4958647	.006024096
167	27889	4657463	12.9228480	5.5068784	.005988024
168	28224	4741632	12.9614814	5.5178484	.005952381
169	28561	4826809	13.0000000	5.5287748	.005917160
170	28900	4913000	13.0384048	5.5396583	.005882353
171	29241	5000211	13.0766968	5.5504991	.005847953
172	29584	5088448	13.1148770	5.5612978	.005813953
173	29929	5177717	13.1529464	5.5720546	.005780347
174	30276	5268024	13.1909060	5.5827702	.005747126
175	30625	5359375	13.2287566	5.5934447	.005714286
176	30976	5451776	13.2664992	5.6040787	.005681818
177	31329	5545233	13.3041347	5.6146724	.005649718
178	31684	5639752	13.3416641	5.6252263	.005617978
179	32041	5735339	13.3790882	5.6357408	.005586592
180	32400	5832000	13.4164079	5.6462162	.005555556
181	32761	5929741	13.4536240	5.6566528	.005524862
182	33124	6028568	13.4907376	5.6670511	.005494505
183	33489	6128487	13.5277493	5.6774114	.005464481
184	33856	6229504	13.5646600	5.6877340	.005434783
185	34225	6331625	13.6014705	5.6980192	.005405405
186	34596	6434856	13.6381817	5.7082675	.005376344

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
187	34969	6539203	13.6747943	5.7184791	.005347594
188	35344	6644672	13.7113092	5.7286543	.005319149
189	35721	6751269	13.7477271	5.7387936	.005291005
190	36100	6859000	13.7840488	5.7488971	.005263158
191	36481	6967871	13.8202750	5.7589652	.005235602
192	36864	7077888	13.8564065	5.7689982	.005208333
193	37249	7189057	13.8924440	5.7789966	.005181347
194	37636	7301384	13.9283883	5.7889604	.005154639
195	38025	7414875	13.9642400	5.7988900	.005128205
196	38416	7529536	14.0000000	5.8087857	.005102041
197	38809	7645373	14.0356688	5.8186479	.005076142
198	39204	7762392	14.0712473	5.8284767	.005050505
199	39601	7880599	14.1067360	5.8382725	.005025126
200	40000	8000000	14.1421356	5.8480355	.005000000
201	40401	8120601	14.1774469	5.8577660	.004975124
202	40804	8242408	14.2126704	5.8674643	.004950495
203	41209	8365427	14.2478068	5.8771307	.004926108
204	41616	8489664	14.2828569	5.8867653	.004901961
205	42025	8615125	14.3178211	5.8963685	.004878049
206	42436	8741816	14.3527001	5.9059406	.004854369
207	42849	8869743	14.3874946	5.9154817	.004830918
208	43264	8998912	14.4222051	5.9249921	.004807692
209	43681	9129329	14.4568323	5.9344721	.004784689
210	44100	9261000	14.4913767	5.9439220	.004761905
211	44521	9393931	14.5258390	5.9533418	.004739336
212	44944	9528128	14.5602198	5.9627320	.004716981
213	45369	9663597	14.5945195	5.9720926	.004694836
214	45796	9800344	14.6287388	5.9814240	.004672897
215	46225	9938375	14.6628783	5.9907264	.004651163
216	46656	10077696	14.6969385	6.0000000	.004629630
217	47089	10218313	14.7309199	6.0092450	.004608295
218	47524	10360232	14.7648231	6.0184617	.004587156
219	47961	10503459	14.7986486	6.0276502	.004566210
220	48400	10648000	14.8323970	6.0368107	.004545455
221	48841	10793861	14.8660687	6.0459435	.004524887
222	49284	10941048	14.8996644	6.0550489	.004504505
223	49729	11089567	14.9331845	6.0641270	.004484305
224	50176	11239424	14.9666295	6.0731779	.004464286
225	50625	11390625	15.0000000	6.0822020	.004444444
226	51076	11543176	15.0332964	6.0911994	.004424779
227	51529	11697083	15.0665192	6.1001702	.004405286
228	51984	11852352	15.0996689	6.1091147	.004385965
229	52441	12008989	15.1327460	6.1180332	.004366812
230	52900	12167000	15.1657509	6.1269257	.004347826
231	53361	12326391	15.1986842	6.1357924	.004329004
232	53824	12487168	15.2315462	6.1446337	.004310345
233	54289	12649337	15.2643375	6.1534495	.004291845
234	54756	12812904	15.2970585	6.1622401	.004273504
235	55225	12977875	15.3297097	6.1710058	.004255319
236	55696	13144256	15.3622915	6.1797466	.004237288
237	56169	13312053	15.3948043	6.1884628	.004219409
238	56644	13481272	15.4272486	6.1971544	.004201681
239	57121	13651919	15.4596248	6.2058218	.004184100
240	57600	13824000	15.4919334	6.2144650	.004166667
241	58081	13997521	15.5241747	6.2230843	.004149378
242	58564	14172488	15.5563492	6.2316797	.004132231
243	59049	14348907	15.5884573	6.2402515	.004115226
244	59536	14526784	15.6204994	6.2487998	.004098361
245	60025	14706125	15.6524758	6.2573248	.0040816
246	60516	14886936	15.6843871	6.2658266	.004065
247	61009	15069223	15.7162336	6.2743054	.004048
248	61504	15252992	15.7480157	6.2827613	.00403

TABLE XXIII.—SQUARES, CUBES, SQUARE ROOTS,

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
249	62001	15438249	15.7797338	6.2911946	.004016064
250	62500	15625000	15.8113883	6.2996053	.004000000
251	63001	15813251	15.8429795	6.3079935	.003984064
252	63504	16003008	15.8745079	6.3163596	.003968254
253	64009	16194277	15.9059737	6.3247035	.003952569
254	64516	16387064	15.9373775	6.3330256	.003937008
255	65025	16581375	15.9687194	6.3413257	.003921569
256	65536	16777216	16.0000000	6.3496042	.003906250
257	66049	16974593	16.0312195	6.3578611	.003891051
258	66564	17173512	16.0623784	6.3660968	.003875969
259	67081	17373979	16.0934769	6.3743111	.003861004
260	67600	17576000	16.1245155	6.3825043	.003846154
261	68121	17779581	16.1554944	6.3906765	.003831418
262	68644	17984728	16.1864141	6.3988279	.003816794
263	69169	18191447	16.2172747	6.4069585	.003802281
264	69696	18399744	16.2480768	6.4150687	.003787879
265	70225	18609625	16.2788206	6.4231583	.003773585
266	70756	18821096	16.3095064	6.4312276	.003759398
267	71289	19034163	16.3401346	6.4392767	.003745318
268	71824	19248832	16.3707055	6.4473057	.003731343
269	72361	19465109	16.4012195	6.4553148	.003717472
270	72900	19683000	16.4316767	6.4633041	.003703704
271	73441	19902511	16.4620776	6.4712736	.003690037
272	73984	20123648	16.4924225	6.4792236	.003676471
273	74529	20346417	16.5227116	6.4871541	.003663004
274	75076	20570824	16.5529454	6.4950653	.003649635
275	75625	20796875	16.5831240	6.5029572	.003636364
276	76176	21024576	16.6132477	6.5108300	.003623188
277	76729	21253933	16.6433170	6.5186839	.003610108
278	77284	21484952	16.6733320	6.5265189	.003597122
279	77841	21717639	16.7032931	6.5343351	.003584229
280	78400	21952000	16.7332005	6.5421326	.003571429
281	78961	22188041	16.7630546	6.5499116	.003558719
282	79524	22425768	16.7928556	6.5576722	.003546099
283	80089	22665187	16.8226038	6.5654144	.003533569
284	80656	22906304	16.8522995	6.5731385	.003521127
285	81225	23149125	16.8819430	6.5808443	.003508772
286	81796	23393656	16.9115345	6.5885323	.003496503
287	82369	23639903	16.9410743	6.5962023	.003484321
288	82944	23887872	16.9705627	6.6038545	.003472222
289	83521	24137569	17.0000000	6.6114890	.003460208
290	84100	24389000	17.0293864	6.6191060	.003448276
291	84681	24642171	17.0587221	6.6267054	.003436426
292	85264	24897088	17.0880075	6.6342874	.003424658
293	85849	25153757	17.1172428	6.6418522	.003412969
294	86436	25412184	17.1464282	6.6493998	.003401361
295	87025	25672375	17.1755640	6.6569302	.003389831
296	87616	25934336	17.2046505	6.6644437	.003378378
297	88209	26198073	17.2336879	6.6719403	.003367003
298	88804	26463592	17.2626765	6.6794200	.003355705
299	89401	26730899	17.2916165	6.6868831	.003344482
300	90000	27000000	17.3205081	6.6943295	.003333333
301	90601	27270901	17.3493516	6.7017593	.003322259
302	91204	27543608	17.3781472	6.7091729	.003311258
303	91809	27818127	17.4068952	6.7165700	.003300330
304	92416	28094464	17.4355958	6.7239508	.003289474
305	93025	28372625	17.4642492	6.7313155	.003278689
306	93636	28652616	17.4928557	6.7386641	.003267974
307	94249	28934443	17.5214155	6.7459967	.003257329
08	94864	29218112	17.5499288	6.7533134	.003246753
9	95481	29503629	17.5783958	6.7606143	.003236246
	96100	29791000	17.6068169	6.7678995	.00322580

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
311	96721	30080231	17.6351921	6.7751690	.003215434
312	97344	30371328	17.6635217	6.7824229	.003205128
313	97969	30664297	17.6918060	6.7896613	.003194888
314	98596	30959144	17.7200451	6.7968844	.003184713
315	99225	31255875	17.7482393	6.8040921	.003174603
316	99856	31554496	17.7763888	6.8112847	.003164557
317	100489	31855013	17.8044938	6.8184620	.003154574
318	101124	32157432	17.8325545	6.8256242	.003144654
319	101761	32461759	17.8605711	6.8327714	.003134796
320	102400	32768000	17.8885438	6.8399037	.003125000
321	103041	33076161	17.9164729	6.8470213	.003115265
322	103684	33386248	17.9443584	6.8541240	.003105590
323	104329	33698267	17.9722008	6.8612120	.003095975
324	104976	34012224	18.0000000	6.8682855	.003086420
325	105625	34328125	18.0277564	6.8753443	.003076923
326	106276	34645976	18.0554701	6.8823888	.003067485
327	106929	34965783	18.0831413	6.8894188	.003058104
328	107584	35287552	18.1107703	6.8964345	.003048780
329	108241	35611289	18.1383571	6.9034359	.003039514
330	108900	35937000	18.1659021	6.9104232	.003030303
331	109561	36264691	18.1934054	6.9173964	.003021148
332	110224	36594368	18.2208672	6.9243556	.003012048
333	110889	36926037	18.2482876	6.9313008	.003003003
334	111556	37259704	18.2756669	6.9382321	.002994012
335	112225	37595375	18.3030052	6.9451496	.002985075
336	112896	37933056	18.3303028	6.9520533	.002976190
337	113569	38272753	18.3575598	6.9589434	.002967359
338	114244	38614472	18.3847763	6.9658198	.002958580
339	114921	38958219	18.4119526	6.9726826	.002949853
340	115600	39304000	18.4390889	6.9795321	.002941176
341	116281	39651821	18.4661853	6.9863681	.002932551
342	116964	40001688	18.4932420	6.9931906	.002923977
343	117649	40353607	18.5202592	7.0000000	.002915452
344	118336	40707584	18.5472370	7.0067962	.002906977
345	119025	41063625	18.5741756	7 0135791	.002898551
346	119716	41421736	18.6010752	7.0203490	.002890173
347	120409	41781923	18.6279360	7.0271058	.002881844
348	121104	42144192	18.6547581	7.0338497	.002873563
349	121801	42508549	18.6815417	7.0405806	.002865330
350	122500	42875000	18.7082869	7.0472987	.002857143
351	123201	43243551	18.7349940	7.0540041	.002849003
352	123904	43614208	18.7616630	7.0606967	.002840909
353	124609	43986977	18.7882942	7.0673767	.002832861
354	125316	44361864	18.8148877	7.0740440	.002824859
355	126025	44738875	18.8414437	7.0806988	.002816901
356	126736	45118016	18.8679623	7.0873411	.002808989
357	127449	45499293	18.8944436	7.0939709	.002801120
358	128164	45882712	18.9208879	7.1005885	.002793296
359	128881	46268279	18.9472953	7.1071937	.002785515
360	129600	46656000	18.9736660	7.1137866	.002777778
361	130321	47045881	19.0000000	7.1203674	.002770083
362	131044	47437928	19.0262976	7.1269360	.002762431
363	131769	47832147	19.0525589	7.1334925	.002754821
364	132496	48228544	19.0787840	7.1400370	.002747253
365	133225	48627125	19.1049732	7.1465695	.002739726
366	133956	49027896	19.1311265	7.1530901	.002732240
367	134689	49430863	19.1572441	7.1595988	.002724796
368	135424	49836032	19.1833261	7.1660057	.002717391
369	*136161*	50243409	19.2093727	7.1725809	.00271002
370	*136900*	50653000	19.2353841	7.1790544	.0027027
371	*137641*	51064811	19.2613603	7.1855162	.002695
372	*138384*	51478848	19.2873015	7.1919663	.00268

TABLE XXIII.—SQUARES, CUBES, SQUARE ROOTS,

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
373	139129	51895117	19.3132079	7.1984050	.002680965
374	139876	52313624	19.3390796	7.2048322	.002673797
375	140625	52734375	19.3649167	7.2112479	.002666667
376	141376	53157376	19.3907194	7.2176522	.002659574
377	142129	53582633	19.4164878	7.2240450	.002652520
378	142884	54010152	19.4422221	7.2304268	.002645503
379	143641	54439939	19.4679223	7.2367972	.002638522
380	144400	54872000	19.4935887	7.2431565	.002631579
381	145161	55306341	19.5192213	7.2495045	.002624672
382	145924	55742968	19.5448203	7.2558415	.002617801
383	146689	56181887	19.5703858	7.2621675	.002610966
384	147456	56623104	19.5959179	7.2684824	.002604167
385	148225	57066625	19.6214169	7.2747864	.002597403
386	148996	57512456	19.6468827	7.2810794	.002590674
387	149769	57960603	19.6723156	7.2873617	.002583979
388	150544	58411072	19.6977156	7.2936330	.002577320
389	151321	58863869	19.7230829	7.2998936	.002570694
390	152100	59319000	19.7484177	7.3061436	.002564103
391	152881	59776471	19.7737199	7.3123828	.002557545
392	153664	60236288	19.7989899	7.3186114	.002551020
393	154449	60698457	19.8242276	7.3248295	.002544529
394	155236	61162984	19.8494332	7.3310369	.002538071
395	156025	61629875	19.8746069	7.3372339	.002531646
396	156816	62099136	19.8997487	7.3434205	.002525253
397	157609	62570773	19.9248588	7.3495966	.002518892
398	158404	63044792	19.9499373	7.3557624	.002512563
399	159201	63521199	19.9749844	7.3619178	.002506266
400	160000	64000000	20.0000000	7.3680630	.002500000
401	160801	64481201	20.0249844	7.3741979	.002493766
402	161604	64964808	20.0499377	7.3803227	.002487562
403	162409	65450827	20.0748599	7.3864373	.002481390
404	163216	65939264	20.0997512	7.3925418	.002475248
405	164025	66430125	20.1246118	7.3986363	.002469136
406	164836	66923416	20.1494417	7.4047206	.002463054
407	165649	67419143	20.1742410	7.4107950	.002457002
408	166464	67917312	20.1990099	7.4168595	.002450980
409	167281	68417929	20.2237484	7.4229142	.002444988
410	168100	68921000	20.2484567	7.4289589	.002439024
411	168921	69426531	20.2731349	7.4349938	.002433090
412	169744	69934528	20.2977831	7.4410189	.002427184
413	170569	70444997	20.3224014	7.4470342	.002421308
414	171396	70957944	20.3469899	7.4530399	.002415459
415	172225	71473375	20.3715488	7.4590359	.002409639
416	173056	71991296	20.3960781	7.4650223	.002403846
417	173889	72511713	20.4205779	7.4709991	.002398082
418	174724	73034632	20.4450483	7.4769664	.002392344
419	175561	73560059	20.4694895	7.4829242	.002386635
420	176400	74088000	20.4939015	7.4888724	.002380952
421	177241	74618461	20.5182845	7.4948113	.002375297
422	178084	75151448	20.5426386	7.5007406	.002369668
423	178929	75686967	20.5669638	7.5066607	.002364066
424	179776	76225024	20.5912603	7.5125715	.002358491
425	180625	76765625	20.6155281	7.5184730	.002352941
426	181476	77308776	20.6397674	7.5243652	.002347418
427	182329	77854483	20.6639783	7.5302482	.002341920
428	183184	78402752	20.6881609	7.5361221	.002336449
429	184041	78953589	20.7123152	7.5419867	.002331002
430	184900	79507000	20.7364414	7.5478423	.002325581
21	185761	80062991	20.7605395	7.5536888	.002320186
	186624	80621568	20.7846097	7.5595263	.002314815
	187489	81182737	20.8086520	7.5653548	.00230946
	188356	81746504	20.8326667	7.5711743	.00230414

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
435	189225	82312875	20.8566536	7.5769849	.002298851
436	190096	82881856	20.8806130	7.5827865	.002293578
437	190969	83453453	20.9045450	7.5885793	.002288330
438	191844	84027672	20.9284495	7.5943633	.002283105
439	192721	84604519	20.9523268	7.6001385	.002277904
440	193600	85184000	20.9761770	7.6059049	.002272727
441	194481	85766121	21.0000000	7.6116626	.002267574
442	195364	86350888	21.0237960	7.6174116	.002262443
443	196249	86938307	21.0475652	7.6231519	.002257336
444	197136	87528384	21.0713075	7.6288837	.002252252
445	198025	88121125	21.0950231	7.6346067	.002247191
446	198916	88716536	21.1187121	7.6403213	.002242152
447	199809	89314623	21.1423745	7.6460272	.002237136
448	200704	89915392	21.1660105	7.6517247	.002232143
449	201601	90518849	21.1896201	7.6574133	.002227171
450	202500	91125000	21.2132034	7.6630943	.002222222
451	203401	91733851	21.2367606	7.6687665	.002217295
452	204304	92345408	21.2602916	7.6744303	.002212389
453	205209	92959677	21.2837967	7.6800857	.002207506
454	206116	93576664	21.3072758	7.6857328	.002202643
455	207025	94196375	21.3307290	7.6913717	.002197802
456	207936	94818816	21.3541565	7.6970023	.002192982
457	208849	95443993.	21.3775583	7.7026246	.002188184
458	209764	96071912	21.4009346	7.7082388	.002183406
459	210681	96702579	21.4242853	7.7138448	.002178649
460	211600	97336000	21.4476106	7.7194426	.002173913
461	212521	97972181	21.4709106	7.7250325	.002169197
462	213444	98611128	21.4941853	7.7306141	.002164502
463	214369	99252847	21.5174348	7.7361877	.002159827
464	215296	99897344	21.5406592	7.7417532	.002155172
465	216225	100544625	21.5638587	7.7473109	.002150538
466	217156	101194696	21.5870331	7.7528606	.002145923
467	218089	101847563	21.6101828	7.7584023	.002141328
468	219024	102503232	21.6333077	7.7639361	.002136752
469	219961	103161709	21.6564078	7.7694620	.002132196
470	220900	103823000	21.6794834	7.7749801	.002127660
471	221841	104487111	21.7025344	7.7804904	.002123142
472	222784	105154048	21.7255610	7.7859928	.002118644
473	223729	105823817	21.7485632	7.7914875	.002114165
474	224676	106496424	21.7715411	7.7969745	.002109705
475	225625	107171875	21.7944947	7.8024538	.002105263
476	226576	107850176	21.8174242	7.8079254	.002100840
477	227529	108531333	21.8403297	7.8133892	.002096436
478	228484	109215352	21 8632111	7.8188456	.002092050
479	229441	109902239	21.8860686	7.8242942	.002087683
480	230400	110592000	21.9089023	7.8297353	.002088333
481	231361	111284641	21.9317122	7.8351688	.002079002
482	232324	111980168	21.9544984	7.8405949	.002074689
483	233289	112678587	21.9772610	7.8460134	.002070393
484	234256	113379904	22.0000000	7.8514244	.002066116
485	235225	114084125	22.0227155	7.8568281	.002061856
486	236196	114791256	22.0454077	7.8622242	.002057613
487	237169	115501303	22.0680765	7.8676130	.002053388
488	238144	116214272	22.0907220	7.8729944	.002049180
489	239121	116930169	22.1133444	7.8783684	.00
490	240100	117649000	22.1359436	7.8837352	
491	241081	118370771	22.1585198	7.8890946	
492	242064	119095488	22.1810730	7.8944468	
493	243049	119823157	22.2036033	7.8997917	
494	244036	120553784	22.2261108	7.9051294	
495	245025	121287375	22.2485955	7.9104599	
496	246016	122023936	22.2710575	7.9157832	

TABLE XXIII.—SQUARES, CUBES, SQUARE ROOTS,

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
497	247009	122763473	22.2934968	7.9210994	.002012072
498	248004	124505992	22.3159136	7.9264085	.002008032
499	249001	124251499	22.3383079	7.9317104	.002004008
500	250000	125000000	22.3606798	7.9370053	.002000000
501	251001	125751501	22.3830293	7.9422931	.001996008
502	252004	126506008	22.4053565	7.9475739	.001992032
503	253009	127263527	22.4276615	7.9528477	.001988072
504	254016	128024064	22.4499443	7.9581144	.001984127
505	255025	128787625	22.4722051	7.9633743	.001980198
506	256036	129554216	22.4944438	7.9686271	.001976285
507	257049	130323843	22.5166605	7.9738731	.001972387
508	258064	131096512	22.5388553	7.9791122	.001968504
509	259081	131872229	22.5610283	7.9843444	.001964637
510	260100	132651000	22.5831796	7.9895697	.001960784
511	261121	133432831	22.6053091	7.9947883	.001956947
512	262144	134217728	22.6274170	8.0000000	.001953125
513	263169	135005697	22.6495033	8.0052049	.001949318
514	264196	135796744	22.6715681	8.0104032	.001945525
515	265225	136590875	22.6936114	8.0155946	.001941748
516	266256	137388096	22.7156334	8.0207794	.001937984
517	267289	138188413	22.7376340	8.0259574	.001934236
518	268324	138991832	22.7596134	8.0311287	.001930502
519	269361	139798359	22.7815715	8.0362935	.001926782
520	270400	140608000	22.8035085	8.0414515	.001923077
521	271441	141420761	22.8254244	8.0466030	.001919386
522	272484	142236648	22.8473193	8.0517479	.001915709
523	273529	143055667	22.8691933	8.0568862	.001912046
524	274576	143877824	22.8910463	8.0620180	.001908397
525	275625	144703125	22.9128785	8.0671432	.001904762
526	276676	145531576	22.9346899	8.0722620	.001901141
527	277729	146363183	22.9564806	8.0773743	.001897533
528	278784	147197952	22.9782506	8.0824800	.001893939
529	279841	148035889	23.0000000	8.0875794	.001890359
530	280900	148877000	23.0217289	8.0926723	.001886792
531	281961	149721291	23.0434372	8.0977589	.001883239
532	283024	150568768	23.0651252	8.1028390	.001879699
533	284089	151419437	23.0867928	8.1079128	.001876173
534	285156	152273304	23.1084400	8.1129803	.001872659
535	286225	153130375	23.1300670	8.1180414	.001869159
536	287296	153990656	23.1516738	8.1230962	.001865672
537	288369	154854153	23.1732605	8.1281447	.001862197
538	289444	155720872	23.1948270	8.1331870	.001858736
539	290521	156590819	23.2163735	8.1382230	.001855288
540	291600	157464000	23.2379001	8.1432529	.001851852
541	292681	158340421	23.2594067	8.1482765	.001848429
542	293764	159220088	23.2808935	8.1532939	.001845018
543	294849	160103007	23.3023604	8.1583051	.001841621
544	295936	160989184	23.3238076	8.1633102	.001838235
545	297025	161878625	23.3452351	8.1683092	.001834862
546	298116	162771336	23.3666429	8.1733020	.001831502
547	299209	163667323	23.3880311	8.1782888	.001828154
548	300304	164566592	23.4093998	8.1832695	.001824818
549	301401	165469149	23.4307490	8.1882441	.001821494
550	302500	166375000	23.4520788	8.1932127	.001818182
551	303601	167284151	23.4733892	8.1981752	.001814882
552	304704	168196608	23.4946802	8.2031319	.001811594
553	305809	169112377	23.5159520	8.2080825	.001808318
554	306916	170031464	23.5372046	8.2130271	.001805054
555	308025	170953875	23.5584380	8.2179657	.001801802
556	309136	171879616	23.5796522	8.2228985	.001798561
557	310249	172808693	23.6008474	8.2278254	.001795332
558	311364	173741112	23.6220236	8.2327463	.00179211

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
559	312481	174676879	23.6431808	8.2376614	.001788909
560	313600	175616000	23.6643191	8.2425706	.001785714
561	314721	176558481	23.6854386	8.2474740	.001782531
562	315844	177504328	23.7065392	8.2523715	.001779359
563	316969	178453547	23.7276210	8.2572633	.001776199
564	318096	179406144	23.7486842	8.2621492	.001773050
565	319225	180362125	23.7697286	8.2670294	.001769912
566	320356	181321496	23.7907545	8.2719039	.001766784
567	321489	182284263	23.8117618	8.2767726	.001763668
568	322624	183250432	23.8327506	8.2816355	.001760563
569	323761	184220009	23.8537209	8.2864928	.001757469
570	324900	185193000	23.8746728	8.2913444	.001754386
571	326041	186169411	23.8956063	8.2961903	.001751313
572	327184	187149248	23.9165215	8.3010304	.001748252
573	328329	188132517	23.9374184	8.3058651	.001745201
574	329476	189119224	23.9582971	8.3106941	.001742160
575	330625	190109375	23.9791576	8.3155175	.001739130
576	331776	191102976	24.0000000	8.3203353	.001736111
577	332929	192100033	24.0208243	8.3251475	.001733102
578	334084	193100552	24.0416306	8.3299542	.001730104
579	335241	194104539	24.0624188	8.3347553	.001727116
580	336400	195112000	24.0831891	8.3395509	.001724138
581	337561	196122941	24.1039416	8.3443410	.001721170
582	338724	197137368	24.1246762	8.3491256	.001718213
583	339889	198155287	24.1453929	8.3539047	.001715266
584	341056	199176704	24.1660919	8.3586784	.001712329
585	342225	200201625	24.1867732	8.3634466	.001709402
586	343396	201230056	24.2074369	8.3682095	.001706485
587	344569	202262003	24.2280829	8.3729668	.001703578
588	345744	203297472	24.2487113	8.3777188	.001700680
589	346921	204336469	24.2693222	8.3824653	.001697793
590	348100	205379000	24.2899156	8.3872065	.001694915
591	349281	206425071	24.3104916	8.3919423	.001692047
592	350464	207474688	24.3310501	8.3966729	.001689189
593	351649	208527857	24.3515913	8.4013981	.001686341
594	352836	209584584	24.3721152	8.4061180	.001683502
595	354025	210644875	24.3926218	8.4108326	.001680672
596	355216	211708736	24.4131112	8.4155419	.001677852
597	356409	212776173	24.4335834	8.4202460	.001675042
598	357604	213847192	24.4540385	8.4249448	.001672241
599	358801	214921799	24.4744765	8.4296383	.001669449
600	360000	216000000	24.4948974	8.4343267	.001666667
601	361201	217081801	24.5153013	8.4390098	.001663894
602	362404	218167208	24.5356883	8.4436877	.001661130
603	363609	219256227	24.5560583	8.4483605	.001658375
604	364816	220348864	24.5764115	8.4530281	.001655629
605	366025	221445125	24.5967478	8.4576906	.001652893
606	367236	222545016	24.6170673	8.4623479	.001650165
607	368449	223648543	24.6373700	8.4670001	.001647446
608	369664	224755712	24.6576560	8.4716471	.001644737
609	370881	225866529	24.6779254	8.4762892	.001642036
610	372100	226981000	24.6981781	8.4809261	.001639344
611	373321	228099131	24.7184142	8.4855579	.001636661
612	374544	229220928	24.7386338	8.4901848	[illegible]
613	375769	230346397	24.7588368	8.4948065	[illegible]
614	376996	231475544	24.7790234	8.4994233	[illegible]
615	378225	232608375	24.7991935	8.5040350	[illegible]
616	379456	233744896	24.8193473	8.5086417	[illegible]
617	380689	234885113	24.8394847	8.5132435	[illegible]
618	381924	236029032	24.8596058	8.517840	[illegible]
619	383161	237176659	24.8797106	8.52243	[illegible]
620	384400	238328000	24.8997992	8.5270	[illegible]

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
621	385641	239483061	24.9198716	8.5316009	.001610306
622	386884	240641848	24.9399278	8.5361780	.001607717
623	388129	241804367	24.9599679	8.5407501	.001605136
624	389376	242970624	24.9799920	8.5453173	.001602564
625	390625	244140625	25.0000000	8.5498797	.001600000
626	391876	245314376	25.0199920	8.5544372	.001597444
627	393129	246491883	25.0399681	8.5589899	.001594896
628	394384	247673152	25.0599282	8.5635377	.001592357
629	395641	248858189	25.0798724	8.5680807	.001589825
630	396900	250047000	25.0998008	8.5726189	.001587302
631	398161	251239591	25.1197134	8.5771523	.001584786
632	399424	252435968	25.1396102	8.5816800	.001582278
633	400689	253636137	25.1594913	8.5862047	.001579779
634	401956	254840104	25.1793566	8.5907238	.001577287
635	403225	256047875	25.1992063	8.5952380	.001574803
636	404496	257259456	25.2190404	8.5997476	.001572327
637	405769	258474853	25.2388589	8.6042525	.001569859
638	407044	259694072	25.2586619	8.6087526	.001567398
639	408321	260917119	25.2784493	8.6132480	.001564945
640	409600	262144000	25.2982213	8.6177388	.001562500
641	410881	263374721	25.3179778	8.6222248	.001560062
642	412164	264609288	25.3377189	8.6267063	.001557632
643	413449	265847707	25.3574447	8.6311830	.001555210
644	414736	267089984	25.3771551	8.6356551	.001552795
645	416025	268336125	25.3968502	8.6401226	.001550388
646	417316	269586136	25.4165301	8.6445855	.001547988
647	418609	270840023	25.4361947	8.6490437	.001545595
648	419904	272097792	25.4558441	8.6534974	.001543210
649	421201	273359449	25.4754784	8.6579465	.001540832
650	422500	274625000	25.4950976	8.6623911	.001538462
651	423801	275894451	25.5147016	8.6668310	.001536098
652	425104	277167808	25.5342907	8.6712665	.001533742
653	426409	278445077	25.5538647	8.6756974	.001531394
654	427716	279726264	25.5734237	8.6801237	.001529052
655	429025	281011375	25.5929678	8.6845456	.001526718
656	430336	282300416	25.6124969	8.6889630	.001524390
657	431649	283593393	25.6320112	8.6933759	.001522070
658	432964	284890312	25.6515107	8.6977843	.001519757
659	434281	286191179	25.6709953	8.7021882	.001517451
660	435600	287496000	25.6904652	8.7065877	.001515152
661	436921	288804781	25.7099203	8.7109827	.001512859
662	438244	290117528	25.7293607	8.7153734	.001510574
663	439569	291434247	25.7487864	8.7197596	.001508296
664	440896	292754944	25.7681975	8.7241414	.001506024
665	442225	294079625	25.7875939	8.7285187	.001503759
666	443556	295408296	25.8069758	8.7328918	.001501502
667	444889	296740963	25.8263431	8.7372604	.001499250
668	446224	298077632	25.8456960	8.7416246	.001497006
669	447561	299418309	25.8650343	8.7459846	.001494768
670	448900	300763000	25.8843582	8.7503401	.001492537
671	450241	302111711	25.9036677	8.7546913	.001490313
672	451584	303464448	25.9229628	8.7590383	.001488095
673	452929	304821217	25.9422435	8.7633809	.001485884
674	454276	306182024	25.9615100	8.7677192	.001483680
675	455625	307546875	25.9807621	8.7720532	.001481481
676	456976	308915776	26.0000000	8.7763830	.001479290
677	458329	310288733	26.0192237	8.7807084	.001477105
678	459684	311665752	26.0384331	8.7850296	.001474926
679	461041	313046839	26.0576284	8.7893466	.001472754
	462400	314432000	26.0768096	8.7936593	.001470588
	463761	315821241	26.0959767	8.7979679	.001468429
	465124	317214568	26.1151297	8.8022721	.00146627

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
683	466489	318611987	26.1342687	8.8065722	.001464129
684	467856	320013504	26.1533937	8.8108681	.001461988
685	469225	321419125	26.1725047	8.8151598	.001459854
686	470596	322828856	26.1916017	8.8194474	.001457726
687	471969	324242703	26.2106848	8.8237307	.001455604
688	473344	325660672	26.2297541	8.8280099	.001453488
689	474721	327082769	26.2488095	8.8322850	.001451379
690	476100	328509000	26.2678511	8.8365559	.001449275
691	477481	329939371	26.2868789	8.8408227	.001447178
692	478864	331373888	26.3058929	8.8450854	.001445087
693	480249	332812557	26.3248932	8.8493440	.001443001
694	481636	334255384	26.3438797	8.8535985	.001440922
695	483025	335702375	26.3628527	8.8578489	.001438849
696	484416	337153536	26.3818119	8.8620952	.001436782
697	485809	338608873	26.4007576	8.8663375	.001434720
698	487204	340068392	26.4196896	8.8705757	.001432665
699	488601	341532099	26.4386081	8.8748099	.001430615
700	490000	343000000	26.4575131	8.8790400	.001428571
701	491401	344472101	26.4764046	8.8832661	.001426534
702	492804	345948408	26.4952826	8.8874882	.001424501
703	494209	347428927	26.5141472	8.8917063	.001422475
704	495616	348913664	26.5329983	8.8959204	.001420455
705	497025	350402625	26.5518361	8.9001304	.001418440
706	498436	351895816	26.5706605	8.9043366	.001416431
707	499849	353393243	26.5894716	8.9085387	.001414427
708	501264	354894912	26.6082694	8.9127369	.001412429
709	502681	356400829	26 6270539	8.9169311	.001410437
710	504100	357911000	26.6458252	8.9211214	.001408451
711	505521	359425431	26.6645833	8.9253078	.001406470
712	506944	360944128	26.6833281	8.9294902	.001404494
713	508369	362467097	26.7020598	8.9336687	.001402525
714	509796	363994344	26.7207784	8.9378433	.001400560
715	511225	365525875	26.7394839	8.9420140	.001398601
716	512656	367061696	26.7581763	8.9461809	.001396648
717	514089	368601813	26.7768557	8.9503438	.001394700
718	515524	370146232	26.7955220	8.9545029	.001392758
719	516961	371694959	26.8141754	8.9586581	.001390821
720	518400	373248000	26.8328157	8.9628095	.001388889
721	519841	374805361	26.8514432	8.9669570	.001386963
722	521284	376367048	26.8700577	8.9711007	.001385042
723	522729	377933067	26.8886593	8.9752406	.001383126
724	524176	379503424	26.9072481	8.9793766	.001381215
725	525625	381078125	26.9258240	8.9835089	.001379310
726	527076	382657176	26.9443872	8.9876373	.001377410
727	528529	384240583	26.9629375	8.9917620	.001375516
728	529984	385828352	26.9814751	8.9958829	.001373626
729	531441	387420489	27.0000000	9.0000000	.001371742
730	532900	389017000	27.0185122	9.0041134	.001369863
731	534361	390617891	27.0370117	9.0082229	.001367989
732	535824	392223168	27.0554985	9.0123288	.001366120
733	537289	393832837	27.0739727	9.0164309	.001364256
734	538756	395446904	27.0924344	9.0205293	.001362398
735	540225	397065375	27.1108834	9.0246239	.001360544
736	541696	398688256	27.1293199	9.0287149	.001358696
737	543169	400315553	27.1477439	9.0328021	.001356852
738	544644	401947272	27.1661554	9.0368857	.001355014
739	546121	403583419	27.1845544	9.0409655	.001353180
740	547600	405224000	27.2029410	9.0450419	.001351351
741	549081	406869021	27.2213152	9.0491142	.001349528
742	550564	408518488	27.2396769	9.0531831	.001347709
743	552049	410172407	27.2580263	9.0572482	.001345895
744	553536	411830784	27.2763634	9.0613098	.00134408

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
745	555025	413493625	27.2946881	9.0653677	.001342282
746	556516	415160936	27.3130006	9.0694220	.001340483
747	558009	416832723	27.3313007	9.0734726	.001338688
748	559504	418508992	27.3495887	9.0775197	.001336898
749	561001	420189749	27.3678644	9.0815631	.001335113
750	562500	421875000	27.3861279	9.0856030	.001333333
751	564001	423564751	27.4043792	9.0896392	.001331558
752	565504	425259008	27.4226184	9.0936719	.001329787
753	567009	426957777	27.4408455	9.0977010	.001328021
754	568516	428661064	27.4590604	9.1017265	.001326260
755	570025	430368875	27.4772633	9.1057485	.001324503
756	571536	432081216	27.4954542	9.1097669	.001322751
757	573049	433798093	27.5136330	9.1137818	.001321004
758	574564	435519512	27.5317998	9.1177931	.001319261
759	576081	437245479	27.5499546	9.1218010	.001317523
760	577600	438976000	27.5680975	9.1258053	.001315789
761	579121	440711081	27.5862284	9.1298061	.001314060
762	580644	442450728	27.6043475	9.1338034	.001312336
763	582169	444194947	27.6224546	9.1377971	.001310616
764	583696	445943744	27.6405499	9.1417874	.001308901
765	585225	447697125	27.6586334	9.1457742	.001307190
766	586756	449455096	27.6767050	9.1497576	.001305483
767	588289	451217663	27.6947648	9.1537375	.001303781
768	589824	452984832	27.7128129	9.1577139	.001302083
769	591361	454756609	27.7308492	9.1616869	.001300390
770	592900	456533000	27.7488739	9.1656565	.001298701
771	594441	458314011	27.7668868	9.1696225	.001297017
772	595984	460099648	27.7848880	9.1735852	.001295337
773	597529	461889917	27.8028775	9.1775445	.001293661
774	599076	463684824	27.8208555	9.1815003	.001291990
775	600625	465484375	27.8388218	9.1854527	.001290323
776	602176	467288576	27.8567766	9.1894018	.001288660
777	603729	469097433	27.8747197	9.1933474	.001287001
778	605284	470910952	27.8926514	9.1972897	.001285347
779	606841	472729139	27.9105715	9.2012286	.001283697
780	608400	474552000	27.9284801	9.2051641	.001282051
781	609961	476379541	27.9463772	9.2090962	.001280410
782	611524	478211768	27.9642629	9.2130250	.001278772
783	613089	480048687	27.9821372	9.2169505	.001277139
784	614656	481890304	28.0000000	9.2208726	.001275510
785	616225	483736625	28.0178515	9.2247914	.001273885
786	617796	485587656	28.0356915	9.2287068	.001272265
787	619369	487443403	28.0535203	9.2326189	.001270648
788	620944	489303872	28.0713377	9.2365277	.001269036
789	622521	491169069	28.0891438	9.2404333	.001267427
790	624100	493039000	28.1069386	9.2443355	.001265823
791	625681	494913671	28.1247222	9.2482344	.001264223
792	627264	496793088	28.1424946	9.2521300	.001262626
793	628849	498677257	28.1602557	9.2560224	.001261034
794	630436	500566184	28.1780056	9.2599114	.001259446
795	632025	502459875	28.1957444	9.2637973	.001257862
796	633616	504358336	28.2134720	9.2676798	.001256281
797	635209	506261573	28.2311884	9.2715592	.001254705
798	636804	508169592	28.2488938	9.2754352	.001253133
799	638401	510082399	28.2665881	9.2793081	.001251564
800	640000	512000000	28.2842712	9.2831777	.001250000
801	641601	513922401	28.3019434	9.2870440	.001248439
802	643204	515849608	28.3196045	9.2909072	.001246883
803	644809	517781627	28.3372546	9.2947671	.001245330
804	646416	519718464	28.3548938	9.2986239	.001243781
805	648025	521660125	28.3725219	9.3024775	.001242236
806	649636	523606616	28.3901391	9.3063278	.001240695

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
807	651249	525557943	28.4077454	9.3101750	.001239157
808	652864	527514112	28.4253408	9.3140190	.001237624
809	654481	529475129	28.4429253	9.3178599	.001236094
810	656100	531441000	28.4604989	9.3216975	.001234568
811	657721	533411731	28.4780617	9.3255320	.001233046
812	659344	535387328	28.4956137	9.3293634	.001231527
813	660969	537367797	28.5131549	9.3331916	.001230012
814	662596	539353144	28.5306852	9.3370167	.001228501
815	664225	541343375	28.5482048	9.3408386	.001226994
816	665856	543338496	28.5657137	9.3446575	.001225490
817	667489	545338513	28.5832119	9.3484731	.001223990
818	669124	547343432	28.6006993	9.3522857	.001222494
819	670761	549353259	28.6181760	9.3560952	.001221001
820	672400	551368000	28.6356421	9.3599016	.001219512
821	674041	553387661	28.6530976	9.3637049	.001218027
822	675684	555412248	28.6705424	9.3675051	.001216545
823	677329	557441767	28.6879766	9.3713022	.001215067
824	678976	559476224	28.7054002	9.3750963	.001213592
825	680625	561515625	28.7228132	9.3788873	.001212121
826	682276	563559976	28.7402157	9.3826752	.001210654
827	683929	565609283	28.7576077	9.3864600	.001209190
828	685584	567663552	28.7749891	9.3902419	.001207729
829	687241	569722789	28.7923601	9.3940206	.001206273
830	688900	571787000	28.8097206	9.3977964	.001204819
831	690561	573856191	28.8270706	9.4015691	.001203369
832	692224	575930368	28.8444102	9.4053387	.001201923
833	693889	578009537	28.8617394	9.4091054	.001200480
834	695556	580093704	28.8790582	9.4128690	.001199041
835	697225	582182875	28.8963666	9.4166297	.001197605
836	698896	584277056	28.9136646	9.4203873	.001196172
837	700569	586376253	28.9309523	9.4241420	.001194743
838	702244	588480472	28.9482297	9.4278936	.001193317
839	703921	590589719	28.9654967	9.4316423	.001191895
840	705600	592704000	28.9827535	9.4353880	.001190476
841	707281	594823321	29.0000000	9.4391307	.001189061
842	708964	596947688	29.0172363	9.4428704	.001187648
843	710649	599077107	29.0344623	9.4466072	.001186240
844	712336	601211584	29.0516781	9.4503410	.001184834
845	714025	603351125	29.0688837	9.4540719	.001183432
846	715716	605495736	29.0860791	9.4577999	.001182033
847	717409	607645423	29.1032644	9.4615249	.001180638
848	719104	609800192	29.1204396	9.4652470	.001179245
849	720801	611960049	29.1376046	9.4689661	.001177856
850	722500	614125000	29.1547595	9.4726824	.001176471
851	724201	616295051	29.1719043	9.4763957	.001175088
852	725904	618470208	29.1890390	9.4801061	.001173709
853	727609	620650477	29.2061637	9.4838136	.001172333
854	729316	622835864	29.2232784	9.4875182	.001170960
855	731025	625026375	29.2403830	9.4912200	.001169591
856	732736	627222016	29.2574777	9.4949188	.001168224
857	734449	629422793	29.2745623	9.4986147	.001166861
858	736164	631628712	29.2916370	9.5023078	.001165501
859	737881	633839779	29.3087018	9.5059980	.001164144
860	739600	636056000	29.3257566	9.5096854	.001162791
861	741321	638277381	29.3428015	9.5133699	.001161440
862	743044	640503928	29.3598365	9.5170515	.001160093
863	744769	642735647	29.3768616	9.5207303	.001158749
864	746496	644972544	29.3938769	9.5244063	.001157407
865	748225	647214625	29.4108823	9.5280794	.001156
866	749956	649461896	29.4278779	9.5317497	.001154
867	751689	651714363	29.4448637	9.5354172	.00115
868	753424	653972032	29.4618397	9.5390818	.0011

TABLE XXIII.—SQUARES, CUBES, SQUARE ROOTS,

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
869	755161	656234909	29.4788059	9.5427437	.001150748
870	756900	658503000	29.4957624	9.5464027	.001149425
871	758641	660776311	29.5127091	9.5500589	.001148106
872	760384	663054848	29.5296461	9.5537123	.001146789
873	762129	665338617	29.5465734	9.5573630	.001145475
874	763876	667627624	29.5634910	9.5610108	.001144165
875	765625	669921875	29.5803989	9.5646559	.001142857
876	767376	672221376	29.5972972	9.5682982	.001141553
877	769129	674526133	29.6141858	9.5719377	.001140251
878	770884	676836152	29.6310648	9.5755745	.001138952
879	772641	679151439	29.6479342	9.5792085	.001137656
880	774400	681472000	29.6647939	9.5828397	.001136364
881	776161	683797841	29.6816442	9.5864682	.001135074
882	777924	686128968	29.6984848	9.5900939	.001133787
883	779689	688465387	29.7153159	9.5937169	.001132503
884	781456	690807104	29.7321375	9.5973373	.001131222
885	783225	693154125	29.7489496	9.6009548	.001129944
886	784996	695506456	29.7657521	9.6045696	.001128668
887	786769	697864103	29.7825452	9.6081817	.001127396
888	788544	700227072	29.7993289	9.6117911	.001126126
889	790321	702595369	29.8161030	9.6153977	.001124859
890	792100	704969000	29.8328678	9.6190017	.001123596
891	793881	707347971	29.8496231	9.6226030	.001122334
892	795664	709732288	29.8663690	9.6262016	.001121076
893	797449	712121957	29.8831056	9.6297975	.001119821
894	799236	714516984	29.8998328	9.6333907	.001118568
895	801025	716917375	29.9165506	9.6369812	.001117318
896	802816	719323136	29.9332591	9.6405690	.001116071
897	804609	721734273	29.9499583	9.6441542	.001114827
898	806404	724150792	29.9666481	9.6477367	.001113586
899	808201	726572699	29.9833287	9.6513166	.001112347
900	810000	729000000	30.0000000	9.6548938	.001111111
901	811801	731432701	30.0166620	9.6584684	.001109878
902	813604	733870808	30.0333148	9.6620403	.001108647
903	815409	736314327	30.0499584	9.6656096	.001107420
904	817216	738763264	30.0665928	9.6691762	.001106195
905	819025	741217625	30.0832179	9.6727403	.001104972
906	820836	743677416	30.0998339	9.6763017	.001103753
907	822649	746142643	30.1164407	9.6798604	.001102536
908	824464	748613312	30.1330383	9.6834166	.001101322
909	826281	751089429	30.1496269	9.6869701	.001100110
910	828100	753571000	30.1662063	9.6905211	.001098901
911	829921	756058031	30.1827765	9.6940694	.001097695
912	831744	758550528	30.1993377	9.6976151	.001096491
913	833569	761048497	30.2158899	9.7011583	.001095290
914	835396	763551944	30.2324329	9.7046989	.001094092
915	837225	766060875	30.2489669	9.7082369	.001092896
916	839056	768575296	30.2654919	9.7117723	.001091703
917	840889	771095213	30.2820079	9.7153051	.001090513
918	842724	773620632	30.2985148	9.7188354	.001089325
919	844561	776151559	30.3150128	9.7223631	.001088139
920	846400	778688000	30.3315018	9.7258883	.001086957
921	848241	781229961	30.3479818	9.7294109	.001085776
922	850084	783777448	30.3644529	9.7329309	.001084599
923	851929	786330467	30.3809151	9.7364484	.001083423
924	853776	788889024	30.3973683	9.7399634	.001082251
925	855625	791453125	30.4138127	9.7434758	.001081081
926	857476	794022776	30.4302481	9.7469857	.001079914
927	859329	796597983	30.4466747	9.7504930	.001078749
928	861184	799178752	30.4630924	9.7539979	.001077586
929	863041	801765089	30.4795013	9.7575002	.00107642
930	864900	804357000	30.4959014	9.7610001	.0010752

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
931	866761	806954491	30.5122926	9.7644974	.001074114
932	868624	809557568	30.5286750	9.7679922	.001072961
933	870489	812166237	30.5450487	9.7714845	.001071811
934	872356	814780504	30.5614136	9.7749743	.001070664
935	874225	817400375	30.5777697	9.7784616	.001069519
936	876096	820025856	30.5941171	9.7819466	.001068376
937	877969	822656953	30.6104557	9.7854288	.001067236
938	879844	825293672	30.6267857	9.7889087	.001066098
939	881721	827936019	30.6431069	9.7923861	.001064963
940	883600	830584000	30.6594194	9.7958611	.001063830
941	885481	833237621	30.6757233	9.7993336	.001062699
942	887364	835896888	30.6920185	9.8028036	.001061571
943	889249	838561807	30.7083051	9.8062711	.001060445
944	891136	841232384	30.7245830	9.8097362	.001059322
945	893025	843908625	30.7408523	9.8131989	.001058201
946	894916	846590536	30.7571130	9.8166591	.001057082
947	896809	849278123	30.7733651	9.8201169	.001055966
948	898704	851971392	30.7896086	9.8235723	.001054852
949	900601	854670349	30.8058436	9.8270252	.001053741
950	902500	857375000	30.8220700	9.8304757	.001052632
951	904401	860085351	30.8382879	9.8339238	.001051525
952	906304	862801408	30.8544972	9.8373695	.001050420
953	908209	865523177	30.8706981	9.8408127	.001049318
954	910116	868250664	30.8868904	9.8442536	.001048218
955	912025	870983875	30.9030743	9.8476920	.001047120
956	913936	873722816	30.9192497	9.8511280	.001046025
957	915849	876467493	30.9354166	9.8545617	.001044932
958	917764	879217912	30.9515751	9.8579929	.001043841
959	919681	881974079	30.9677251	9.8614218	.001042753
960	921600	884736000	30.9838668	9.8648483	.001041667
961	923521	887503681	31.0000000	9.8682724	.001040583
962	925444	890277128	31.0161248	9.8716941	.001039501
963	927369	893056347	31.0322413	9.8751135	.001038422
964	929296	895841344	31.0483494	9.8785305	.001037344
965	931225	898632125	31.0644491	9.8819451	.001036269
966	933156	901428696	31.0805405	9.8853574	.001035197
967	935089	904231063	31.0966236	9.8887673	.001034126
968	937024	907039232	31.1126984	9.8921749	.001033058
969	938961	909853209	31.1287648	9.8955801	.001031992
970	940900	912673000	31.1448230	9.8989830	.001030928
971	942841	915498611	31.1608729	9.9023835	.001029866
972	944784	918330048	31.1769145	9.9057817	.001028807
973	946729	921167317	31.1929479	9.9091776	.001027749
974	948676	924010424	31.2089731	9.9125712	.001026694
975	950625	926859375	31.2249900	9.9159624	.001025641
976	952576	929714176	31.2409987	9.9193513	.001024590
977	954529	932574833	31.2569992	9.9227379	.001023541
978	956484	935441352	31.2729915	9.9261222	.001022495
979	958441	938313739	31.2889757	9.9295042	.001021450
980	960400	941192000	31.3049517	9.9328839	.001020408
981	962361	944076141	31.3209195	9.9362613	.001019368
982	964324	946966168	31.3368792	9.9396363	.001018330
983	966289	949862087	31.3528308	9.9430092	.001017294
984	968256	952763904	31.3687743	9.9463797	.001016260
985	970225	955671625	31.3847097	9.9497479	.001015228
986	972196	958585256	31.4006369	9.9531138	.001014199
987	974169	961504803	31.4165561	9.9564775	.001013171
988	976144	964430272	31.4324673	9.9598389	.001012146
989	978121	967361669	31.4483704	9.9631981	.0010111
990	980100	970299000	31.4642654	9.9665549	.00101
991	982081	973242271	31.4801525	9.9699095	.0010
992	984064	976191488	31.4960315	9.9732619	.001

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
993	986049	979146657	31.5119025	9.9766120	.001007049
994	988036	982107784	31.5277655	9.9799599	.001006036
995	990025	985074875	31.5436206	9.9833055	.001005025
996	992016	988047936	31.5594677	9.9866488	.001004016
997	994009	991026973	31.5753068	9.9899900	.001003009
998	996004	994011992	31.5911380	9.9933289	.001002004
999	998001	997002999	31.6069613	9.9966656	.001001001
1000	1000000	1000000000	31.6227766	10.0000000	.001000000
1001	1002001	1003003001	31.6385840	10.0033322	.0009990010
1002	1004004	1006012008	31.6543836	10.0066622	.0009980040
1003	1006009	1009027027	31.6701752	10.0099899	.0009970090
1004	1008016	1012048064	31.6859590	10.0133155	.0009960159
1005	1010025	1015075125	31.7017349	10.0166389	.0009950249
1006	1012036	1018108216	31.7175030	10.0199601	.0009940358
1007	1014049	1021147343	31.7332633	10.0232791	.0009930487
1008	1016064	1024192512	31.7490157	10.0265958	.0009920635
1009	1018081	1027243729	31.7647603	10.0299104	.0009910803
1010	1020100	1030301000	31.7804972	10.0332228	.0009900990
1011	1022121	1033364331	31.7962262	10.0365330	.0009891197
1012	1024144	1036433728	31.8119474	10.0398410	.0009881423
1013	1026169	1039509197	31.8276609	10.0431469	.0009871668
1014	1028196	1042590744	31.8433666	10.0464506	.0009861933
1015	1030225	1045678375	31.8590646	10.0497521	.0009852217
1016	1032256	1048772096	31.8747549	10.0530514	.0009842520
1017	1034289	1051871913	31.8904374	10.0563485	.0009832842
1018	1036324	1054977832	31.9061123	10.0596435	.0009823183
1019	1038361	1058089859	31.9217794	10.0629364	.0009813543
1020	1040400	1061208000	31.9374388	10.0662271	.0009803922
1021	1042441	1064332261	31.9530906	10.0695156	.0009794319
1022	1044484	1067462648	31.9687347	10.0728020	.0009784736
1023	1046529	1070599167	31.9843712	10.0760863	.0009775171
1024	1048576	1073741824	32.0000000	10.0793684	.0009765625
1025	1050625	1076890625	32.0156212	10.0826484	.0009756098
1026	1052676	1080045576	32.0312348	10.0859262	.0009746589
1027	1054729	1083206683	32.0468407	10.0892019	.0009737098
1028	1056784	1086373952	32.0624391	10.0924755	.0009727626
1029	1058841	1089547389	32.0780298	10.0957469	.0009718173
1030	1060900	1092727000	32.0936131	10.0990163	.0009708738
1031	1062961	1095912791	32.1091887	10.1022835	.0009699321
1032	1065024	1099104768	32.1247568	10.1055487	.00 9689922
1033	1067089	1102302937	32.1403173	10.1088117	.0009680542
1034	1069156	1105507304	32.1558704	10.1120726	.0009671180
1035	1071225	1108717875	32.1714159	10.1153314	.0009661836
1036	1073296	1111934656	32.1869539	10.1185882	.0009652510
1037	1075369	1115157653	32.2024844	10.1218428	.0009643202
1038	1077444	1118386872	32.2180074	10.1250953	.0009633911
1039	1079521	1121622319	32.2335229	10.1283457	.0009624639
1040	1081600	1124864000	32.2490310	10.1315941	.0009615385
1041	1083681	1128111921	32.2645316	10.1348403	.0009606148
1042	1085764	1131366088	32.2800248	10.1380845	.0009596929
1043	1087849	1134626507	32.2955105	10.1413266	.0009587738
1044	1089936	1137893184	32.3109888	10.1445667	.0009578544
1045	1092025	1141166125	32.3264598	10.1478047	.0009569378
1046	1094116	1144445336	32.3419233	10.1510406	.0009560229
1047	1096209	1147730823	32.3573794	10.1542744	.0009551098
1048	1098304	1151022592	32.3728281	10.1575062	.0009541985
1049	1100401	1154320649	32.3882695	10.1607359	.0009532888
1050	*1102500*	1157625000	32.4037035	10.1639636	.0009523810
1051	*1104601*	1160935651	32.4191301	10.1671893	.0009514748
1052	*1106704*	1164252608	32.4345495	10.1704129	.0009505703
1053	*1108809*	1167575877	32.4499615	10.1736344	.0009496676
1054	*1110916*	1170905464	32.4653662	10.1768539	.0009487666

TABLE XXIV.—LOGARITHMS OF NUMBERS.

No. 100 L. 000.] [No. 109 L. 040.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
100	000000	0434	0868	1301	1734	2166	2598	3029	3461	3891	432
1	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
2	8600	9026	9451	9876	0300	0724	1147	1570	1993	2415	424
3	012837	3259	3680	4100	4521	4940	5360	5779	6197	6616	420
4	7033	7451	7868	8284	8700	9116	9532	9947	0361	0775	416
5	021189	1603	2016	2428	2841	3252	3664	4075	4486	4896	412
6	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
7	9384	9789	0195	0600	1004	1408	1812	2216	2619	3021	404
8	033424	3826	4227	4628	5029	5430	5830	6230	6629	7028	400
9	7426	7825	8223	8620	9017	9414	9811				
	04							0207	0602	0998	397

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
434	43.4	86.8	130.2	173.6	217.0	260.4	303.8	347.2	390.6
433	43.3	86.6	129.9	173.2	216.5	259.8	303.1	346.4	389.7
432	43.2	86.4	129.6	172.8	216.0	259.2	302.4	345.6	388.8
431	43.1	86.2	129.3	172.4	215.5	258.6	301.7	344.8	387.9
430	43.0	86.0	129.0	172.0	215.0	258.0	301.0	344.0	387.0
429	42.9	85.8	128.7	171.6	214.5	257.4	300.3	343.2	386.1
428	42.8	85.6	128.4	171.2	214.0	256.8	299.6	342.4	385.2
427	42.7	85.4	128.1	170.8	213.5	256.2	298.9	341.6	384.3
426	42.6	85.2	127.8	170.4	213.0	255.6	298.2	340.8	383.4
425	42.5	85.0	127.5	170.0	212.5	255.0	297.5	340.0	382.5
424	42.4	84.8	127.2	169.6	212.0	254.4	296.8	339.2	381.6
423	42.3	84.6	126.9	169.2	211.5	253.8	296.1	338.4	380.7
422	42.2	84.4	126.6	168.8	211.0	253.2	295.4	337.6	379.8
421	42.1	84.2	126.3	168.4	210.5	252.6	294.7	336.8	378.9
420	42.0	84.0	126.0	168.0	210.0	252.0	294.0	336.0	378.0
419	41.9	83.8	125.7	167.6	209.5	251.4	293.3	335.2	377.1
418	41.8	83.6	125.4	167.2	209.0	250.8	292.6	334.4	376.2
417	41.7	83.4	125.1	166.8	208.5	250.2	291.9	333.6	375.3
416	41.6	83.2	124.8	166.4	208.0	249.6	291.2	332.8	374.4
415	41.5	83.0	124.5	166.0	207.5	249.0	290.5	332.0	373.5
414	41.4	82.8	124.2	165.6	207.0	248.4	289.8	331.2	372.6
413	41.3	82.6	123.9	165.2	206.5	247.8	289.1	330.4	371.7
412	41.2	82.4	123.6	164.8	206.0	247.2	288.4	329.6	370.8
411	41.1	82.2	123.3	164.4	205.5	246.6	287.7	328.8	369.9
410	41.0	82.0	123.0	164.0	205.0	246.0	287.0	328.0	369.0
409	40.9	81.8	122.7	163.6	204.5	245.4	286.3	327.2	368.1
408	40.8	81.6	122.4	163.2	204.0	244.8	285.6	326.4	367.2
407	40.7	81.4	122.1	162.8	203.5	244.2	284.9	325.6	366.3
406	40.6	81.2	121.8	162.4	203.0	243.6	284.2	324.8	365.4
405	40.5	81.0	121.5	162.0	202.5	243.0	283.5	324.0	364.5
404	40.4	80.8	121.2	161.6	202.0	242.4	282.8	323.2	363.6
403	40.3	80.6	120.9	161.2	201.5	241.8	282.1	322.4	362.7
402	40.2	80.4	120.6	160.8	201.0	241.2	281.4	321.6	361.8
401	40.1	80.2	120.3	160.4	200.5	240.6	280.7	320.8	360.9
400	40.0	80·0	120.0	160.0	200.0	240.0	280.0	320.0	360.0
399	39.9	79.8	119.7	159.6	199.5	239.4	279.3	319.2	359.
398	39.8	79.6	119.4	159.2	199.0	238.8	278.6	318.4	35
397	39.7	79.4	119.1	158.8	198.5	238.2	277.9	317.6	
396	39.6	79.2	118.8	158.4	198.0	237.6	277.2	316.8	
395	39.5	79.0	118.5	158.0	197.5	237.0	276.5	316.0	

N.	0	1	2	3	4	5	6	7	8	9	Diff.
110	041393	1787	2182	2576	2969	3362	3755	4148	4540	4932	393
1	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	390
2	9218	9606	9993	0380	0766	1153	1538	1924	2309	2694	386
3	053078	3463	3846	4230	4613	4996	5378	5760	6142	6524	383
4	6905	7286	7666	8046	8426	8805	9185	9563	9942	0320	379
5	060698	1075	1452	1829	2206	2582	2958	3333	3709	4083	376
6	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	373
7	8186	8557	8928	9298	9668	0038	0407	0776	1145	1514	370
8	071882	2250	2617	2985	3352	3718	4085	4451	4816	5182	366
9	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
395	39.5	79.0	118.5	158.0	197.5	237.0	276.5	316.0	355.5
394	39.4	78.8	118.2	157.6	197.0	236.4	275.8	315.2	354.6
393	39.3	78.6	117.9	157.2	196.5	235.8	275.1	314.4	353.7
392	39.2	78.4	117.6	156.8	196.0	235.2	274.4	313.6	352.8
391	39.1	78.2	117.3	156.4	195.5	234.6	273.7	312.8	351.9
390	39.0	78.0	117.0	156.0	195.0	234.0	273.0	312.0	351.0
389	38.9	77.8	116.7	155.6	194.5	233.4	272.3	311.2	350.1
388	38.8	77.6	116.4	155.2	194.0	232.8	271.6	310.4	349.2
387	38.7	77.4	116.1	154.8	193.5	232.2	270.9	309.6	348.3
386	38.6	77.2	115.8	154.4	193.0	231.6	270.2	308.8	347.4
385	38.5	77.0	115.5	154.0	192.5	231.0	269.5	308.0	346.5
384	38.4	76.8	115.2	153.6	192.0	230.4	268.8	307.2	345.6
383	38.3	76.6	114.9	153.2	191.5	229.8	268.1	306.4	344.7
382	38.2	76.4	114.6	152.8	191.0	229.2	267.4	305.6	343.8
381	38.1	76.2	114.3	152.4	190.5	228.6	266.7	304.8	342.9
380	38.0	76.0	114.0	152.0	190.0	228.0	266.0	304.0	342.0
379	37.9	75.8	113.7	151.6	189.5	227.4	265.3	303.2	341.1
378	37.8	75.6	113.4	151.2	189.0	226.8	264.6	302.4	340.2
377	37.7	75.4	113.1	150.8	188.5	226.2	263.9	301.6	339.3
376	37.6	75.2	112.8	150.4	188.0	225.6	263.2	300.8	338.4
375	37.5	75.0	112.5	150.0	187.5	225.0	262.5	300.0	337.5
374	37.4	74.8	112.2	149.6	187.0	224.4	261.8	299.2	336.6
373	37.3	74.6	111.9	149.2	186.5	223.8	261.1	298.4	335.7
372	37.2	74.4	111.6	148.8	186.0	223.2	260.4	297.6	334.8
371	37.1	74.2	111.3	148.4	185.5	222.6	259.7	296.8	333.9
370	37.0	74.0	111.0	148.0	185.0	222.0	259.0	296.0	333.0
369	36.9	73.8	110.7	147.6	184.5	221.4	258.3	295.2	332.1
368	36.8	73.6	110.4	147.2	184.0	220.8	257.6	294.4	331.2
367	36.7	73.4	110.1	146.8	183.5	220.2	256.9	293.6	330.3
366	36.6	73.2	109.8	146.4	183.0	219.6	256.2	292.8	329.4
365	36.5	73.0	109.5	146.0	182.5	219.0	255.7	292.0	328.5
364	36.4	72.8	109.2	145.6	182.0	218.4	254.8	291.2	327.6
363	36.3	72.6	108.9	145.2	181.5	217.8	254.1	290.4	326.7
362	36.2	72.4	108.6	144.8	181.0	217.2	253.4	289.6	325.8
361	36.1	72.2	108.3	144.4	180.5	216.6	252.7	288.8	324.9
360	36.0	72.0	108.0	144.0	180.0	216.0	252.0	288.0	324.0
359	35.9	71.8	107.7	143.6	179.5	215.4	251.3	287.2	323.1
358	35.8	71.6	107.4	143.2	179.0	214.8	250.6	286.4	322.2
357	35.7	71.4	107.1	142.8	178.5	214.2	249.9	285.6	321.3
56	35.6	71.2	106.8	142.4	178.0	213.6	249.2	284.8	320

N.	0	1	2	3	4	5	6	7	8	9	Diff.
120	079181	9543	9904	0266	0626	0987	1347	1707	2067	2426	360
1	082785	3144	3503	3861	4219	4576	4934	5291	5647	6004	357
2	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
3	9905	0258	0611	0963	1315	1667	2018	2370	2721	3071	352
4	093422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
5	6910	7257	7604	7951	8298	8644	8990	9335	9681	0026	346
6	100371	0715	1059	1403	1747	2091	2434	2777	3119	3462	343
7	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	341
8	7210	7549	7888	8227	8565	8903	9241	9579	9916	0253	338
9	110590	0926	1263	1599	1934	2270	2605	2940	3275	3609	335
130	3943	4277	4611	4944	5278	5611	5943	6276	6608	6940	333
1	7271	7603	7934	8265	8595	8926	9256	9586	9915	0245	330
2	120574	0903	1231	1560	1888	2216	2544	2871	3198	3525	328
3	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
4	7105 13	7429	7753	8076	8399	8722	9045	9368	9690	0012	323

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
355	35.5	71.0	106.5	142.0	177.5	213.0	248.5	284.0	319.5
354	35.4	70.8	106.2	141.6	177.0	212.4	247.8	283.2	318.6
353	35.3	70.6	105.9	141.2	176.5	211.8	247.1	282.4	317.7
352	35.2	70.4	105.6	140.8	176.0	211.2	246.4	281.6	316.8
351	35.1	70.2	105.3	140.4	175.5	210.6	245.7	280.8	315.9
350	35.0	70.0	105.0	140.0	175.0	210.0	245.0	280.0	315.0
349	34.9	69.8	104.7	139.6	174.5	209.4	244.3	279.2	314.1
348	34.8	69.6	104.4	139.2	174.0	208.8	243.6	278.4	313.2
347	34.7	69.4	104.1	138.8	173.5	208.2	242.9	277.6	312.3
346	34.6	69.2	103.8	138.4	173.0	207.6	242.2	276.8	311.4
345	34.5	69.0	103.5	138.0	172.5	207.0	241.5	276.0	310.5
344	34.4	68.8	103.2	137.6	172.0	206.4	240.8	275.2	309.6
343	34.3	68.6	102.9	137.2	171.5	205.8	240.1	274.4	308.7
342	34.2	68.4	102.6	136.8	171.0	205.2	239.4	273.6	307.8
341	34.1	68.2	102.3	136.4	170.5	204.6	238.7	272.8	306.9
340	34.0	68.0	102.0	136.0	170.0	204.0	238.0	272.0	306.0
339	33.9	67.8	101.7	135.6	169.5	203.4	237.3	271.2	305.1
338	33.8	67.6	101.4	135.2	169.0	202.8	236.6	270.4	304.2
337	33.7	67.4	101.1	134.8	168.5	202.2	235.9	269.6	303.3
336	33.6	67.2	100.8	134.4	168.0	201.6	235.2	268.8	302.4
335	33.5	67.0	100.5	134.0	167.5	201.0	234.5	268.0	301.5
334	33.4	66.8	100.2	133.6	167.0	200.4	233.8	267.2	300.6
333	33.3	66.6	99.9	133.2	166.5	199.8	233.1	266.4	299.7
332	33.2	66.4	99.6	132.8	166.0	199.2	232.4	265.6	298.8
331	33.1	66.2	99.3	132.4	165.5	198.6	231.7	264.8	297.9
330	33.0	66.0	99.0	132.0	165.0	198.0	231.0	264.0	297.0
329	32.9	65.8	98.7	131.6	164.5	197.4	230.3	263.2	296.1
328	32.8	65.6	98.4	131.2	164.0	196.8	229.6	262.4	295.2
327	32.7	65.4	98.1	130.8	163.5	196.2	228.9	261.6	294.3
326	32.6	65.2	97.8	130.4	163.0	195.6	228.2	260.8	293.4
325	32.5	65.0	97.5	130.0	162.5	195.0	227.5	260.0	292.5
324	32.4	64.8	97.2	129.6	162.0	194.4	226.8	259.2	291.
323	32.3	64.6	96.9	129.2	161.5	193.8	226.1	258.4	290
322	32.2	64.4	96.6	128.8	161.0	193.2	225.4	257.6	2

N.	0	1	2	3	4	5	6	7	8	9	Diff.
135	130334	0655	0977	1298	1619	1939	2260	2580	2900	3219	321
6	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
7	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	316
8	9879	0194	0508	0822	1136	1450	1763	2076	2389	2702	314
9	143015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
140	6128	6438	6748	7058	7367	7676	7985	8294	8603	8911	309
1	9219	9527	9835	0142	0449	0756	1063	1370	1676	1982	307
2	152288	2594	2900	3205	3510	3815	4120	4424	4728	5032	305
3	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
4	8362	8664	8965	9266	9567	9868	0168	0469	0769	1068	301
5	161368	1667	1967	2266	2564	2863	3161	3460	3758	4055	299
6	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
7	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
8	170262	0555	0848	1141	1434	1726	2019	2311	2603	2895	293
9	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
321	32.1	64.2	96.3	128.4	160.5	192.6	224.7	256.8	288.9
320	32.0	64.0	96.0	128.0	160.0	192.0	224.0	256.0	288.0
319	31.9	63.8	95.7	127.6	159.5	191.4	223.3	255.2	287.1
318	31.8	63.6	95.4	127.2	159.0	190.8	222.6	251.4	286.2
317	31.7	63.4	95.1	126.8	158.5	190.2	221.9	253.6	285.3
316	31.6	63.2	94.8	126.4	158.0	189.6	221.2	252.8	284.4
315	31.5	63.0	94.5	126.0	157.5	189.0	220.5	252.0	283.5
314	31.4	62.8	94.2	125.6	157.0	188.4	219.8	251.2	282.6
313	31.3	62.6	93.9	125.2	156.5	187.8	219.1	250.4	281.7
312	31.2	62.4	93.6	124.8	156.0	187.2	218.4	249.6	280.8
311	31.1	62.2	93.3	124.4	155.5	186.6	217.7	248.8	279.9
310	31.0	62.0	93.0	124.0	155.0	186.0	217.0	248.0	279.0
309	30.9	61.8	92.7	123.6	154.5	185.4	216.3	247.2	278.1
308	30.8	61.6	92.4	123.2	154.0	184.8	215.6	246.4	277.2
307	30.7	61.4	92.1	122.8	153.5	184.2	214.9	245.6	276.3
306	30.6	61.2	91.8	122.4	153.0	183.6	214.2	244.8	275.4
305	30.5	61.0	91.5	122.0	152.5	183.0	213.5	244.0	274.5
304	30.4	60.8	91.2	121.6	152.0	182.4	212.8	243.2	273.6
303	30.3	60.6	90.9	121.2	151.5	181.8	212.1	242.4	272.7
302	30.2	60.4	90.6	120.8	151.0	181.2	211.4	241.6	271.8
301	30.1	60.2	90.3	120.4	150.5	180.6	210.7	240.8	270.9
300	30.0	60.0	90.0	120.0	150.0	180.0	210.0	240.0	270.0
299	29.9	59.8	89.7	119.6	149.5	179.4	209.3	239.2	269.1
298	29.8	59.6	89.4	119.2	149.0	178.8	208.6	238.4	268.2
297	29.7	59.4	89.1	118.8	148.5	178.2	207.9	237.6	267.3
296	29.6	59.2	88.8	118.4	148.0	177.6	207.2	236.8	266.4
295	29.5	59.0	88.5	118.0	147.5	177.0	206.5	236.0	265.5
294	29.4	58.8	88.2	117.6	147.0	176.4	205.8	235.2	264.6
293	29.3	58.6	87.9	117.2	146.5	175.8	205.1	234.4	263.7
292	29.2	58.4	87.6	116.8	146.0	175.2	204.4	233.6	262.8
291	29.1	58.2	87.3	116.4	145.5	174.6	203.7	232.8	261.9
290	29.0	58.0	87.0	116.0	145.0	174.0	203.0	232.0	261.0
289	28.9	57.8	86.7	115.6	144.5	173.4	202.3	231.2	260.1
288	28.8	57.6	86.4	115.2	144.0	172.8	201.6	230.4	259.2
287	28.7	57.4	86.1	114.8	143.5	172.2	200.9	229.6	258.
286	28.6	57.2	85.8	114.4	143.0	171.6	200.2	228.8	257

N.	0	1	2	3	4	5	6	7	8	9	Diff.
150	176091	6381	6670	6959	7248	7536	7825	8113	8401	8689	289
1	8977	9264	9552	9839							
					0126	0413	0699	0986	1272	1558	287
2	181844	2129	2415	2700	2985	3270	3555	3839	4123	4407	285
3	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
4	7521	7803	8084	8366	8647	8928	9209	9490	9771		
										0051	281
5	190332	0612	0892	1171	1451	1730	2010	2289	2567	2846	279
6	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
7	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
8	8657	8932	9206	9481	9755						
						0029	0303	0577	0850	1124	274
9	201397	1670	1943	2216	2488	2761	3033	3305	3577	3848	272
160	4120	4391	4663	4934	5204	5475	5746	6016	6286	6556	271
1	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
2	9515	9783									
			0051	0319	0586	0853	1121	1388	1654	1921	267
3	212188	2454	2720	2986	3252	3518	3783	4049	4314	4579	266
4	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
5	7484	7747	8010	8273	8536	8798	9060	9323	9585	9846	262
6	220108	0370	0631	0892	1153	1414	1675	1936	2196	2456	261
7	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
8	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
9	7887	8144	8400	8657	8913	9170	9426	9682	9938		
	23									0193	256

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
285	28.5	57.0	85.5	114.0	142.5	171.0	199.5	228.0	256.5
284	28.4	56.8	85.2	113.6	142.0	170.4	198.8	227.2	255.6
283	28.3	56.6	84.9	113.2	141.5	169.8	198.1	226.4	254.7
282	28.2	56.4	84.6	112.8	141.0	169.2	197.4	225.6	253.8
281	28.1	56.2	84.3	112 4	140.5	168.6	196.7	224.8	252.9
280	28.0	56.0	84.0	112.0	140.0	168.0	196.0	224.0	252.0
279	27.9	55.8	83.7	111.6	139.5	167.4	195.3	223.2	251.1
278	27.8	55.6	83.4	111.2	139.0	166.8	194.6	222.4	250.2
277	27.7	55.4	83.1	110.8	138.5	166.2	193.9	221.6	249.3
276	27.6	55.2	82.8	110.4	138.0	165.6	193.2	220.8	248.4
275	27.5	55.0	82.5	110.0	137.5	165.0	192.5	220.0	247.5
274	27.4	54.8	82.2	109.6	137.0	164.4	191.8	219.2	246.6
273	27.3	54.6	81.9	109.2	136.5	163.8	191.1	218.4	245.7
272	27.2	54.4	81.6	108.8	136.0	163.2	190.4	217.6	244.8
271	27.1	54.2	81.3	108.4	135.5	162.6	189.7	216.8	243.9
270	27.0	54.0	81.0	108.0	135.0	162.0	189.0	216.0	243.0
269	26.9	53.8	80.7	107.6	134.5	161.4	188.3	215.2	242.1
268	26.8	53.6	80.4	107.2	134.0	160.8	187.6	214.4	241.2
267	26.7	53.4	80.1	106.8	133.5	160.2	186.9	213.6	240.3
266	26.6	53.2	79.8	106.4	133.0	159.6	186.2	212.8	239.4
265	26.5	53.0	79.5	106.0	132.5	159.0	185.5	212.0	238.5
264	26.4	52.8	79.2	105.6	132.0	158.4	184.8	211.2	237.6
263	26.3	52.6	78.9	105.2	131.5	157.8	184.1	210.4	236.7
262	26.2	52.4	78.6	104.8	131.0	157.2	183.4	209.6	235.8
261	26.1	52.2	78.3	104.4	130.5	156.6	182.7	208.8	234.9
260	26.0	52.0	78.0	104.0	130.0	156.0	182.0	208.0	234.0
259	25.9	51.8	77.7	103.6	129.5	155.4	181.3	207.2	233.1
258	25.8	51.6	77.4	103.2	129.0	154.8	180.6	206.4	232.2
257	25.7	51.4	77.1	102.8	128.5	154.2	179.9	205.6	231
256	25.6	51.2	76.8	102.4	128.0	153.6	179.2	204.8	23
255	25.5	51.0	76.5	102.0	127.5	153.0	178.5	204.0	2

TABLE XXIV.—LOGARITHMS OF NUMBERS.

No. 170 L. 230.] [No. 189 L. 278

N.	0	1	2	3	4	5	6	7	8	9	Diff.
170	230449	0704	0960	1215	1470	1724	1979	2234	2488	2742	255
1	2996	3250	3504	3757	4011	4264	4517	4770	5023	5276	253
2	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
3	8046	8297	8548	8799	9049	9299	9550	9800	0050	0300	250
4	240549	0799	1048	1297	1546	1795	2044	2293	2541	2790	249
5	3038	3286	3534	3782	4030	4277	4525	4772	5019	5266	248
6	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
7	7973	8219	8464	8709	8954	9198	9443	9687	9932	0176	245
8	250420	0664	0908	1151	1395	1638	1881	2125	2368	2610	243
9	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
180	5273	5514	5755	5996	6237	6477	6718	6958	7198	7439	241
1	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
2	260071	0310	0548	0787	1025	1263	1501	1739	1976	2214	238
3	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
4	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
5	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279	234
6	9513	9746	9980	0213	0446	0679	0912	1144	1377	1600	233
7	271842	2074	2306	2538	2770	3001	3233	3464	3696	3927	232
8	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
9	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
255	25.5	51.0	76.5	102.0	127.5	153.0	178.5	204.0	229.5
254	25.4	50.8	76.2	101.6	127.0	152.4	177.8	203.2	228.6
253	25.3	50.6	75.9	101.2	126.5	151.8	177.1	202.4	227.7
252	25.2	50.4	75.6	100.8	126.0	151.2	176.4	201.6	226.8
251	25.1	50.2	75.3	100.4	125.5	150.6	175.7	200.8	225.9
250	25.0	50.0	75.0	100.0	125.0	150.0	175.0	200.0	225.0
249	24.9	49.8	74.7	99.6	124.5	149.4	174.3	199.2	224.1
248	24.8	49.6	74.4	99.2	124.0	148.8	173.6	198.4	223.2
247	24.7	49.4	74.1	98.8	123.5	148.2	172.9	197.6	222.3
246	24.6	49.2	73.8	98.4	123.0	147.6	172.2	196.8	221.4
245	24.5	49.0	73.5	98.0	122.5	147.0	171.5	196.0	220.5
244	24.4	48.8	73.2	97.6	122.0	146.4	170.8	195.2	219.6
243	24.3	48.6	72.9	97.2	121.5	145.8	170.1	194.4	218.7
242	24.2	48.4	72.6	96.8	121.0	145.2	169.4	193.6	217.8
241	24.1	48.2	72.3	96.4	120.5	144.6	168.7	192.8	216.9
240	24.0	48.0	72.0	96.0	120.0	144.0	168.0	192.0	216.0
239	23.9	47.8	71.7	95.6	119.5	143.4	167.3	191.2	215.1
238	23.8	47.6	71.4	95.2	119.0	142.8	166.6	190.4	214.2
237	23.7	47.4	71.1	94.8	118.5	142.2	165.9	189.6	213.3
236	23.6	47.2	70.8	94.4	118.0	141.6	165.2	188.8	212.4
235	23.5	47.0	70.5	94.0	117.5	141.0	164.5	188.0	211.5
234	23.4	46.8	70.2	93.6	117.0	140.4	163.8	187.2	210.6
233	23.3	46.6	69.9	93.2	116.5	139.8	163.1	186.4	209.7
232	23.2	46.4	69.6	92.8	116.0	139.2	162.4	185.6	208.8
231	23.1	46.2	69.3	92.4	115.5	138.6	161.7	184.8	207.9
230	23.0	46.0	69.0	92.0	115.0	138.0	161.0	184.0	207.0
229	22.9	45.8	68.7	91.6	114.5	137.4	160.3	183.2	206
228	22.8	45.6	68.4	91.2	114.0	136.8	159.6	182.4	205
227	22.7	45.4	68.1	90.8	113.5	136.2	158.9	181.6	20
226	22.6	45.2	67.8	90.4	113.0	135.6	158.2	180.8	2

N.	0	1	2	3	4	5	6	7	8	9	Diff.
190	278754	8982	9211	9439	9667	9895	0123	0351	0578	0806	228
1	281033	1261	1488	1715	1942	2169	2396	2622	2849	3075	227
2	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
3	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
4	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
5	290035	0257	0480	0702	0925	1147	1369	1591	1813	2034	222
6	2256	2478	2699	2920	3141	3363	3584	3804	4025	4246	221
7	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
8	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
9	8853	9071	9289	9507	9725	9943	0161	0378	0595	0813	218
200	301030	1247	1464	1681	1898	2114	2331	2547	2764	2980	217
1	3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
2	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	215
3	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417	213
4	9630	9843	0056	0268	0481	0693	0906	1118	1330	1542	212
5	311754	1966	2177	2389	2600	2812	3023	3234	3445	3656	211
6	3867	4078	4289	4499	4710	4920	5130	5340	5551	5760	210
7	5970	6180	6390	6599	6809	7018	7227	7436	7646	7854	209
8	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
9	320146	0354	0562	0769	0977	1184	1391	1598	1805	2012	207
210	2219	2426	2633	2839	3046	3252	3458	3665	3871	4077	206
1	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131	205
2	6336	6541	6745	6950	7155	7359	7563	7767	7972	8176	204
3	8380	8583	8787	8991	9194	9398	9601	9805	0008	0211	203
4	330414	0617	0819	1022	1225	1427	1630	1832	2034	2236	202

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
225	22.5	45.0	67.5	90.0	112.5	135.0	157.5	180.0	202.5
224	22.4	44.8	67.2	89.6	112.0	134.4	156.8	179.2	201.6
223	22.3	44.6	66.9	89.2	111.5	133.8	156.1	178.4	200.7
222	22.2	44.4	66.6	88.8	111.0	133.2	155.4	177.6	199.8
221	22.1	44.2	66.3	88.4	110.5	132.6	154.7	176.8	198.9
220	22.0	44.0	66.0	88.0	110.0	132.0	154.0	176.0	198.0
219	21.9	43.8	65.7	87.6	109.5	131.4	153.3	175.2	197.1
218	21.8	43.6	65.4	87.2	109.0	130.8	152.6	174.4	196.2
217	21.7	43.4	65.1	86.8	108.5	130.2	151.9	173.6	195.3
216	21.6	43.2	64.8	86.4	108.0	129.6	151.2	172.8	194.4
215	21.5	43.0	64.5	86.0	107.5	129.0	150.5	172.0	193.5
214	21.4	42.8	64.2	85.6	107.0	128.4	149.8	171.2	192.6
213	21.3	42.6	63.9	85.2	106.5	127.8	149.1	170.4	191.7
212	21.2	42.4	63.6	84.8	106.0	127.2	148.4	169.6	190.8
211	21.1	42.2	63.3	84.4	105.5	126.6	147.7	168.8	189.9
210	21.0	42.0	63.0	84.0	105.0	126.0	147.0	168.0	189.0
209	20.9	41.8	62.7	83.6	104.5	125.4	146.3	167.2	188.1
208	20.8	41.6	62.4	83.2	104.0	124.8	145.6	166.4	187.2
207	20.7	41.4	62.1	82.8	103.5	124.2	144.9	165.6	186.3
206	20.6	41.2	61.8	82.4	103.0	123.6	144.2	164.8	185.
205	20.5	41.0	61.5	82.0	102.5	123.0	143.5	164.0	184
204	20.4	40.8	61.2	81.6	102.0	122.4	142.8	163.2	18
203	20.3	40.6	60.9	81.2	101.5	121.8	142.1	162.4	
202	20.2	40.4	60.6	80.8	101.0	121.2	141.4	161.6	

TABLE XXIV.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
215	332438	2640	2842	3044	3246	3447	3649	3850	4051	4253	202
6	4454	4655	4856	5057	5257	5458	5658	5859	6059	6260	201
7	6460	6660	6860	7060	7260	7459	7659	7858	8058	8257	200
8	8456	8656	8855	9054	9253	9451	9650	9849			
									0047	0246	199
9	340444	0642	0841	1039	1237	1435	1632	1830	2028	2225	198
220	2423	2620	2817	3014	3212	3409	3606	3802	3999	4196	197
1	4392	4589	4785	4981	5178	5374	5570	5766	5962	6157	196
2	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
3	8305	8500	8694	8889	9083	9278	9472	9666	9860		
										0054	194
4	350248	0442	0636	0829	1023	1216	1410	1603	1796	1989	193
5	2183	2375	2568	2761	2954	3147	3339	3532	3724	3916	193
6	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	192
7	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	191
8	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	190
9	9835										
		0025	0215	0404	0593	0783	0972	1161	1350	1539	189
230	361728	1917	2105	2294	2482	2671	2859	3048	3236	3424	188
1	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301	188
2	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	187
3	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
4	9216	9401	9587	9772	9958						
						0143	0328	0513	0698	0883	185
5	371068	1253	1437	1622	1806	1991	2175	2360	2544	2728	184
6	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	184
7	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
8	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
9	8398	8580	8761	8943	9124	9306	9487	9668	9849		
	38									0030	181

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
202	20.2	40.4	60.6	80.8	101.0	121.2	141.4	161.6	181.8
201	20.1	40.2	60.3	80.4	100.5	120.6	140.7	160.8	180.9
200	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0	180.0
199	19.9	39.8	59.7	79.6	99.5	119.4	139.3	159.2	179.1
198	19.8	39.6	59.4	79.2	99.0	118.8	138.6	158.4	178.2
197	19.7	39.4	59.1	78.8	98.5	118.2	137.9	157.6	177.3
196	19.6	39.2	58.8	78.4	98.0	117.6	137.2	156.8	176.4
195	19.5	39.0	58.5	78.0	97.5	117.0	136.5	156.0	175.5
194	19.4	38.8	58.2	77.6	97.0	116.4	135.8	155.2	174.6
193	19.3	38.6	57.9	77.2	96.5	115.8	135.1	154.4	173.7
192	19.2	38.4	57.6	76.8	96.0	115.2	134.4	153.6	172.8
191	19.1	38.2	57.3	76.4	95.5	114.6	133.7	152.8	171.9
190	19.0	38.0	57.0	76.0	95.0	114.0	133.0	152.0	171.0
189	18.9	37.8	56.7	75.6	94.5	113.4	132.3	151.2	170.1
188	18.8	37.6	56.4	75.2	94.0	112.8	131.6	150.4	169.2
187	18.7	37.4	56.1	74.8	93.5	112.2	130.9	149.6	168.3
186	18.6	37.2	55.8	74.4	93.0	111.6	130.2	148.8	167.4
185	18.5	37.0	55.5	74.0	92.5	111.0	129.5	148.0	166.5
184	18.4	36.8	55.2	73.6	92.0	110.4	128.8	147.2	165.6
183	18.3	36.6	54.9	73.2	91.5	109.8	128.1	146.4	164.7
182	18.2	36.4	54.6	72.8	91.0	109.2	127.4	145.6	163.8
181	18.1	36.2	54.3	72.4	90.5	108.6	126.7	144.8	162.9
180	18.0	36.0	54.0	72.0	90.0	108.0	126.0	144.0	162.
179	17.9	35.8	53.7	71.6	89.5	107.4	125.3	143.2	161

TABLE XXIV.—LOGARITHMS OF NUMBERS.

No. 240 L. 380.] [No. 269 L. 431.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
240	380211	0392	0573	0754	0934	1115	1296	1476	1656	1837	181
1	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	180
2	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428	179
3	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
4	7390	7568	7746	7924	8101	8279	8456	8634	8811	8989	178
5	9166	9343	9520	9698	9875	0051	0228	0405	0582	0759	177
6	390935	1112	1288	1464	1641	1817	1993	2169	2345	2521	176
7	2697	2873	3048	3224	3400	3575	3751	3926	4101	4277	176
8	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
9	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
250	7940	8114	8287	8461	8634	8808	8081	9154	9328	9501	173
1	9674	9847	0020	0192	0365	0538	0711	0883	1056	1228	173
2	401401	1573	1745	1917	2089	2261	2433	2605	2777	2949	172
3	3121	3292	3464	3635	3807	3978	4149	4320	4492	4663	171
4	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
5	6540	6710	6881	7051	7221	7391	7561	7731	7901	8070	170
6	8240	8410	8579	8749	8918	9087	9257	9426	9595	9764	169
7	9933	0102	0271	0440	0609	0777	0946	1114	1283	1451	169
8	411620	1788	1956	2124	2293	2461	2629	2796	2964	3132	168
9	3300	3467	3635	3803	3970	4137	4305	4472	4639	4806	167
260	4973	5140	5307	5474	5641	5808	5974	6141	6308	6474	167
1	6641	6807	6973	7139	7306	7472	7638	7804	7970	8135	166
2	8301	8467	8633	8798	8964	9129	9295	9460	9625	9791	165
3	9956	0121	0286	0451	0616	0781	0945	1110	1275	1439	165
4	421604	1768	1933	2097	2261	2426	2590	2754	2918	3082	164
5	3246	3410	3574	3737	3901	4065	4228	4392	4555	4718	164
6	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349	163
7	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	162
8	8135	8297	8459	8621	8783	8944	9106	9268	9429	9591	162
9	9752 43	9914	0075	0236	0398	0559	0720	0881	1042	1203	161

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
178	17.8	35.6	53.4	71.2	89.0	106.8	124.6	142.4	160.2
177	17.7	35.4	53.1	70.8	88.5	106.2	123.9	141.6	159.3
176	17.6	35.2	52.8	70.4	88.0	105.6	123.2	140.8	158.4
175	17.5	35.0	52.5	70.0	7.5	105.0	122.5	140.0	157.5
174	17.4	34.8	52.2	69.6	87.0	104.4	121.8	139.2	156.6
173	17.3	34.6	51.9	69.2	86.5	103.8	121.1	138.4	155.7
172	17.2	34.4	51.6	68.8	86.0	103.2	120.4	137.6	154.8
171	17.1	34.2	51.3	68.4	85.5	102.6	119.7	136.8	153.9
170	17.0	34.0	51.0	68.0	85.0	102.0	119.0	136.0	153.0
169	16.9	33.8	50.7	67.6	84.5	101.4	118.3	135.2	152.1
168	16.8	33.6	50.4	67.2	84.0	100.8	117.6	134.4	151.2
167	16.7	33.4	50.1	66.8	83.5	100.2	116.9	133.6	150.3
166	16.6	33.2	49.8	66.4	83.0	99.6	116.2	132.8	149.4
165	16.5	33.0	49.5	66.0	82.5	99.0	115.5	132.0	148.5
164	16.4	32.8	49.2	65.6	82.0	98.4	114.8	131.2	147.6
163	16.3	32.6	48.9	65.2	81.5	97.8	114.1	130.4	146.
162	16.2	32.4	48.5	64.8	81.0	97.2	113.4	129.6	14
161	16.1	32.2	48.3	64.4	80.5	96.6	112.7	128.8	1

N.	0	1	2	3	4	5	6	7	8	9	Diff.
270	431364	1525	1685	1846	2007	2167	2328	2488	2649	2809	161
1	2969	3130	3290	3450	3610	3770	3930	4090	4249	4409	160
2	4569	4729	4888	5048	5207	5367	5526	5685	5844	6004	159
3	6163	6322	6481	6640	6799	6957	7116	7275	7433	7592	159
4	7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	158
5	9333	9491	9648	9806	9964	0122	0279	0437	0594	0752	158
6	440909	1066	1224	1381	1538	1695	1852	2009	2166	2323	157
7	2480	2637	2793	2950	3106	3263	3419	3576	3732	3889	157
8	4045	4201	4357	4513	4669	4825	4981	5137	5293	5449	156
9	5604	5760	5915	6071	6226	6382	6537	6692	6848	7003	155
280	7158	7313	7468	7623	7778	7933	8088	8242	8397	8552	155
1	8706	8861	9015	9170	9324	9478	9633	9787	9941	0095	154
2	450249	0403	0557	0711	0865	1018	1172	1326	1479	1633	154
3	1786	1940	2093	2247	2400	2553	2706	2859	3012	3165	153
4	3318	3471	3624	3777	3930	4082	4235	4387	4540	4692	153
5	4845	4997	5150	5302	5454	5606	5758	5910	6062	6214	152
6	6366	6518	6670	6821	6973	7125	7276	7428	7579	7731	152
7	7882	8033	8184	8336	8487	8638	8789	8940	9091	9242	151
8	9392	9543	9694	9845	9995	0146	0296	0447	0597	0748	151
9	460898	1048	1198	1348	1499	1649	1799	1948	2098	2248	150
290	2398	2548	2697	2847	2997	3146	3296	3445	3594	3744	150
1	3893	4042	4191	4340	4490	4639	4788	4936	5085	5234	149
2	5383	5532	5680	5829	5977	6126	6274	6423	6571	6719	149
3	6868	7016	7164	7312	7460	7608	7756	7904	8052	8200	148
4	8347	8495	8643	8790	8938	9085	9233	9380	9527	9675	148
5	9822	9969	0116	0263	0410	0557	0704	0851	0998	1145	147
6	471292	1438	1585	1732	1878	2025	2171	2318	2464	2610	146
7	2756	2903	3049	3195	3341	3487	3633	3779	3925	4071	146
8	4216	4362	4508	4653	4799	4944	5090	5235	5381	5526	146
9	5671	5816	5962	6107	6252	6397	6542	6687	6832	6976	145

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
161	16.1	32.2	48.3	64.4	80.5	96.6	112.7	128.8	144.9
160	16.0	32.0	48.0	64.0	80.0	96.0	112.0	128.0	144.0
159	15.9	31.8	47.7	63.6	79.5	95.4	111.3	127.2	143.1
158	15.8	31.6	47.4	63.2	79.0	94.8	110.6	126.4	142.2
157	15.7	31.4	47.1	62.8	78.5	94.2	109.9	125.6	141.3
156	15.6	31.2	46.8	62.4	78.0	93.6	109.2	124.8	140.4
155	15.5	31.0	46.5	62.0	77.5	93.0	108.5	124.0	139.5
154	15.4	30.8	46.2	61.6	77.0	92.4	107.8	123.2	138.6
153	15.3	30.6	45.9	61.2	76.5	91.8	107.1	122.4	137.7
152	15.2	30.4	45.6	60.8	76.0	91.2	106.4	121.6	136.8
151	15.1	30.2	45.3	60.4	75.5	90.6	105.7	120.8	135.9
150	15.0	30.0	45.0	60.0	75.0	90.0	105.0	120.0	135.0
149	14.9	29.8	44.7	59.6	74.5	89.4	104.3	119.2	134.1
148	14.8	29.6	44.4	59.2	74.0	88.8	103.6	118.4	133.2
147	14.7	29.4	44.1	58.8	73.5	88.2	102.9	117.6	132.3
146	14.6	29.2	43.8	58.4	73.0	87.6	102.2	116.8	131.4
145	*14.5*	29.0	43.5	58.0	72.5	87.0	101.5	116.0	130.5
144	*14.4*	28.8	43.2	57.6	72.0	86.4	100.8	115.2	129.6
143	*14.3*	28.6	42.9	57.2	71.5	85.8	100.1	114.4	128.7
142	*14.2*	28.4	42.6	56.8	71.0	85.2	99.4	113.6	127.8
141	*14.1*	28.2	42.3	56.4	70.5	84.6	98.7	112.8	126.9
140	*14.0*	28.0	42.0	56.0	70.0	84.0	98.0	112.0	126.

TABLE XXIV.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
300	477121	7266	7411	7555	7700	7844	7989	8133	8278	8422	145
1	8566	8711	8855	8999	9143	9287	9431	9575	9719	9863	144
2	480007	0151	0294	0438	0582	0725	0869	1012	1156	1299	144
3	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
4	2874	3016	3159	3302	3445	3587	3730	3872	4015	4157	143
5	4300	4442	4585	4727	4869	5011	5153	5295	5437	5579	142
6	5721	5863	6005	6147	6289	6430	6572	6714	6855	6997	142
7	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410	141
8	8551	8692	8833	8974	9114	9255	9396	9537	9677	9818	141
9	9958	0099	0239	0380	0520	0661	0801	0941	1081	1222	140
310	491362	1502	1642	1782	1922	2062	2201	2341	2481	2621	140
1	2760	2900	3040	3179	3319	3458	3597	3737	3876	4015	139
2	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406	139
3	5544	5683	5822	5960	6099	6238	6376	6515	6653	6791	139
4	6930	7068	7206	7344	7483	7621	7759	7897	8035	8173	138
5	8311	8448	8586	8724	8862	8999	9137	9275	9412	9550	138
6	9687	9824	9962	0099	0236	0374	0511	0648	0785	0922	137
7	501059	1196	1333	1470	1607	1744	1880	2017	2154	2291	137
8	2427	2564	2700	2837	2973	3109	3246	3382	3518	3655	136
9	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014	136
320	5150	5286	5421	5557	5693	5828	5964	6099	6234	6370	136
1	6505	6640	6776	6911	7046	7181	7316	7451	7586	7721	135
2	7856	7991	8126	8260	8395	8530	8664	8799	8934	9068	135
3	9203	9337	9471	9606	9740	9874	0009	0143	0277	0411	134
4	510545	0679	0813	0947	1081	1215	1349	1482	1616	1750	134
5	1883	2017	2151	2284	2418	2551	2684	2818	2951	3084	133
6	3218	3351	3484	3617	3750	3883	4016	4149	4282	4415	133
7	4548	4681	4813	4946	5079	5211	5344	5476	5609	5741	133
8	5874	6006	6139	6271	6403	6535	6668	6800	6932	7064	132
9	7196	7328	7460	7592	7724	7855	7987	8119	8251	8382	132
330	8514	8646	8777	8909	9040	9171	9303	9434	9566	9697	131
1	9828	9959	0090	0221	0353	0484	0615	0745	0876	1007	131
2	521138	1269	1400	1530	1661	1792	1922	2053	2183	2314	131
3	2444	2575	2705	2835	2966	3096	3226	3356	3486	3616	130
4	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915	130
5	5045	5174	5304	5434	5563	5693	5822	5951	6081	6210	129
6	6339	6469	6598	6727	6856	6985	7114	7243	7372	7501	129
7	7630	7759	7888	8016	8145	8274	8402	8531	8660	8788	129
8	8917	9045	9174	9302	9430	9559	9687	9815	9943	0072	128
9	530200	0328	0456	0584	0712	0840	0968	1096	1223	1351	128

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
139	13.9	27.8	41.7	55.6	69.5	83.4	97.3	111.2	125.1
138	13.8	27.6	41.4	55.2	69.0	82.8	96.6	110.4	124.2
137	13.7	27.4	41.1	54.8	68.5	82.2	95.9	109.6	123.3
136	13.6	27.2	40.8	54.4	68.0	81.6	95.2	108.8	122.4
135	13.5	27.0	40.5	54.0	67.5	81.0	94.5	108.0	121.5
134	13.4	26.8	40.2	53.6	67.0	80.4	93.8	107.2	120.6
133	13.3	26.6	39.9	53.2	66.5	79.8	93.1	106.4	119.7
132	13.2	26.4	39.6	52.8	66.0	79.2	92.4	105.6	118.8
131	*13.1*	*26.2*	39.3	52.4	65.5	78.6	91.7	104.8	117.
130	*13.0*	*26.0*	39.0	52.0	65.0	78.0	91.0	104.0	117
129	*12.9*	25.8	38.7	51.6	64.5	77.4	90.3	103.2	11
128	*12.8*	25.6	38.4	51.2	64.0	76.8	89.6	102.4	1
127	*12.7*	25.4	38.1	50.8	63.5	76.2	88.9	101.6	

No. 340 L. 531.] [No. 379 L. 579.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
340	531479	1607	1734	1862	1990	2117	2245	2372	2500	2627	128
1	2754	2882	3009	3136	3264	3391	3518	3645	3772	3899	127
2	4026	4153	4280	4407	4534	4661	4787	4914	5041	5167	127
3	5294	5421	5547	5674	5800	5927	6053	6180	6306	6432	126
4	6558	6685	6811	6937	7063	7189	7315	7441	7567	7693	126
5	7819	7945	8071	8197	8322	8448	8574	8699	8825	8951	126
6	9076	9202	9327	9452	9578	9703	9829	9954	0079	0204	125
7	540329	0455	0580	0705	0830	0955	1080	1205	1330	1454	125
8	1579	1704	1829	1953	2078	2203	2327	2452	2576	2701	125
9	2825	2950	3074	3199	3323	3447	3571	3696	3820	3944	124
350	4068	4192	4316	4440	4564	4688	4812	4936	5060	5183	124
1	5307	5431	5555	5678	5802	5925	6049	6172	6296	6419	124
2	6543	6666	6789	6913	7036	7159	7282	7405	7529	7652	123
3	7775	7898	8021	8144	8267	8389	8512	8635	8758	8881	123
4	9003	9126	9249	9371	9494	9616	9739	9861	9984	0106	123
5	550228	0351	0473	0595	0717	0840	0962	1084	1206	1328	122
6	1450	1572	1694	1816	1938	2060	2181	2303	2425	2547	122
7	2668	2790	2911	3033	3155	3276	3398	3519	3640	3762	121
8	3883	4004	4126	4247	4368	4489	4610	4731	4852	4973	121
9	5094	5215	5336	5457	5578	5699	5820	5940	6061	6182	121
360	6303	6423	6544	6664	6785	6905	7026	7146	7267	7387	120
1	7507	7627	7748	7868	7988	8108	8228	8349	8469	8589	120
2	8709	8829	8948	9068	9188	9308	9428	9548	9667	9787	120
3	9907	0026	0146	0265	0385	0504	0624	0743	0863	0982	119
4	561101	1221	1340	1459	1578	1698	1817	1936	2055	2174	119
5	2293	2412	2531	2650	2769	2887	3006	3125	3244	3362	119
6	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548	119
7	4666	4784	4903	5021	5139	5257	5376	5494	5612	5730	118
8	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	118
9	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
370	8202	8319	8436	8554	8671	8788	8905	9023	9140	9257	117
1	9374	9491	9608	9725	9842	9959	0076	0193	0309	0426	117
2	570543	0660	0776	0893	1010	1126	1243	1359	1476	1592	117
3	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
4	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915	116
5	4031	4147	4263	4379	4494	4610	4726	4841	4957	5072	116
6	5188	5303	5419	5534	5650	5765	5880	5996	6111	6226	115
7	6341	6457	6572	6687	6802	6917	7032	7147	7262	7377	115
8	7492	7607	7722	7836	7951	8066	8181	8295	8410	8525	115
9	8639	8754	8868	8983	9097	9212	9326	9441	9555	9669	114

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
128	12.8	25.6	38.4	51.2	64.0	76.8	89.6	102.4	115.2
127	12.7	25.4	38.1	50.8	63.5	76.2	88.9	101.6	114.3
126	12.6	25.2	37.8	50.4	63.0	75.6	88.2	100.8	113.4
125	12.5	25.0	37.5	50.0	62.5	75.0	87.5	100.0	112.5
124	12.4	24.8	37.2	49.6	62.0	74.4	86.8	99.2	111.6
123	*12.3*	*24.6*	36.9	49.2	61.5	73.8	86.1	98.4	110.7
122	*12.2*	*24.4*	36.6	48.8	61.0	73.2	85.4	97.6	109.8
121	*12.1*	*24.2*	36.3	48.4	60.5	72.6	84.7	96.8	108.9
120	*12.0*	*24.0*	36.0	48.0	60.0	72.0	84.0	96.0	108.0
119	*11.9*	23.8	35.7	47.6	59.5	71.4	83.3	95.2	107.1

TABLE XXIV.—LOGARITHMS OF NUMBERS.

No. 380. L. 579.] [No. 414 L. 617.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
380	579784	9898	0012	0126	0241	0355	0469	0583	0697	0811	114
1	580925	1039	1153	1267	1381	1495	1608	1722	1836	1950	
2	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085	
3	3199	3312	3426	3539	3652	3765	3879	3992	4105	4218	
4	4331	4444	4557	4670	4783	4896	5009	5122	5235	5348	113
5	5461	5574	5686	5799	5912	6024	6137	6250	6362	6475	
6	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599	
7	7711	7823	7935	8047	8160	8272	8384	8496	8608	8720	112
8	8832	8944	9056	9167	9279	9391	9503	9615	9726	9838	
9	9950	0061	0173	0284	0396	0507	0619	0730	0842	0953	
390	591065	1176	1287	1399	1510	1621	1732	1843	1955	2066	
1	2177	2288	2399	2510	2621	2732	2843	2954	3064	3175	111
2	3286	3397	3508	3618	3729	3840	3950	4061	4171	4282	
3	4393	4503	4614	4724	4834	4945	5055	5165	5276	5386	
4	5496	5606	5717	5827	5937	6047	6157	6267	6377	6487	
5	6597	6707	6817	6927	7037	7146	7256	7366	7476	7586	110
6	7695	7805	7914	8024	8134	8243	8353	8462	8572	8681	
7	8791	8900	9009	9119	9228	9337	9446	9556	9665	9774	
8	9883	9992	0101	0210	0319	0428	0537	0646	0755	0864	109
9	600973	1082	1191	1299	1408	1517	1625	1734	1843	1951	
400	2060	2169	2277	2386	2494	2603	2711	2819	2928	3036	
1	3144	3253	3361	3469	3577	3686	3794	3902	4010	4118	108
2	4226	4334	4442	4550	4658	4766	4874	4982	5089	5197	
3	5305	5413	5521	5628	5736	5844	5951	6059	6166	6274	
4	6381	6489	6596	6704	6811	6919	7026	7133	7241	7348	
5	7455	7562	7669	7777	7884	7991	8098	8205	8312	8419	107
6	8526	8633	8740	8847	8954	9061	9167	9274	9381	9488	
7	9594	9701	9808	9914	0021	0128	0234	0341	0447	0554	
8	610660	0767	0873	0979	1086	1192	1298	1405	1511	1617	
9	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678	106
410	2784	2890	2996	3102	3207	3313	3419	3525	3630	3736	
1	3842	3947	4053	4159	4264	4370	4475	4581	4686	4792	
2	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845	
3	5950	6055	6160	6265	6370	6476	6581	6686	6790	6895	105
4	7000	7105	7210	7315	7420	7525	7629	7734	7839	7943	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
118	11.8	23.6	35.4	47.2	59.0	70.8	82.6	94.4	106.2
117	11.7	23.4	35.1	46.8	58.5	70.2	81.9	93.6	105.3
116	11.6	23.2	34.8	46.4	58.0	69.6	81.2	92.8	104.4
115	11.5	23.0	34.5	46.0	57.5	69.0	80.5	92.0	103.5
114	11.4	22.8	34.2	45.6	57.0	68.4	79.8	91.2	102.6
113	11.3	22.6	33.9	45.2	56.5	67.8	79.1	90.4	101.7
112	11.2	22.4	33.6	44.8	56.0	67.2	78.4	89.6	100.8
111	11.1	22.2	33.3	44.4	55.5	66.6	77.7	88.8	99.9
110	11.0	22.0	33.0	44.0	55.0	66.0	77.0	88.0	99.0
109	10.9	21.8	32.7	43.6	54.5	65.4	76.3	87.2	98.1
108	10.8	21.6	32.4	43.2	54.0	64.8	75.6	86.4	97.2
107	10.7	21.4	32.1	42.8	53.5	64.2	74.9	85.6	96.3
106	10.6	21.2	31.8	42.4	53.0	63.6	74.2	84.8	95.
105	10.5	21.0	31.5	42.0	52.5	63.0	73.5	84.0	94
105	10.5	21.0	31.5	42.0	52.5	63.0	73.5	84.0	9
104	10.4	20.8	31.2	41.6	52.0	62.4	72.8	83.2	

N.	0	1	2	3	4	5	6	7	8	9	Diff.
415	618048	8153	8257	8362	8466	8571	8676	8780	8884	8989	105
6	9093	9198	9302	9406	9511	9615	9719	9824	9928	0032	
7	620136	0240	0344	0448	0552	0656	0760	0864	0968	1072	104
8	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110	
9	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146	
420	3249	3353	3456	3559	3663	3766	3869	3973	4076	4179	
1	4282	4385	4488	4591	4695	4798	4901	5004	5107	5210	103
2	5312	5415	5518	5621	5724	5827	5929	6032	6135	6238	
3	6340	6443	6546	6648	6751	6853	6956	7058	7161	7263	
4	7366	7468	7571	7673	7775	7878	7980	8082	8185	8287	
5	8389	8491	8593	8695	8797	8900	9002	9104	9206	9308	102
6	9410	9512	9613	9715	9817	9919	0021	0123	0224	0326	
7	630428	0530	0631	0733	0835	0936	1038	1139	1241	1342	
8	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356	
9	2457	2559	2660	2761	2862	2963	3064	3165	3266	3367	
430	3468	3569	3670	3771	3872	3973	4074	4175	4276	4376	101
1	4477	4578	4679	4779	4880	4981	5081	5182	5283	5383	
2	5484	5584	5685	5785	5886	5986	6087	6187	6287	6388	
3	6488	6588	6688	6789	6889	6989	7089	7189	7290	7390	
4	7490	7590	7690	7790	7890	7990	8090	8190	8290	8389	100
5	8489	8589	8689	8789	8888	8988	9088	9188	9287	9387	
6	9486	9586	9686	9785	9885	9984	0084	0183	0283	0382	
7	640481	0581	0680	0779	0879	0978	1077	1177	1276	1375	
8	1474	1573	1672	1771	1871	1970	2069	2168	2267	2366	
9	2465	2563	2662	2761	2860	2959	3058	3156	3255	3354	99
440	3453	3551	3650	3749	3847	3946	4044	4143	4242	4340	
1	4439	4537	4636	4734	4832	4931	5029	5127	5226	5324	
2	5422	5521	5619	5717	5815	5913	6011	6110	6208	6306	
3	6404	6502	6600	6698	6796	6894	6992	7089	7187	7285	98
4	7383	7481	7579	7676	7774	7872	7969	8067	8165	8262	
5	8360	8458	8555	8653	8750	8848	8945	9043	9140	9237	
6	9335	9432	9530	9627	9724	9821	9919	0016	0113	0210	
7	650308	0405	0502	0599	0696	0793	0890	0987	1084	1181	
8	1278	1375	1472	1569	1666	1762	1859	1956	2053	2150	97
9	2246	2343	2440	2536	2633	2730	2826	2923	3019	3116	
450	3213	3309	3405	3502	3598	3695	3791	3888	3984	4080	
1	4177	4273	4369	4465	4562	4658	4754	4850	4946	5042	
2	5138	5235	5331	5427	5523	5619	5715	5810	5906	6002	96
3	6098	6194	6290	6386	6482	6577	6673	6769	6864	6960	
4	7056	7152	7247	7343	7438	7534	7629	7725	7820	7916	
5	8011	8107	8202	8298	8393	8488	8584	8679	8774	8870	
6	8965	9060	9155	9250	9346	9441	9536	9631	9726	9821	
7	9916	0011	0106	0201	0296	0391	0486	0581	0676	0771	95
8	660865	0960	1055	1150	1245	1339	1434	1529	1623	1718	
9	1813	1907	2002	2096	2191	2286	2380	2475	2569	2663	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
105	10.5	21.0	31.5	42.0	52.5	63.0	73.5	84.0	94.5
104	10.4	20.8	31.2	41.6	52.0	62.4	72.8	83.2	93.6
103	10.3	20.6	30.9	41.2	51.5	61.8	72.1	82.4	92.7
102	10.2	20.4	30.6	40.8	51.0	61.2	71.4	81.6	91.8
101	10.1	20.2	30.3	40.4	50.5	60.6	70.7	80.8	90.9
100	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0
99	9.9	19.8	29.7	39.6	49.5	59.4	69.3	79.2	89.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
460	662758	2852	2947	3041	3135	3230	3324	3418	3512	3607	
1	3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	
2	4642	4736	4830	4924	5018	5112	5206	5299	5393	5487	94
3	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	
4	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	
5	7453	7546	7640	7733	7826	7920	8013	8106	8199	8293	
6	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	
7	9317	9410	9503	9596	9689	9782	9875	9967	0060	0153	93
8	670246	0339	0431	0524	0617	0710	0802	0895	0988	1080	
9	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	
470	2098	2190	2283	2375	2467	2560	2652	2744	2836	2929	
1	3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	
2	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
3	4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	
4	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	
5	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	
6	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	
7	8518	8609	8700	8791	8882	8973	9064	9155	9246	9337	91
8	9428	9519	9610	9700	9791	9882	9973	0063	0154	0245	
9	680336	0426	0517	0607	0698	0789	0879	0970	1060	1151	
480	1241	1332	1422	1513	1603	1693	1784	1874	1964	2055	
1	2145	2235	2326	2416	2506	2596	2686	2777	2867	2957	
2	3047	3137	3227	3317	3407	3497	3587	3677	3767	3857	90
3	3947	4037	4127	4217	4307	4396	4486	4576	4666	4756	
4	4845	4935	5025	5114	5204	5294	5383	5473	5563	5652	
5	5742	5831	5921	6010	6100	6189	6279	6368	6458	6547	
6	6636	6726	6815	6904	6994	7083	7172	7261	7351	7440	
7	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
8	8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	
9	9309	9398	9486	9575	9664	9753	9841	9930	0019	0107	
490	690196	0285	0373	0462	0550	0639	0728	0816	0905	0993	
1	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	
2	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	
3	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
4	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	
5	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	
6	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	
7	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	
8	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	
9	8100	8188	8275	8362	8449	8535	8622	8709	8796	8883	87

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
98	9.8	19.6	29.4	39.2	49.0	58.8	68.6	78.4	88.2
97	9.7	19.4	29.1	38.8	48.5	58.2	67.9	77.6	87.3
96	9.6	19.2	28.8	38.4	48.0	57.6	67.2	76.8	86.4
95	9.5	19.0	28.5	38.0	47.5	57.0	66.5	76.0	85.5
94	9.4	18.8	28.2	37.6	47.0	56.4	65.8	75.2	84.6
93	9.3	18.6	27.9	37.2	46.5	55.8	65.1	74.4	83.7
92	9.2	18.4	27.6	36.8	46.0	55.2	64.4	73.6	82.8
91	9.1	18.2	27.3	36.4	45.5	54.6	63.7	72.8	81.
90	9.0	18.0	27.0	36.0	45.0	54.0	63.0	72.0	81
89	8.9	17.8	26.7	35.6	44.5	53.4	62.3	71.2	8
88	8.8	17.6	26.4	35.2	44.0	52.8	61.6	70.4	
87	8.7	17.4	26.1	34.8	43.5	52.2	60.9	69.6	
86	8.6	17.2	25.8	34.4	43.0	51.6	60.2	68.8	

N.	0	1	2	3	4	5	6	7	8	9	Diff.
500	698970	9057	9144	9231	9317	9404	9491	9578	9664	9751	
1	9838	9924	0011	0098	0184	0271	0358	0444	0531	0617	
2	700704	0790	0877	0963	1050	1136	1222	1309	1395	1482	
3	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	
4	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	
5	3291	3377	3463	3549	3635	3721	3807	3893	3979	4065	86
6	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	
7	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	
8	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	
9	6718	6803	6888	6974	7059	7144	7229	7315	7400	7485	
510	7570	7655	7740	7826	7911	7996	8081	8166	8251	8336	85
1	8421	8506	8591	8676	8761	8846	8931	9015	9100	9185	
2	9270	9355	9440	9524	9609	9694	9779	9863	9948	0033	
3	710117	0202	0287	0371	0456	0540	0625	0710	0794	0879	
4	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	
5	1807	1892	1976	2060	2144	2229	2313	2397	2481	2566	
6	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	
7	3491	3575	3659	3742	3826	3910	3994	4078	4162	4246	84
8	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	
9	5167	5251	5335	5418	5502	5586	5669	5753	5836	5920	
520	6003	6087	6170	6254	6337	6421	6504	6588	6671	6754	
1	6838	6921	7004	7088	7171	7254	7338	7421	7504	7587	
2	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	
3	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
4	9331	9414	9497	9580	9663	9745	9828	9911	9994	0077	
5	720159	0242	0325	0407	0490	0573	0655	0738	0821	0903	
6	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	
7	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	
8	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	
9	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
530	4276	4358	4440	4522	4604	4685	4767	4849	4931	5013	
1	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	
2	5912	5993	6075	6156	6238	6320	6401	6483	6564	6646	
3	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	
4	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	
5	8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	
6	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
7	9974	0055	0136	0217	0298	0378	0459	0540	0621	0702	
8	730782	0863	0944	1024	1105	1186	1266	1347	1428	1508	
9	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	
540	2394	2474	2555	2635	2715	2796	2876	2956	3037	3117	
1	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	
2	3999	4079	4160	4240	4320	4400	4480	4560	4640	4720	80
3	4800	4880	4960	5040	5120	5200	5279	5359	5439	5519	
4	5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
87	8.7	17.4	26.1	34.8	43.5	52.2	60.9	69.6	78.3
86	8.6	17.2	25.8	34.4	43.0	51.6	60.2	68.8	77.4
85	8.5	17.0	25.5	34.0	42.5	51.0	59.5	68.0	76.5
84	8.4	16.8	25.2	33.6	42.0	50.4	58.8	67.2	75.

No. 545 L. 736.] [No. 584 L. 767.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
545	736397	6476	6556	6635	6715	6795	6874	6954	7034	7113	
6	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	
7	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	
8	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	
9	9572	9651	9731	9810	9889	9968					
							0047	0126	0205	0284	79
550	740363	0442	0521	0600	0678	0757	0836	0915	0994	1073	
1	1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	
2	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	
3	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	
4	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	
5	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	
6	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
7	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	
8	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	
9	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	
560	8188	8266	8343	8421	8498	8576	8653	8731	8808	8885	
1	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	
2	9736	9814	9891	9968							
					0045	0123	0200	0277	0354	0431	
3	750508	0586	0663	0740	0817	0894	0971	1048	1125	1202	
4	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
5	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	
6	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	
7	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	
8	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	
9	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	
570	5875	5951	6027	6103	6180	6256	6332	6408	6484	6560	
1	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
2	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	
3	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	
4	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	
5	9668	9743	9819	9894	9970						
						0045	0121	0196	0272	0347	
6	760422	0498	0573	0649	0724	0799	0875	0950	1025	1101	
7	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	
8	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
9	2679	2754	2829	2904	2978	3053	3128	3203	3278	3353	
580	3428	3503	3578	3653	3727	3802	3877	3952	4027	4101	
1	4176	4251	4326	4400	4475	4550	4624	4699	4774	4848	
2	4923	4998	5072	5147	5221	5296	5370	5445	5520	5594	
3	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	
4	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
83	8.3	16.6	24.9	33.2	41.5	49.8	58.1	66.4	74.7
82	8.2	16.4	24.6	32.8	41.0	49.2	57.4	65.6	73.8
81	8.1	16.2	24.3	32.4	40.5	48.6	56.7	64.8	72.9
80	8.0	16.0	24.0	32.0	40.0	48.0	56.0	64.0	72.0
79	7.9	15.8	23.7	31.6	39.5	47.4	55.3	63.2	71.1
78	7.8	15.6	23.4	31.2	39.0	46.8	54.6	62.4	70.2
77	7.7	15.4	23.1	30.8	38.5	46.2	53.9	61.6	69.
76	7.6	15.2	22.8	30.4	38.0	45.6	53.2	60.8	68
75	7.5	15.0	22.5	30.0	37.5	45.0	52.5	60.0	6
74	7.4	14.8	22.2	29.6	37.0	44.4	51.8	59.2	

N.	0	1	2	3	4	5	6	7	8	9	Diff.
585	767156	7230	7304	7379	7453	7527	7601	7675	7749	7823	
6	7898	7972	8046	8120	8194	8268	8342	8416	8490	8564	74
7	8638	8712	8786	8860	8934	9008	9082	9156	9230	9303	
8	9377	9451	9525	9599	9673	9746	9820	9894	9968	0042	
9	770115	0189	0263	0336	0410	0484	0557	0631	0705	0778	
590	0852	0926	0999	1073	1146	1220	1293	1367	1440	1514	
1	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	
2	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	
3	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	
4	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
5	4517	4590	4663	4736	4809	4882	4955	5028	5100	5173	
6	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	
7	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	
8	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	
9	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	
600	8151	8224	8296	8368	8441	8513	8585	8658	8730	8802	
1	8874	8947	9019	9091	9163	9236	9308	9380	9452	9524	
2	9596	9669	9741	9813	9885	9957	0029	0101	0173	9245	
3	780317	0389	0461	0533	0605	0677	0749	0821	0893	0965	72
4	1037	1109	1181	1253	1324	1396	1468	1540	1612	1684	
5	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	
6	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	
7	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	
8	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	
9	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	
610	5330	5401	5472	5543	5615	5686	5757	5828	5899	5970	
1	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
2	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	
3	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	
4	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	
5	8875	8946	9016	9087	9157	9228	9299	9369	9440	9510	
6	9581	9651	9722	9792	9863	9933	0004	0074	0144	0215	
7	790285	0356	0426	0496	0567	0637	0707	0778	0848	0918	
8	0988	1059	1129	1199	1269	1340	1410	1480	1550	1620	
9	1691	1761	1831	1901	1971	2041	2111	2181	2252	2322	
620	2392	2462	2532	2602	2672	2742	2812	2882	2952	3022	70
1	3092	3162	3231	3301	3371	3441	3511	3581	3651	3721	
2	3790	3860	3930	4000	4070	4139	4209	4279	4349	4418	
3	4488	4558	4627	4697	4767	4836	4906	4976	5045	5115	
4	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	
5	5880	5949	6019	6088	6158	6227	6297	6366	6436	6505	
6	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	
7	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	
8	7960	8029	8098	8167	8236	8305	8374	8443	8513	8582	
9	8651	8720	8789	8858	8927	8996	9065	9134	9203	9272	69

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
75	7.5	15.0	22.5	30.0	37.5	45.0	52.5	60.0	67.5
74	7.4	14.8	22.2	29.6	37.0	44.4	51.8	59.2	66.6
73	7.3	14.6	21.9	29.2	36.5	43.8	51.1	58.4	65.7
72	7.2	14.4	21.6	28.8	36.0	43.2	50.4	57.6	64.8
71	7.1	14.2	21.3	28.4	35.5	42.6	49.7	56.8	63.9
70	7.0	14.0	21.0	28.0	35.0	42.0	49.0	56.0	63.0
69	6.9	13.8	20.7	27.6	34.5	41.4	48.3	55.2	62.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
630	799341	9409	9478	9547	9616	9685	9754	9823	9892	9961	
1	800029	0098	0167	0236	0305	0373	0442	0511	0580	0648	
2	0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	
3	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	
4	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	
5	2774	2842	2910	2979	3047	3116	3184	3252	3321	3389	
6	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	
7	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	
8	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
9	5501	5569	5637	5705	5773	5841	5908	5976	6044	6112	
640	806180	6248	6316	6384	6451	6519	6587	6655	6723	6790	
1	6858	6926	6994	7061	7129	7197	7264	7332	7400	7467	
2	7535	7603	7670	7738	7806	7873	7941	8008	8076	8143	
3	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	
4	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	
5	9560	9627	9694	9762	9829	9896	9964	0031	0098	0165	
6	810233	0300	0367	0434	0501	0569	0636	0703	0770	0837	
7	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
8	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	
9	2245	2312	2379	2445	2512	2579	2646	2713	2780	2847	
650	2913	2980	3047	3114	3181	3247	3314	3381	3448	3514	
1	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	
2	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	
3	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	
4	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	
5	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838	
6	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	
7	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	
8	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	66
9	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	
660	9544	9610	9676	9741	9807	9873	9939	0004	0070	0136	
1	820201	0267	0333	0399	0464	0530	0595	0661	0727	0792	
2	0858	0924	0989	1055	1120	1186	1251	1317	1382	1448	
3	1514	1579	1645	1710	1775	1841	1906	1972	2037	2103	
4	2168	2233	2299	2364	2430	2495	2560	2626	2691	2756	
5	2822	2887	2952	3018	3083	3148	3213	3279	3344	3409	
6	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	
7	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
8	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	
9	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	
670	6075	6140	6204	6269	6334	6399	6464	6528	6593	6658	
1	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	
2	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	
3	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	
4	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
68	6.8	13.6	20.4	27.2	34.0	40.8	47.6	54.4	61.2
67	*6.7*	*13.4*	20.1	26.8	33.5	40.2	46.9	53.6	60.3
66	*6.6*	*13.2*	19.8	26.4	33.0	39.6	46.2	52.8	59.
65	*6.5*	*13.0*	19.5	26.0	32.5	39.0	45.5	52.0	58
64	*6.4*	*12.8*	19.2	25.6	32.0	38.4	44.8	51.2	5

N.	0	1	2	3	4	5	6	7	8	9	Diff.
675	829304	9368	9432	9497	9561	9625	9690	9754	9818	9882	
6	9947	0011	0075	0139	0204	0268	0332	0396	0460	0525	
7	830589	0653	0717	0781	0845	0909	0973	1037	1102	1166	
8	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	64
9	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	
680	2509	2573	2637	2700	2764	2828	2892	2956	3020	3083	
1	3147	3211	3275	3338	3402	3466	3530	3593	3657	3721	
2	3784	3848	3912	3975	4039	4103	4166	4230	4294	4357	
3	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	
4	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	
5	5691	5754	5817	5881	5944	6007	6071	6134	6197	6261	
6	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	
7	6957	7020	7083	7146	7210	7273	7336	7399	7462	7525	
8	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156	
9	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786	63
690	8849	8912	8975	9038	9101	9164	9227	9289	9352	9415	
1	9478	9541	9604	9667	9729	9792	9855	9918	9981	0043	
2	840106	0169	0232	0294	0357	0420	0482	0545	0608	0671	
3	0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	
4	1359	1422	1485	1547	1610	1672	1735	1797	1860	1922	
5	1985	2047	2110	2172	2235	2297	2360	2422	2484	2547	
6	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	
7	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	
8	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	
9	4477	4539	4601	4664	4726	4788	4850	4912	4974	5036	
700	5098	5160	5222	5284	5346	5408	5470	5532	5594	5656	62
1	5718	5780	5842	5904	5966	6028	6090	6151	6213	6275	
2	6337	6399	6461	6523	6585	6646	6708	6770	6832	6894	
3	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	
4	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	
5	8189	8251	8312	8374	8435	8497	8559	8620	8682	8743	
6	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	
7	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	
8	850033	0095	0156	0217	0279	0340	0401	0462	0524	0585	
9	0646	0707	0769	0830	0891	0952	1014	1075	1136	1197	
710	1258	1320	1381	1442	1503	1564	1625	1686	1747	1809	
1	1870	1931	1992	2053	2114	2175	2236	2297	2358	2419	
2	2480	2541	2602	2663	2724	2785	2846	2907	2968	3029	61
3	3090	3150	3211	3272	3333	3394	3455	3516	3577	3637	
4	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	
5	4306	4367	4428	4488	4549	4610	4670	4731	4792	4852	
6	4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	
7	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	
8	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	
9	6729	6789	6850	6910	6970	7031	7091	7152	7212	7272	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
65	6.5	13.0	19.5	26.0	32.5	39.0	45.5	52.0	58.5
64	*6.4*	*12.8*	19.2	25.6	32.0	38.4	44.8	51.2	57.6
63	*6.3*	*12.6*	18.9	25.2	31.5	37.8	44.1	50.4	56.7
2	*6.2*	*12.4*	18.6	24.8	31.0	37.2	43.4	49.6	55.8
	6.1	*12.2*	18.3	24.4	30.5	36.6	42.7	48.8	54.9
	6.0	*12.0*	18.0	24.0	30.0	36.0	42.0	48.0	54.0

N.	0	1	2	3	4	5	6	7	8	9	Diff.
720	857332	7393	7453	7513	7574	7634	7694	7755	7815	7875	
1	7935	7995	8056	8116	8176	8236	8297	8357	8417	8477	
2	8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	
3	9138	9198	9258	9318	9379	9439	9499	9559	9619	9679	60
4	9739	9799	9859	9918	9978	0038	0098	0158	0218	0278	
5	860338	0398	0458	0518	0578	0637	0697	0757	0817	0877	
6	0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	
7	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	
8	2131	2191	2251	2310	2370	2430	2489	2549	2608	2668	
9	2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	
730	3323	3382	3442	3501	3561	3620	3680	3739	3799	3858	
1	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	
2	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	
3	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	
4	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	
5	6287	6346	6405	6465	6524	6583	6642	6701	6760	6819	
6	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
7	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	
8	8056	8115	8174	8233	8292	8350	8409	8468	8527	8586	
9	8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	
740	9232	9290	9349	9408	9466	9525	9584	9642	9701	9760	
1	9818	9877	9935	9994	0053	0111	0170	0228	0287	0345	
2	870404	0462	0521	0579	0638	0696	0755	0813	0872	0930	
3	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	
4	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	
5	2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	
6	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	
7	3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	
8	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
9	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	
750	5061	5119	5177	5235	5293	5351	5409	5466	5524	5582	
1	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	
2	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	
3	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	
4	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	
5	7947	8004	8062	8119	8177	8234	8292	8349	8407	8464	
6	8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	
7	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	
8	9669	9726	9784	9841	9898	9956	0013	0070	0127	0185	
9	880242	0299	0356	0413	0471	0528	0585	0642	0699	0756	
760	0814	0871	0928	0985	1042	1099	1156	1213	1271	1328	
1	1385	1442	1499	1556	1613	1670	1727	1784	1841	1898	
2	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
3	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	
4	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
59	5.9	11.8	17.7	23.6	29.5	35.4	41.3	47.2	53.1
58	5.8	11.6	17.4	23.2	29.0	34.8	40.6	46.4	52.2
57	5.7	11.4	17.1	22.8	28.5	34.2	39.9	45.6	51.
56	5.6	11.2	16.8	22.4	28.0	33.6	39.2	44.8	5

N.	0	1	2	3	4	5	6	7	8	9	Diff.
765	883661	3718	3775	3832	3888	3945	4002	4059	4115	4172	
6	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	
7	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	
8	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	
9	5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	
770	6491	6547	6604	6660	6716	6773	6829	6885	6942	6998	
1	7054	7111	7167	7223	7280	7336	7392	7449	7505	7561	
2	7617	7674	7730	7786	7842	7898	7955	8011	8067	8123	
3	8179	8236	8292	8348	8404	8460	8516	8573	8629	8685	
4	8741	8797	8853	8909	8965	9021	9077	9134	9190	9246	
5	9302	9358	9414	9470	9526	9582	9638	9694	9750	9806	56
6	9862	9918	9974	0030	0086	0141	0197	0253	0309	0365	
7	890421	0477	0533	0589	0645	0700	0756	0812	0868	0924	
8	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482	
9	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	
780	2095	2150	2206	2262	2317	2373	2429	2484	2540	2595	
1	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	
2	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	
3	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	
4	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	
5	4870	4925	4980	5036	5091	5146	5201	5257	5312	5367	
6	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	
7	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	
8	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	
9	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
790	7627	7682	7737	7792	7847	7902	7957	8012	8067	8122	
1	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	
2	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	
3	9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	
4	9821	9875	9930	9985	0039	0094	0149	0203	0258	0312	
5	900367	0422	0476	0531	0586	0640	0695	0749	0804	0859	
6	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	
7	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	
8	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	
9	2547	2601	2655	2710	2764	2818	2873	2927	2981	3036	
800	3090	3144	3199	3253	3307	3361	3416	3470	3524	3578	
1	3633	3687	3741	3795	3849	3904	3958	4012	4066	4120	
2	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	
3	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	
4	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
5	5796	5850	5904	5958	6012	6066	6119	6173	6227	6281	
6	6335	6389	6443	6497	6551	6604	6658	6712	6766	6820	
7	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	
8	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895	
9	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
57	5.7	11.4	17.1	22.8	28.5	34.2	39.9	45.6	51.3
	5.6	11.2	16.8	22.4	28.0	33.6	39.2	44.8	50.4
	5.5	11.0	16.5	22.0	27.5	33.0	38.5	44.0	49.5
	5.4	10.8	16.2	21.6	27.0	32.4	37.8	43.2	48.6

N.	0	1	2	3	4	5	6	7	8	9	Diff.
810	908485	8539	8592	8646	8699	8753	8807	8860	8914	8967	
1	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	
2	9556	9610	9663	9716	9770	9823	9877	9930	9984	0037	
3	910091	0144	0197	0251	0304	0358	0411	0464	0518	0571	
4	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	
5	1158	1211	1264	1317	1371	1424	1477	1530	1584	1637	
6	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	
7	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	
8	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	
9	3284	3337	3390	3443	3496	3549	3602	3655	3708	3761	53
820	3814	3867	3920	3973	4026	4079	4132	4184	4237	4290	
1	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	
2	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	
3	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	
4	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	
5	6454	6507	6559	6612	6664	6717	6770	6822	6875	6927	
6	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	
7	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	
8	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	
9	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	
830	9078	9130	9183	9235	9287	9340	9392	9444	9496	9549	
1	9601	9653	9706	9758	9810	9862	9914	9967	0019	0071	
2	920123	0176	0228	0280	0332	0384	0436	0489	0541	0593	
3	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	
4	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
5	1686	1738	1790	1842	1894	1946	1998	2050	2102	2154	
6	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	
7	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	
8	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	
9	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	
840	4279	4331	4383	4434	4486	4538	4589	4641	4693	4744	
1	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	
2	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	
3	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	
4	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	
5	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	
6	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	
7	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	
8	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	
9	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	
850	9419	9470	9521	9572	9623	9674	9725	9776	9827	9879	51
1	9930	9981	0032	0083	0134	0185	0236	0287	0338	0389	
2	930440	0491	0542	0592	0643	0694	0745	0796	0847	0898	
3	0949	1000	1051	1102	1153	1204	1254	1305	1356	1407	
4	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
53	5.3	10.6	15.9	21.2	26.5	31.8	37.1	42.4	47
52	5.2	10.4	15.6	20.8	26.0	31.2	36.4	41.6	
51	5.1	10.2	15.3	20.4	25.5	30.6	35.7	40.8	
50	5.0	10.0	15.0	20.0	25.0	30.0	35.0	40.0	

N.	0	1	2	3	4	5	6	7	8	9	Diff.
855	931966	2017	2068	2118	2169	2220	2271	2322	2372	2423	
6	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	
7	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	
8	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	
9	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	
860	4498	4549	4599	4650	4700	4751	4801	4852	4902	4953	
1	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	
2	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	
3	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	
4	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	
5	7016	7066	7116	7167	7217	7267	7317	7367	7418	7468	
6	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	
7	8019	8069	8119	8169	8219	8269	8320	8370	8420	8470	50
8	8520	8570	8620	8670	8720	8770	8820	8870	8920	8970	
9	9020	9070	9120	9170	9220	9270	9320	9369	9419	9469	
870	9519	9569	9619	9669	9719	9769	9819	9869	9918	9968	
1	940018	0068	0118	0168	0218	0267	0317	0367	0417	0467	
2	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	
3	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	
4	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	
5	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	
6	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	
7	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	
8	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	
9	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	
880	4483	4532	4581	4631	4680	4729	4779	4828	4877	4927	
1	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	
2	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	
3	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	
4	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	
5	6943	6992	7041	7090	7139	7189	7238	7287	7336	7385	
6	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
7	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	
8	8413	8462	8511	8560	8608	8657	8706	8755	8804	8853	
9	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	
890	9390	9439	9488	9536	9585	9634	9683	9731	9780	9829	
1	9878	9926	9975	0024	0073	0121	0170	0219	0267	0316	
2	950365	0414	0462	0511	0560	0608	0657	0706	0754	0803	
3	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	
4	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	
5	1823	1872	1920	1969	2017	2066	2114	2163	2211	2260	
6	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	
7	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	
8	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	
9	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
	5.1	10.2	15.3	20.4	25.5	30.6	35.7	40.8	45.9
	5.0	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.0
	4.9	9.8	14.7	19.6	24.5	29.4	34.3	39.2	44.1
	.8	9.6	14.4	19.2	24.0	28.8	33.6	38.4	43.2

N.	0	1	2	3	4	5	6	7	8	9	Diff.
900	954243	4291	4339	4387	4435	4484	4532	4580	4628	4677	
1	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	
2	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	
3	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	
4	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	
5	6649	6697	6745	6793	6840	6888	6936	6984	7032	7080	48
6	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	
7	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	
8	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	
9	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	
910	9041	9089	9137	9185	9232	9280	9328	9375	9423	9471	
1	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	
2	9995	0042	0090	0138	0185	0233	0280	0328	0376	0423	
3	960471	0518	0566	0613	0661	0709	0756	0804	0851	0899	
4	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	
5	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	
6	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	
7	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	
8	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	
9	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	
920	3788	3835	3882	3929	3977	4024	4071	4118	4165	4212	
1	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	
2	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	
3	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	
4	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
5	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	
6	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	
7	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	
8	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	
9	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	
930	8483	8530	8576	8623	8670	8716	8763	8810	8856	8903	
1	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	
2	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	
3	9882	9928	9975	0021	0068	0114	0161	0207	0254	0300	
4	970347	0393	0440	0486	0533	0579	0626	0672	0719	0765	
5	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229	
6	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	
7	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	
8	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	
9	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	
940	3128	3174	3220	3266	3313	3359	3405	3451	3497	3543	
1	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	
2	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	
3	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	
4	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
47	4.7	9.4	14.1	18.8	23.5	28.2	32.9	37.6	
46	4.6	9.2	13.8	18.4	23.0	27.6	32.2	36.8	

N.	0	1	2	3	4	5	6	7	8	9	Diff.
945	975432	5478	5524	5570	5616	5662	5707	5753	5799	5845	
6	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	
7	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	
8	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	
9	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	
950	7724	7769	7815	7861	7906	7952	7998	8043	8089	8135	
1	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	
2	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	
3	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	
4	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	
5	980003	0049	0094	0140	0185	0231	0276	0322	0367	0412	
6	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	
7	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	
8	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	
9	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	
960	2271	2316	2362	2407	2452	2497	2543	2588	2633	2678	
1	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	
2	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	
3	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	
4	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	
5	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	4
6	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	
7	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	
8	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	
9	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	
970	6772	6817	6861	6906	6951	6996	7040	7085	7130	7175	
1	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	
2	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	
3	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	
4	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	
5	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	
6	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	
7	9895	9939	9983	0028	0072	0117	0161	0206	0250	0294	
8	990339	0383	0428	0472	0516	0561	0605	0650	0694	0738	
9	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	
980	1226	1270	1315	1359	1403	1448	1492	1536	1580	1625	
1	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	
2	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	
3	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	
4	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	
5	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	
6	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	
7	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	4
8	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	
9	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
46	4.6	9.2	13.8	18.4	23.0	27.6	32.2	36.8	
45	4.5	9.0	13.5	18.0	22.5	27.0	31.5	36.0	
44	4.4	8.8	13.2	17.6	22.0	26.4	30.8	35.2	
	4.3	8.6	12.9	17.2	21.5	25.8	30.1	34.4	

N.	0	1	2	3	4	5	6	7	8	9	Diff.
990	995635	5679	5723	5767	5811	5854	5898	5942	5986	6030	
1	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
2	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	
3	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	
4	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	
5	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	
6	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	
7	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	
8	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	
9	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43

LOGARITHMS OF NUMBERS FROM 1 TO 100.

N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
1	0.000000	21	1.322219	41	1.612784	61	1.785330	81	1.908485
2	0.301030	22	1.342423	42	1.623249	62	1.792392	82	1.913814
3	0.477121	23	1.361728	43	1.633468	63	1.799341	83	1.919078
4	0.602060	24	1.380211	44	1.643453	64	1.806180	84	1.924279
5	0.698970	25	1.397940	45	1.653213	65	1.812913	85	1.929419
6	0.778151	26	1.414973	46	1.662758	66	1.819544	86	1.934498
7	0.845098	27	1.431364	47	1.672098	67	1.826075	87	1.939519
8	0.903090	28	1.447158	48	1.681241	68	1.832509	88	1.944483
9	0.954243	29	1.462398	49	1.690196	69	1.838849	89	1.949390
10	1.000000	30	1.477121	50	1.698970	70	1.845098	90	1.954243
11	1.041393	31	1.491362	51	1.707570	71	1.851258	91	1.959041
12	1.079181	32	1.505150	52	1.716003	72	1.857332	92	1.963788
13	1.113943	33	1.518514	53	1.724276	73	1.863323	93	1.968483
14	1.146128	34	1.531479	54	1.732394	74	1.869232	94	1.973128
15	1.176091	35	1.544068	55	1.740363	75	1.875061	95	1.977724
16	1.204120	36	1.556302	56	1.748188	76	1.880814	96	1.982271
17	1.230449	37	1.568202	57	1.755875	77	1.886491	97	1.986772
18	1.255273	38	1.579784	58	1.763428	78	1.892095	98	1.991226
19	1.278754	39	1.591065	59	1.770852	79	1.897627	99	1.995635
20	1.301030	40	1.602060	60	1.778151	80	1.903090	100	2.000000

	Value at 0°.	Sign in 1st Quad.	Value at 90°.	Sign in 2d Quad.	Value at 180°.	Sign in 3d Quad.	Value at 270°	Sign in 4th Quad.	Value at 360°.
Sin........	O	+	R	+	O	−	R	−	O
Tan.......	O	+	∞	−	O	+	∞	−	O
Sec.......	R	+	∞	−	R	−	∞	+	R
Versin....	O	+	R	+	2 R	+	R	+	O
Cos.......	R	+	O	−	R	−	O	+	R
Cot.... ...	∞	+	O	−	∞	+	O	−	∞
Cosec.....	∞	+	R	+	∞	−	R	−	∞

R signifies equal to rad; ∞ signifies infinite; O signifies evanescent.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
990	995635	5679	5723	5767	5811	5854	5898	5942	5986	6030	
1	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
2	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	
3	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	
4	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	
5	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	
6	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	
7	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	
8	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	
9	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43

LOGARITHMS OF NUMBERS FROM 1 TO 100.

N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
1	0.000000	21	1.322219	41	1.612784	61	1.785330	81	1.908485
2	0.301030	22	1.342423	42	1.623249	62	1.792392	82	1.913814
3	0.477121	23	1.361728	43	1.633468	63	1.799341	83	1.919078
4	0.602060	24	1.380211	44	1.643453	64	1.806180	84	1.924279
5	0.698970	25	1.397940	45	1.653213	65	1.812913	85	1.929419
6	0.778151	26	1.414973	46	1.662758	66	1.819544	86	1.934498
7	0.845098	27	1.431364	47	1.672098	67	1.826075	87	1.939519
8	0.903090	28	1.447158	48	1.681241	68	1.832509	88	1.944483
9	0.954243	29	1.462398	49	1.690196	69	1.838849	89	1.949390
10	1.000000	30	1.477121	50	1.698970	70	1.845098	90	1.954243
11	1.041393	31	1.491362	51	1.707570	71	1.851258	91	1.959041
12	1.079181	32	1.505150	52	1.716003	72	1.857332	92	1.963788
13	1.113943	33	1.518514	53	1.724276	73	1.863323	93	1.968483
14	1.146128	34	1.531479	54	1.732394	74	1.869232	94	1.973128
15	1.176091	35	1.544068	55	1.740363	75	1.875061	95	1.977724
16	1.204120	36	1.556302	56	1.748188	76	1.880814	96	1.982271
17	1.230449	37	1.568202	57	1.755875	77	1.886491	97	1.986772
18	1.255273	38	1.579784	58	1.763428	78	1.892095	98	1.991226
19	1.278754	39	1.591065	59	1.770852	79	1.897627	99	1.995635
20	1.301030	40	1.602060	60	1.778151	80	1.903090	100	2.000000

	Value at 0°.	Sign in 1st Quad.	Value at 90°.	Sign in 2d Quad.	Value at 180°.	Sign in 3d Quad.	Value at 270°	Sign in 4th Quad.	Value at 360°.
Sin........	O	+	R	+	O	−	R	−	O
Tan.......	O	+	∞	−	O	+	∞	−	O
Sec.......	R	+	∞	−	R	−	∞	+	R
Versin....	O	+	R	+	2 R	+	R	+	O
Cos.......	R	+	O	−	R	−	O	+	R
Cot.... ...	∞	+	O	−	∞	+	O	−	∞
Cosec.....	∞	+	R	+	∞	−	R	−	∞

R signifies equal to rad; ∞ signifies infinite; O signifies evanescent.

″	′	Sine.	$q-l$		Tang.	Cotang.	$q+l$	D 1″	Cosine.	′
			4.685				15.314			
0	0	Inf. neg.	575	575	Inf. neg.	Inf. pos.	425		ten	60
60	1	6.463726	575	575	6.463726	13.536274	425		ten	59
120	2	.764756	575	575	.764756	.235244	425		ten	58
180	3	6.940847	575	575	6.940847	13.059153	425		ten	57
240	4	7.065786	575	575	7.065786	12.934214	425		ten	56
300	5	.162696	575	575	.162696	.837304	425		ten	55
360	6	.241877	575	575	.241878	.758122	425	.02	9.999999	54
420	7	.308824	575	575	.308825	.691175	425	.00	.999999	53
480	8	.366816	574	576	.366817	.633183	424	.00	.999999	52
540	9	.417968	574	576	.417970	.582030	424	.00	.999999	51
600	10	.463726	574	576	.463727	.536273	424	.02	.999998	50
660	11	7.505118	574	576	7.505120	12.494880	424	.00	9.999998	49
720	12	.542906	574	577	.542909	.457091	423	.02	.999997	48
780	13	.577668	574	577	.577672	.422328	423	.00	.999997	47
840	14	.609853	574	577	.609857	.390143	423	.02	.999996	46
900	15	.639816	573	578	.639820	.360180	422	.00	.999996	45
960	16	.667845	573	578	.667849	.332151	422	.02	.999995	44
1020	17	.694173	573	578	.694179	.305821	422	.00	.999995	43
1080	18	.718997	573	579	.719003	.280997	421	.02	.999994	42
1140	19	.742478	573	579	.742484	.257516	421	.02	.999993	41
1200	20	.764754	572	580	.764761	.235239	420	.00	.999993	40
1260	21	7.785943	572	580	7.785951	12.214049	420	.02	9.999992	39
1320	22	.806146	572	581	.806155	.193845	419	.02	.999991	38
1380	23	.825451	572	581	.825460	.174540	419	.02	.999990	37
1440	24	.843934	571	582	.843944	.156056	418	.02	.999989	36
1500	25	.861662	571	583	.861674	.138326	417	.00	.999989	35
1560	26	.878695	571	583	.878708	.121292	417	.02	.999988	34
1620	27	.895085	570	584	.895099	.104901	416	.02	.999987	33
1680	28	.910879	570	584	.910894	.089106	416	.02	.999986	32
1740	29	.926119	570	585	.926134	.073866	415	.02	.999985	31
1800	30	.940842	569	586	.940858	.059142	414	.03	.999983	30
1860	31	7.955082	569	587	7.955100	12.044900	413	.02	9.999982	29
1920	32	.968870	569	587	.968889	.031111	413	.02	.999981	28
1980	33	.982233	568	588	.982253	.017747	412	.02	.999980	27
2040	34	7.995198	568	589	7.995219	12.004781	411	.02	.999979	26
2100	35	8.007787	567	590	8.007809	11.992191	410	.03	.999977	25
2160	36	.020021	567	591	.020044	.979956	409	.02	.999976	24
2220	37	.031919	566	592	.031945	.968055	408	.02	.999975	23
2280	38	.043501	566	593	.043527	.956473	407	.03	.999973	22
2340	39	.054781	566	593	.054809	.945191	407	.02	.999972	21
2400	40	.065776	565	594	.065806	.934194	406	.02	.999971	20
2460	41	8.076500	565	595	8.076531	11.923469	405	.03	9.999969	19
2520	42	.086965	564	596	.086997	.913003	404	.02	.999968	18
2580	43	.097183	564	598	.097217	.902783	402	.03	.999966	17
2640	44	.107167	563	599	.107203	.892797	401	.03	.999964	16
2700	45	.116926	562	600	.116963	.883037	400	.02	.999963	15
2760	46	.126471	562	601	.126510	.873490	399	.03	.999961	14
2820	47	.135810	561	602	.135851	.864149	398	.03	.999959	13
2880	48	.144953	561	603	.144996	.855004	397	.02	.999958	12
2940	49	.153907	560	604	.153952	.846048	396	.03	.999956	11
3000	50	.162681	560	605	.162727	.837273	395	.03	.999954	10
3060	51	8.171280	559	607	8.171328	11.828672	393	.03	9.999952	9
3120	52	.179713	558	608	.179763	.820237	392	.03	.999950	8
3180	53	.187985	558	609	.188036	.811964	391	.03	.999948	7
3240	54	.196102	557	611	.196156	.803844	389	.03	.999946	6
3300	55	.204070	556	612	.204126	.795874	388	.03	.999944	5
3360	56	.211895	556	613	.211953	.788047	387	.03	.999942	4
3420	57	.219581	555	615	.219641	.780359	385	.03	.999940	3
3480	58	.227134	554	616	.227195	.772805	384	.03	.999938	2
3540	59	.234557	554	618	.234621	.765379	382	.03	.999936	
3600	60	8.241855	553	619	8.241921	11.758079	381	.03	9.999934	
			4.685				15.314			
″	′	Cosine.	$q-l$		Cotang.	Tang.	$q+l$	D 1″	Sin	

1° TABLE XXV.—LOGARITHMIC SINES, 178°

″	′	Sine.	q − l		Tang.	Cotang.	q + l	D 1″	Cosine.	′
			4.685				15.314			
3600	0	8.241855	553	619	8.241921	11.758079	381	.03	9.999934	60
3660	1	.249033	552	620	.249102	.750898	380	.05	.999932	59
3720	2	.256094	551	622	.256165	.743835	378	.03	.999929	58
3780	3	.263042	551	623	.263115	.736885	377	.03	.999927	57
3840	4	.269881	550	625	.269956	.730044	375	.05	.999925	56
3900	5	.276614	549	627	.276691	.723309	373	.02	.999922	55
3960	6	.283243	548	628	.283323	.716677	372	.03	.999920	54
4020	7	.289773	547	630	.289856	.710144	370	.05	.999918	53
4080	8	.296207	546	632	.296292	.703708	368	.03	.999915	52
4140	9	.302546	546	633	.302634	.697366	367	.05	.999913	51
4200	10	.308794	545	635	.308884	.691116	365	.05	.999910	50
4260	11	8.314954	544	637	8.315046	11.684954	363	.03	9.999907	49
4320	12	.321027	543	638	.321122	.678878	362	.05	.999905	48
4380	13	.327016	542	640	.327114	.672886	360	.05	.999902	47
4440	14	.332924	541	642	.333025	.666975	358	.03	.999899	46
4500	15	.338753	540	644	.338856	.661144	356	.05	.999897	45
4560	16	.344504	539	646	.344610	.655390	354	.05	.999894	44
4620	17	.350181	539	648	.350289	.649711	352	.05	.999891	43
4680	18	.355783	538	649	.355895	.644105	351	.05	.999888	42
4740	19	.361315	537	651	.361430	.638570	349	.05	.999885	41
4800	20	.366777	536	653	.366895	.633105	347	.05	.999882	40
4860	21	8.372171	535	655	8.372292	11.627708	345	.05	9.999879	39
4920	22	.377499	534	657	.377622	.622378	343	.05	.999876	38
4980	23	.382762	533	659	.382889	.617111	341	.05	.999873	37
5040	24	.387962	532	661	.388092	.611908	339	.05	.999870	36
5100	25	.393101	531	663	.393234	.606766	337	.05	.999867	35
5160	26	.398179	530	666	.398315	.601685	334	.05	.999864	34
5220	27	.403199	529	668	.403338	.596662	332	.05	.999861	33
5280	28	.408161	527	670	.408304	.591696	330	.07	.999858	32
5340	29	.413068	526	672	.413213	.586787	328	.05	.999854	31
5400	30	.417919	525	674	.418068	.581932	326	.05	.999851	30
5460	31	8.422717	524	676	8.422869	11.577131	324	.07	9.999848	29
5520	32	.427462	523	679	.427618	.572382	321	.05	.999844	28
5580	33	.432156	522	681	.432315	.567685	319	.05	.999841	27
5640	34	.436800	521	683	.436962	.563038	317	.07	.999838	26
5700	35	.441394	520	685	.441560	.558440	315	.05	.999834	25
5760	36	.445941	518	688	.446110	.553890	312	.07	.999831	24
5820	37	.450440	517	690	.450613	.549387	310	.05	.999827	23
5880	38	.454893	516	693	.455070	.544930	307	.07	.999824	22
5940	39	.459301	515	695	.459481	.540519	305	.07	.999820	21
6000	40	.463665	514	697	.463849	.536151	303	.05	.999816	20
6060	41	8.467985	512	700	8.468172	11.531828	300	.07	9.999813	19
6120	42	.472263	511	702	.472454	.527546	298	.07	.999809	18
6180	43	.476498	510	705	.476693	.523307	295	.07	.999805	17
6240	44	.480693	509	707	.480892	.519108	293	.07	.999801	16
6300	45	.484848	507	710	.485050	.514950	290	.05	.999797	15
6360	46	.488963	506	713	.489170	.510830	287	.07	.999794	14
6420	47	.493040	505	715	.493250	.506750	285	.07	.999790	13
6480	48	.497078	503	718	.497293	.502707	282	.07	.999786	12
6540	49	.501080	502	720	.501298	.498702	280	.07	.999782	11
6600	50	.505045	501	723	.505267	.494733	277	.07	.999778	10
6660	51	8.508974	499	726	8.509200	11.490800	274	.08	9.999774	9
6720	52	.512867	498	729	.513098	.486902	271	.07	.999769	8
6780	53	.516726	497	731	.516961	.483039	269	.07	.999765	7
6840	54	.520551	495	734	.520790	.479210	266	.07	.999761	6
6900	55	.524343	494	737	.524586	.475414	263	.07	.999757	5
6960	56	.528102	492	740	.528349	.471651	260	.08	.999753	4
7020	57	.531828	491	743	.532080	.467920	257	.07	.999748	3
7080	58	.535523	490	745	.535779	.464221	255	.07	.999744	2
7140	59	.539186	488	748	.539447	.460553	252	.08	.999740	1
7200	60	8.542819	487	751	8.543084	11.456916	249		9.999735	0
			4.685				15.314			
″	′	Cosine.	q − l		Cotang.	Tang.	q + l	D 1″	Sine.	

'	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	'
0	8.542819	60.05	9.999735	.07	8.543084	60.12	11.456916	60
1	.546422	59.55	.999731	.08	.546691	59.62	.453309	59
2	.549995	59.07	.999726	.07	.550268	59.15	.449732	58
3	.553539	58.58	.999722	.08	.553817	58.65	.446183	57
4	.557054	58.10	.999717	.07	.557336	58.20	.442664	56
5	.560540	57.65	.999713	.08	.560828	57.72	.439172	55
6	.563999	57.20	.999708	.07	.564291	57.27	.435709	54
7	.567431	56.75	.999704	.08	.567727	56.88	.432273	53
8	.570836	56.30	.999699	.08	.571137	56.38	.428863	52
9	.574214	55.87	.999694	.08	.574520	55.95	.425480	51
10	.577566	55.43	.999689	.07	.577877	55.52	.422123	50
11	8.580892	55.02	9.999685	.08	8.581208	55.10	11.418792	49
12	.584193	54.60	.999680	.08	.584514	54.68	.415486	48
13	.587469	54.20	.999675	.08	.587795	54.27	.412205	47
14	.590721	53.78	.999670	.08	.591051	53.87	.408949	46
15	.593948	53.40	.999665	.08	.594283	53.48	.405717	45
16	.597152	53.00	.999660	.08	.597492	53.08	.402508	44
17	.600332	52.62	.999655	.08	.600677	52.70	.399323	43
18	.603489	52.23	.999650	.08	.603839	52.32	.396161	42
19	.606623	51.85	.999645	.08	.606978	51.93	.393022	41
20	.609734	51.48	.999640	.08	.610094	51.58	.389906	40
21	8.612823	51.13	9.999635	.10	8.613189	51.22	11.386811	39
22	.615891	50.77	.999629	.08	.616262	50.85	.383738	38
23	.618937	50.42	.999624	.08	.619313	50.50	.380687	37
24	.621962	50.05	.999619	.08	.622343	50.15	.377657	36
25	.624965	49.72	.999614	.10	.625352	49.80	.374648	35
26	.627948	49.38	.999608	.08	.628340	49.47	.371660	34
27	.630911	49.05	.999603	.10	.631308	49.13	.368692	33
28	.633854	48.70	.999597	.08	.634256	48.80	.365744	32
29	.636776	48.40	.999592	.10	.637184	48.48	.362816	31
30	.639680	48.05	.999586	.08	.640093	48.15	.359907	30
31	8.642563	47.75	9.999581	.10	8.642982	47.85	11.357018	29
32	.645428	47.43	.999575	.08	.645853	47.52	.354147	28
33	.648274	47.13	.999570	.10	.648704	47.22	.351296	27
34	.651102	46.82	.999564	.10	.651537	46.92	.348463	26
35	.653911	46.52	.999558	.08	.654352	46.62	.345648	25
36	.656702	46.22	.999553	.10	.657149	46.32	.342851	24
37	.659475	45.92	.999547	.10	.659928	46.02	.340072	23
38	.662230	45.63	.999541	.10	.662689	45.73	.337311	22
39	.664968	45.35	.999535	.10	.665433	45.45	.334567	21
40	.667689	45.07	.999529	.08	.668160	45.17	.331840	20
41	8.670393	44.78	9.999524	.10	8.670870	44.88	11.329130	19
42	.673080	44.52	.999518	.10	.673563	44.60	.326437	18
43	.675751	44.23	.999512	.10	.676239	44.35	.323761	17
44	.678405	43.97	.999506	.10	.678900	44.07	.321100	16
45	.681043	43.70	.999500	.12	.681544	43.80	.318456	15
46	.683665	43.45	.999493	.10	.684172	43.53	.315828	14
47	.686272	43.18	.999487	.10	.686784	43.28	.313216	13
48	.688863	42.92	.999481	.10	.689381	43.03	.310619	12
49	.691438	42.67	.999475	.10	.691963	42.77	.308037	11
50	.693998	42.42	.999469	.10	.694529	42.53	.305471	10
51	8.696543	42.17	9.999463	.12	8.697081	42.27	11.302919	9
52	.699073	41.93	.999456	.10	.699617	42.03	.300383	8
53	.701589	41.68	.999450	.12	.702139	41.78	.297861	7
54	.704090	41.45	.999443	.10	.704646	41.57	.295354	6
55	.706577	41.20	.999437	.10	.707140	41.30	.292860	5
56	.709049	40.97	.999431	.12	.709618	41.08	.290382	4
57	.711507	40.75	.999424	.10	.712083	40.85	.287917	3
58	.713952	40.52	.999418	.12	.714534	40.63	.285466	2
59	.716383	40.28	.999411	.12	.716972	40.40	.283028	1
60	8.718800		9.999404		8.719396		11.280604	
'	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tan	

′	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	′
0	8.718800		9.999404		8.719396		11.280604	60
1	.721204	40.07	.999398	.10	.721806	40.17	.278194	59
2	.723595	39.85	.999391	.12	.724204	39.97	.275796	58
3	.725972	39.62	.999384	.12	.726588	39.73	.273412	57
4	.728337	39.42	.999378	.10	.728959	39.52	.271041	56
5	.730688	39.18	.999371	.12	.731317	39.30	.268683	55
6	.733027	38.98	.999364	.12	.733663	39.10	.266337	54
7	.735354	38.78	.999357	.12	.735996	38.88	.264004	53
8	.737667	38.55	.999350	.12	.738317	38.68	.261683	52
9	.739969	38.37	.999343	.12	.740626	38.48	.259374	51
10	.742259	38.17	.999336	.12	.742922	38.27	.257078	50
11	8.744536	37.95	9.999329	.12	8.745207	38.08	11.254793	49
12	.746802	37.77	.999322	.12	.747479	37.87	.252521	48
13	.749055	37.55	.999315	.12	.749740	37.68	.250260	47
14	.751297	37.37	.999308	.12	.751989	37.48	.248011	46
15	.753528	37.18	.999301	.12	.754227	37.30	.245773	45
16	.755747	36.98	.999294	.12	.756453	37.10	.243547	44
17	.757955	36.80	.999287	.12	.758668	36.92	.241332	43
18	.760151	36.60	.999279	.13	.760872	36.73	.239128	42
19	.762337	36.43	.999272	.12	.763065	36.55	.236935	41
20	.764511	36.23	.999265	.12	.765246	36.35	.234754	40
21	8.766675	36.07	9.999257	.13	8.767417	36.18	11.232583	39
22	.768828	35.88	.999250	.12	.769578	36.02	.230422	38
23	.770970	35.70	.999242	.13	.771727	35.82	.228273	37
24	.773101	35.52	.999235	.12	.773866	35.65	.226134	36
25	.775223	35.37	.999227	.13	.775995	35.48	.224005	35
26	.777333	35.17	.999220	.12	.778114	35.32	.221886	34
27	.779434	35.02	.999212	.13	.780222	35.13	.219778	33
28	.781524	34.83	.999205	.12	.782320	34.97	.217680	32
29	.783605	34.68	.999197	.13	.784408	34.80	.215592	31
30	.785675	34.50	.999189	.13	.786486	34.63	.213514	30
31	8.787736	34.35	9.999181	.13	8.788554	34.47	11.211446	29
32	.789787	34.18	.999174	.12	.790613	34.32	.209387	28
33	.791828	34.02	.999166	.13	.792662	34.15	.207338	27
34	.793859	33.85	.999158	.13	.794701	33.98	.205299	26
35	.795881	33.70	.999150	.13	.796731	33.83	.203269	25
36	.797894	33.55	.999142	.13	.798752	33.68	.201248	24
37	.799897	33.38	.999134	.13	.800763	33.52	.199237	23
38	.801892	33.25	.999126	.13	.802765	33.37	.197235	22
39	.803876	33.07	.999118	.13	.804758	33.22	.195242	21
40	.805852	32.93	.999110	.13	.806742	33.07	.193258	20
41	8.807819	32.78	9.999102	.13	8.808717	32.92	11.191283	19
42	.809777	32.63	.999094	.13	.810683	32.77	.189317	18
43	.811726	32.48	.999086	.13	.812641	32.63	.187359	17
44	.813667	32.35	.999077	.15	.814589	32.47	.185411	16
45	.815599	32.20	.999069	.13	.816529	32.33	.183471	15
46	.817522	32.05	.999061	.13	.818461	32.20	.181539	14
47	.819436	31.90	.999053	.13	.820384	32.05	.179616	13
48	.821343	31.78	.999044	.15	.822298	31.90	.177702	12
49	.823240	31.62	.999036	.13	.824205	31.78	.175795	11
50	.825130	31.50	.999027	.15	.826103	31.63	.173897	10
51	8.827011	31.35	9.999019	.13	8.827992	31.48	11.172008	9
52	.828884	31.22	.999010	.15	.829874	31.37	.170126	8
53	.830749	31.08	.999002	.13	.831748	31.23	.168252	7
54	.832607	30.97	.998993	.15	.833613	31.08	.166387	6
55	.834456	30.82	.998984	.15	.835471	30.97	.164529	5
56	.836297	30.68	.998976	.13	.837321	30.83	.162679	4
57	.838130	30.55	.998967	.15	.839163	30.70	.160837	3
58	.839956	30.43	.998958	.15	.840998	30.58	.159002	2
59	.841774	30.30	.998950	.13	.842825	30.47	.157175	1
60	8.843585	30.18	9.998941	.15	8.844644	30.32	11.155356	0
′	Cosine.	D 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	8.843585	30.03	9.998941	.15	8.844644	30.18	11.155356	60
1	.845387	29.93	.998932	.15	.846455	30.08	.153545	59
2	.847183	29.80	.998923	.15	.848260	29.95	.151740	58
3	.848971	29.67	.998914	.15	.850057	29.82	.149943	57
4	.850751	29.57	.998905	.15	.851846	29.70	.148154	56
5	.852525	29.43	.998896	.15	.853628	29.58	.146372	55
6	.854291	29.30	.998887	.15	.855403	29.47	.144597	54
7	.856049	29.20	.998878	.15	.857171	29.35	.142829	53
8	.857801	29.08	.998869	.15	.858932	29.23	.141068	52
9	.859546	28.95	.998860	.15	.860686	29.12	.139314	51
10	.861283	28.85	.998851	.17	.862433	29.00	.137567	50
11	8.863014	28.73	9.998841	.15	8.864173	28.88	11.135827	49
12	.864738	28.62	.998832	.15	.865906	28.77	.134094	48
13	.866455	28.50	.998823	.17	.867632	28.65	.132368	47
14	.868165	28.38	.998813	.15	.869351	28.55	.130649	46
15	.869868	28.28	.998804	.15	.871064	28.43	.128936	45
16	.871565	28.17	.998795	.17	.872770	28.32	.127230	44
17	.873255	28.05	.998785	.15	.874469	28.22	.125531	43
18	.874938	27.95	.998776	.17	.876162	28.12	.123838	42
19	.876615	27.83	.998766	.15	.877849	28.00	.122151	41
20	.878285	27.73	.998757	.17	.879529	27.88	.120471	40
21	8.879949	27.63	9.998747	.15	8.881202	27.78	11.118798	39
22	.881607	27.52	.998738	.17	.882869	27.68	.117131	38
23	.883258	27.42	.998728	.17	.884530	27.58	.115470	37
24	.884903	27.32	.998718	.17	.886185	27.47	.113815	36
25	.886542	27.20	.998708	.15	.887833	27.38	.112167	35
26	.888174	27.12	.998699	.17	.889476	27.27	.110524	34
27	.889801	27.00	.998689	.17	.891112	27.17	.108888	33
28	.891421	26.90	.998679	.17	.892742	27.07	.107258	32
29	.893035	26.80	.998669	.17	.894366	26.97	.105634	31
30	.894643	26.72	.998659	.17	.895984	26.87	.104016	30
31	8.896246	26.60	9.998649	.17	8.897596	26.78	11.102404	29
32	.897842	26.50	.998639	.17	.899203	26.67	.100797	28
33	.899432	26.42	.998629	.17	.900803	26.58	.099197	27
34	.901017	26.32	.998619	.17	.902398	26.48	.097602	26
35	.902596	26.22	.998609	.17	.903987	26.38	.096013	25
36	.904169	26.12	.998599	.17	.905570	26.28	.094430	24
37	.905736	26.02	.998589	.18	.907147	26.20	.092853	23
38	.907297	25.93	.998578	.17	.908719	26.10	.091281	22
39	.908853	25.85	.998568	.17	.910285	26.02	.089715	21
40	.910404	25.75	.998558	.17	.911846	25.92	.088154	20
41	8.911949	25.65	9.998548	.18	8.913401	25.83	11.086599	19
42	.913488	25.57	.998537	.17	.914951	25.73	.085049	18
43	.915022	25.47	.998527	.18	.916495	25.63	.083505	17
44	.916550	25.38	.998516	.17	.918034	25.57	.081966	16
45	.918073	25.30	.998506	.18	.919568	25.47	.080432	15
46	.919591	25.20	.998495	.17	.921096	25.38	.078904	14
47	.921103	25.12	.998485	.18	.922619	25.28	.077381	13
48	.922610	25.03	.998474	.17	.924136	25.22	.075864	12
49	.924112	24.95	.998464	.18	.925649	25.12	.074351	11
50	.925609	24.85	.998453	.18	.927156	25.03	.072844	10
51	8.927100	24.78	9.998442	.18	8.928658	24.95	11.071342	9
52	.928587	24.68	.998431	.17	.930155	24.87	.069845	8
53	.930068	24.60	.998421	.18	.931647	24.78	.068353	7
54	.931544	24.52	.998410	.18	.933134	24.70	.066866	6
55	.933015	24.43	.998399	.18	.934616	24.62	.065384	5
56	.934481	24.35	.998388	.18	.936093	24.53	.063907	4
57	.935942	24.27	.998377	.18	.937565	24.45	.062435	3
58	.937398	24.20	.998366	.18	.939032	24.37	.060968	2
59	.938850	24.10	.998355	.18	.940494	24.30	.059506	1
60	8.940296		9.998344		8.941952		11.058048	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

TABLE XXV.—LOGARITHMIC SINES,

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.
0	8.940296	24.03	9.998344	.18	8.941952	24.20	11.058048
1	.941738	23.93	.998333	.18	.943404	24.13	.056596
2	.943174	23.87	.998322	.18	.944852	24.05	.055148
3	.944606	23.80	.998311	.18	.946295	23.98	.053705
4	.946034	23.70	.998300	.18	.947734	23.90	.052266
5	.947456	23.63	.998289	.20	.949168	23.82	.050832
6	.948874	23.55	.998277	.18	.950597	23.73	.049403
7	.950287	23.48	.998266	.18	.952021	23.67	.047979
8	.951696	23.40	.998255	.20	.953441	23.58	.046559
9	.953100	23.32	.998243	.18	.954856	23.52	.045144
10	.954499	23.25	.998232	.20	.956267	23.45	.043733
11	8.955894	23.17	9.998220	.18	8.957674	23.35	11.042326
12	.957284	23.10	.998209	.20	.959075	23.30	.040925
13	.958670	23.03	.998197	.18	.960473	23.22	.039527
14	.960052	22.95	.998186	.20	.961866	23.15	.038134
15	.961429	22.87	.998174	.18	.963255	23.07	.036745
16	.962801	22.82	.998163	.20	.964639	23.00	.035361
17	.964170	22.73	.998151	.20	.966019	22.92	.033981
18	.965534	22.65	.998139	.18	.967394	22.87	.032606
19	.966893	22.60	.998128	.20	.968766	22.78	.031234
20	.968249	22.52	.998116	.20	.970133	22.72	.029867
21	8.969600	22.45	9.998104	.20	8.971496	22.65	11.028504
22	.970947	22.37	.998092	.20	.972855	22.57	.027145
23	.972289	22.32	.998080	.20	.974209	22.52	.025791
24	.973628	22.23	.998068	.20	.975560	22.43	.024440
25	.974962	22.18	.998056	.20	.976906	22.37	.023094
26	.976293	22.10	.998044	.20	.978248	22.30	.021752
27	.977619	22.03	.998032	.20	.979586	22.25	.020414
28	.978941	21.97	.998020	.20	.980921	22.17	.019079
29	.980259	21.90	.998008	.20	.982251	22.10	.017749
30	.981573	21.83	.997996	.20	.983577	22.03	.016423
31	8.982883	21.77	9.997984	.20	8.984899	21.97	11.015101
32	.984189	21.72	.997972	.22	.986217	21.92	.013783
33	.985491	21.63	.997959	.20	.987532	21.83	.012468
34	.986789	21.57	.997947	.20	.988842	21.78	.011158
35	.988083	21.52	.997935	.22	.990149	21.70	.009851
36	.989374	21.43	.997922	.20	.991451	21.65	.008549
37	.990660	21.38	.997910	.22	.992750	21.58	.007250
38	.991943	21.32	.997897	.20	.994045	21.53	.005955
39	.993222	21.25	.997885	.22	.995337	21.45	.004663
40	.994497	21.18	.997872	.20	.996624	21.40	.003376
41	8.995768	21.13	9.997860	.22	8.997908	21.33	11.002092
42	.997036	21.05	.997847	.20	8.999188	21.28	11.000812
43	.998299	21.02	.997835	.22	9.000465	21.22	10.999535
44	8.999560	20.93	.997822	.22	.001738	21.15	.998262
45	9.000816	20.88	.997809	.20	.003007	21.08	.996993
46	.002069	20.82	.997797	.22	.004272	21.03	.995728
47	.003318	20.75	.997784	.22	.005534	20.97	.994466
48	.004563	20.70	.997771	.22	.006792	20.92	.993208
49	.005805	20.65	.997758	.22	.008047	20.85	.991953
50	.007044	20.57	.997745	.22	.009298	20.80	.990702
51	9.008278	20.53	9.997732	.22	9.010546	20.73	10.989454
52	.009510	20.45	.997719	.22	.011790	20.68	.988210
53	.010737	20.42	.997706	.22	.013031	20.62	.986969
54	.011962	20.33	.997693	.22	.014268	20.57	.985732
55	.013182	20.30	.997680	.22	.015502	20.50	.984498
56	.014400	20.22	.997667	.22	.016732	20.45	.983268
57	.015613	20.18	.997654	.22	.017959	20.40	.982041
58	.016824	20.12	.997641	.22	.019183	20.33	.980817
59	.018031	20.07	.997628	.23	.020403	20.28	.979597
60	9.019235		9.997614		9.021620		10.978380
	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.019235	20.00	9.997614	.22	9.021620	20.23	10.978380	60
1	.020435	19.95	.997601	.22	.022834	20.17	.977166	59
2	.021632	19.88	.997588	.23	.024044	20.12	.975956	58
3	.022825	19.85	.997574	.22	.025251	20.07	.974749	57
4	.024016	19.78	.997561	.23	.026455	20.00	.973545	56
5	.025203	19.72	.997547	.22	.027655	19.95	.972345	55
6	.026386	19.68	.997534	.23	.028852	19.90	.971148	54
7	.027567	19.62	.997520	.22	.030046	19.85	.969954	53
8	.028744	19.57	.997507	.23	.031237	19.80	.968763	52
9	.029918	19 52	.997493	.22	.032425	19.73	.967575	51
10	.031089	19.47	.997480	.23	.033609	19.70	.966391	50
11	9.032257	19.40	9.997466	.23	9.034791	19.63	10.965209	49
12	.033421	19.35	.997452	.22	.035969	19.58	.964031	48
13	.034582	19.32	.997439	.23	.037144	19.53	.962856	47
14	.035741	19.25	.997425	.23	.038316	19.48	.961684	46
15	.036896	19.20	.997411	.23	.039485	19.43	.960515	45
16	.038048	19.15	.997397	.23	.040651	19.37	.959349	44
17	.039197	19.08	.997383	.23	.041813	19.33	.958187	43
18	.040342	19.05	.997369	.23	.042973	19.28	.957027	42
19	.041485	19.00	.997355	.23	.044130	19.23	.955870	41
20	.042625	18.95	.997341	.23	.045284	19.17	.954716	40
21	9.043762	18.88	9.997327	.23	9.046434	19.13	10.953566	39
22	.044895	18.85	.997313	.23	.047582	19.08	.952418	38
23	.046026	18.80	.997299	.23	.048727	19.03	.951273	37
24	.047154	18.75	.997285	.23	.049869	18.98	.950131	36
25	.048279	18.68	.997271	.23	.051008	18.93	.948992	35
26	.049400	18.65	.997257	.25	.052144	18.88	.947856	34
27	.050519	18.60	.997242	.23	.053277	18 83	.946723	33
28	.051635	18.57	.997228	.23	.054407	18.80	.945593	32
29	.052749	18.50	.997214	.25	.055535	18.73	.944465	31
30	.053859	18.45	.997199	.23	.056659	18.70	.943341	30
31	9.054966	18.42	9.997185	.25	9.057781	18.65	10.942219	29
32	.056071	18.35	.997170	.23	.058900	18.60	.941100	28
33	.057172	18.32	.997156	.25	.060016	18.57	.939984	27
34	.058271	18.27	.997141	.23	.061130	18.50	.938870	26
35	.059367	18.22	.997127	.25	.062240	18.47	.937760	25
36	.060460	18.18	.997112	.23	.063348	18.42	.936652	24
37	.061551	18.13	.997098	.25	.064453	18.38	.935547	23
38	.062639	18.08	.997083	.25	.065556	18.32	.934444	22
39	.063724	18.03	.997068	.25	.066655	18.28	.933345	21
40	.064806	17.98	.997053	.23	.067752	18.25	.932248	20
41	9.065885	17.95	9.997039	.25	9.068846	18.20	10.931154	19
42	.066962	17.90	.997024	.25	.069938	18.15	.930062	18
43	.068036	17.85	.997009	.25	.071027	18.10	.928973	17
44	.069107	17.82	.996994	.25	.072113	18.07	.927887	16
45	.070176	17.77	.996979	.25	.073197	18.02	.926803	15
46	.071242	17.73	.996964	.25	.074278	17.97	.925722	14
47	.072306	17.67	.996949	.25	.075356	17.93	.924644	13
48	.073366	17.63	.996934	.25	.076432	17.88	.923568	12
49	.074424	17.60	.996919	.25	.077505	17.85	.922495	11
50	.075480	17.55	.996904	.25	.078576	17.80	.921424	10
51	9.076533	17.50	9.996889	.25	9.079644	17.77	10.920356	9
52	.077583	17.47	.996874	.27	.080710	17.72	.919290	8
53	.078631	17.42	.996858	.25	.081773	17.67	.918227	7
54	.079676	17.38	.996843	.27	.082833	17.63	.917167	6
55	.080719	17.33	.996828	.27	.083891	17.60	.916109	5
56	.081759	17.30	.996812	.25	.084947	17.55	.915053	4
57	.082797	17.25	.996797	.25	.086000	17.50	.914000	3
58	.083832	17.20	.996782	.27	.087050	17.47	.912950	2
59	.084864	17.17	.996766	.25	.088098	17.43	.911902	1
60	9.085894		9.996751		9.089144		10.910856	
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.085894	17.13	9.996751	.27	9.089144	17.38	10.910856	60
1	.086922	17.08	.996735	.25	.090187	17.35	.909813	59
2	.087947	17.05	.996720	.27	.091228	17.30	.908772	58
3	.088970	17.00	.996704	.27	.092266	17.27	.907734	57
4	.089990	16.97	.996688	.25	.093302	17.23	.906698	56
5	.091008	16.93	.996673	.27	.094336	17.18	.905664	55
6	.092024	16.88	.996657	.27	.095367	17.13	.904633	54
7	.093037	16.83	.996641	.27	.096395	17.12	.903605	53
8	.094047	16.82	.996625	.25	.097422	17.07	.902578	52
9	.095056	16.77	.996610	.27	.098446	17.03	.901554	51
10	.096062	16.72	.996594	.27	.099468	16.98	.900532	50
11	9.097065	16.68	9.996578	.27	9.100487	16.95	10.899513	49
12	.098066	16.65	.996562	.27	.101504	16.92	.898496	48
13	.099065	16.62	.996546	.27	.102519	16.88	.897481	47
14	.100062	16.57	.996530	.27	.103532	16.83	.896468	46
15	.101056	16.53	.996514	.27	.104542	16.80	.895458	45
16	.102048	16.48	.996498	.27	.105550	16.77	.894450	44
17	.103037	16.47	.996482	.28	.106556	16.72	.893444	43
18	.104025	16.42	.996465	.27	.107559	16.68	.892441	42
19	.105010	16.37	.996449	.27	.108560	16.65	.891440	41
20	.105992	16.35	.996433	.27	.109559	16.62	.890441	40
21	9.106973	16.30	9.996417	.28	9.110556	16.58	10.889444	39
22	.107951	16.27	.996400	.27	.111551	16.53	.888449	38
23	.108927	16.23	.996384	.27	.112543	16.50	.887457	37
24	.109901	16.20	.996368	.28	.113533	16.47	.886467	36
25	.110873	16.15	.996351	.27	.114521	16.43	.885479	35
26	.111842	16.12	.996335	.28	.115507	16.40	.884493	34
27	.112809	16.08	.996318	.27	.116491	16.35	.883509	33
28	.113774	16.05	.996302	.28	.117472	16.33	.882528	32
29	.114737	16.02	.996285	.27	.118452	16.28	.881548	31
30	.115698	15.97	.996269	.28	.119429	16.25	.880571	30
31	9.116656	15.95	9.996252	.28	9.120404	16.22	10.879596	29
32	.117613	15.90	.996235	.27	.121377	16.18	.878623	28
33	.118567	15.87	.996219	.28	.122348	16.15	.877652	27
34	.119519	15.83	.996202	.28	.123317	16.12	.876683	26
35	.120469	15.80	.996185	.28	.124284	16.08	.875716	25
36	.121417	15.75	.996168	.28	.125249	16.03	.874751	24
37	.122362	15.73	.996151	.28	.126211	16.02	.873789	23
38	.123306	15.70	.996134	.28	.127172	15.97	.872828	22
39	.124248	15.65	.996117	.28	.128130	15.95	.871870	21
40	.125187	15.63	.996100	.28	.129087	15.90	.870913	20
41	9.126125	15.58	9.996083	.28	9.130041	15.88	10.869959	19
42	.127060	15.55	.996066	.28	.130994	15.83	.869006	18
43	.127993	15.53	.996049	.28	.131944	15.82	.868056	17
44	.128925	15.48	.996032	.28	.132893	15.77	.867107	16
45	.129854	15.45	.996015	.28	.133839	15.75	.866161	15
46	.130781	15.42	.995998	.30	.134784	15.70	.865216	14
47	.131706	15.40	.995980	.28	.135726	15.68	.864274	13
48	.132630	15.35	.995963	.28	.136667	15.63	.863333	12
49	.133551	15.32	.995946	.30	.137605	15.62	.862395	11
50	.134470	15.28	.995928	.28	.138542	15.57	.861458	10
51	9.135387	15.27	9.995911	.28	9.139476	15.55	10.860524	9
52	.136303	15.22	.995894	.30	.140409	15.52	.859591	8
53	.137216	15.20	.995876	.28	.141340	15.48	.858660	7
54	.138128	15.15	.995859	.30	.142269	15.45	.857731	6
55	.139037	15.12	.995841	.30	.143196	15.42	.856804	5
56	.139944	15.10	.995823	.28	.144121	15.38	.855879	4
57	.140850	15.07	.995806	.30	.145044	15.37	.854956	3
58	.141754	15.02	.995788	.28	.145966	15.32	.854034	2
59	.142655	15.00	.995771	.30	.146885	15.30	.853115	1
60	9.143555		9.995753		9.147803		10.852197	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

97°

N.	0	1	2	3	4	5	6	7	8	9	Diff.
990	995635	5679	5723	5767	5811	5854	5898	5942	5986	6030	
1	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
2	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	
3	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	
4	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	
5	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	
6	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	
7	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	
8	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	
9	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43

LOGARITHMS OF NUMBERS FROM 1 TO 100.

N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
1	0.000000	21	1.322219	41	1.612784	61	1.785330	81	1.908485
2	0.301030	22	1.342423	42	1.623249	62	1.792392	82	1.913814
3	0.477121	23	1.361728	43	1.633468	63	1.799341	83	1.919078
4	0.602060	24	1.380211	44	1.643453	64	1.806180	84	1.924279
5	0.698970	25	1.397940	45	1.653213	65	1.812913	85	1.929419
6	0.778151	26	1.414973	46	1.662758	66	1.819544	86	1.934498
7	0.845098	27	1.431364	47	1.672098	67	1.826075	87	1.939519
8	0.903090	28	1.447158	48	1.681241	68	1.832509	88	1.944483
9	0.954243	29	1.462398	49	1.690196	69	1.838849	89	1.949390
10	1.000000	30	1.477121	50	1.698970	70	1.845098	90	1.954243
11	1.041393	31	1.491362	51	1.707570	71	1.851258	91	1.959041
12	1.079181	32	1.505150	52	1.716003	72	1.857332	92	1.963788
13	1.113943	33	1.518514	53	1.724276	73	1.863323	93	1.968483
14	1.146128	34	1.531479	54	1.732394	74	1.869232	94	1.973128
15	1.176091	35	1.544068	55	1.740363	75	1.875061	95	1.977724
16	1.204120	36	1.556302	56	1.748188	76	1.880814	96	1.982271
17	1.230449	37	1.568202	57	1.755875	77	1.886491	97	1.986772
18	1.255273	38	1.579784	58	1.763428	78	1.892095	98	1.991226
19	1.278754	39	1.591065	59	1.770852	79	1.897627	99	1.995635
20	1.301030	40	1.602060	60	1.778151	80	1.903090	100	2.000000

	Value at 0°.	Sign in 1st Quad.	Value at 90°.	Sign in 2d Quad.	Value at 180°.	Sign in 3d Quad.	Value at 270°	Sign in 4th Quad.	Value at 360°.
Sin........	O	+	R	+	O	−	R	−	O
Tan.......	O	+	∞	−	O	+	∞	−	O
Sec	R	+	∞	−	R	−	∞	+	R
Versin....	O	+	R	+	2 R	+	R	+	O
Cos	R	+	O	−	R	−	O	+	R
Cot.... ...	∞	+	O	−	∞	+	O	−	∞
Cosec.....	∞	+	R	+	∞	−	R	−	∞

R signifies equal to rad; ∞ signifies infinite; O signifies evanescent.

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.194332	13.28	9.994620	.33	9.199713	13.60	10.800287	60
1	.195129	13.27	.994600	.33	.200529	13.60	.799471	59
2	.195925	13.23	.994580	.33	.201345	13.57	.798655	58
3	.196719	13.20	.994560	.33	.202159	13.53	.797841	57
4	.197511	13.18	.994540	.35	.202971	13.52	.797029	56
5	.198302	13.15	.994519	.33	.203782	13.50	.796218	55
6	.199091	13.13	.994499	.33	.204592	13.47	.795408	54
7	.199879	13.12	.994479	.33	.205400	13.45	.794600	53
8	.200666	13.08	.994459	.35	.206207	13.43	.793793	52
9	.201451	13.05	.994438	.33	.207013	13.40	.792987	51
10	.202234	13.05	.994418	.33	.207817	13.37	.792183	50
11	9.203017	13.00	9.994398	.35	9.208619	13.35	10.791381	49
12	.203797	13.00	.994377	.33	.209420	13.33	.790580	48
13	.204577	12.95	.994357	.35	.210220	13.30	.789780	47
14	.205354	12.95	.994336	.33	.211018	13.28	.788982	46
15	.206131	12.92	.994316	.35	.211815	13.27	.788185	45
16	.206906	12.88	.994295	.35	.212611	13.23	.787389	44
17	.207679	12.88	.994274	.33	.213405	13.22	.786595	43
18	.208452	12.83	.994254	.35	.214198	13.18	.785802	42
19	.209222	12.83	.994233	.35	.214989	13.18	.785011	41
20	.209992	12.80	.994212	.35	.215780	13.13	.784220	40
21	9.210760	12.77	9.994191	.33	9.216568	13.13	10.783432	39
22	.211526	12.75	.994171	.35	.217356	13.10	.782644	38
23	.212291	12.73	.994150	.35	.218142	13.07	.781858	37
24	.213055	12.72	.994129	.35	.218926	13.07	.781074	36
25	.213818	12.68	.994108	.35	.219710	13.03	.780290	35
26	.214579	12.65	.994087	.35	.220492	13.00	.779508	34
27	.215338	12.65	.994066	.35	.221272	13.00	.778728	33
28	.216097	12.62	.994045	.35	.222052	12.97	.777948	32
29	.216854	12.58	.994024	.35	.222830	12.95	.777170	31
30	.217609	12.57	.994003	.35	.223607	12.92	.776393	30
31	9.218363	12.55	9.993982	.37	9.224382	12.90	10.775618	29
32	.219116	12.53	.993960	.35	.225156	12.88	.774844	28
33	.219868	12.50	.993939	.35	.225929	12.85	.774071	27
34	.220618	12.48	.993918	.35	.226700	12.85	.773300	26
35	.221367	12.47	.993897	.37	.227471	12.80	.772529	25
36	.222115	12.43	.993875	.35	.228239	12.80	.771761	24
37	.222861	12.42	.993854	.37	.229007	12.77	.770993	23
38	.223606	12.38	.993832	.35	.229773	12.77	.770227	22
39	.224349	12.38	.993811	.37	.230539	12.72	.769461	21
40	.225092	12.35	.993789	.35	.231302	12.72	.768698	20
41	9.225833	12.33	9.993768	.37	9.232065	12.68	10.767935	19
42	.226573	12.30	.993746	.35	.232826	12.67	.767174	18
43	.227311	12.28	.993725	.37	.233586	12.65	.766414	17
44	.228048	12.27	.993703	.37	.234345	12.63	.765655	16
45	.228784	12.23	.993681	.35	.235103	12.60	.764897	15
46	.229518	12.23	.993660	.37	.235859	12.58	.764141	14
47	.230252	12.20	.993638	.37	.236614	12.57	.763386	13
48	.230984	12.18	.993616	.37	.237368	12.53	.762632	12
49	.231715	12.15	.993594	.37	.238120	12.53	.761880	11
50	.232444	12.13	.993572	.37	.238872	12.50	.761128	10
51	9.233172	12.12	9.993550	.37	9.239622	12.48	10.760378	9
52	.233899	12.10	.993528	.37	.240371	12.45	.759629	8
53	.234625	12.07	.993506	.37	.241118	12.45	.758882	7
54	.235349	12.07	.993484	.37	.241865	12.42	.758135	6
55	.236073	12.03	.993462	.37	.242610	12.40	.757390	5
56	.236795	12.00	.993440	.37	.243354	12.38	.756646	4
57	.237515	12.00	.993418	.37	.244097	12.37	.755903	3
58	.238235	11.97	.993396	.37	.244839	12.33	.755161	2
59	.238953	11.95	.993374	.38	.245579	12.33	.754421	1
60	9.239670		9.993351		9.246319		10.753681	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	′
9.440338	7.33	9.982842	.62	9.457496	7.95	10.542504	60
.440778	7.33	.982805	.60	.457973	7.93	.542027	59
.441218	7.33	.982769	.60	.458449	7.93	.541551	58
.441658	7.30	.982733	.62	.458925	7.92	.541075	57
.442096	7.32	.982696	.60	.459400	7.92	.540600	56
.442535	7.30	.982660	.60	.459875	7.90	.540125	55
.442973	7.28	.982624	.62	.460349	7.90	.539651	54
.443410	7.28	.982587	.60	.460823	7.90	.539177	53
.443847	7.28	.982551	.62	.461297	7.88	.538703	52
.444284	7.27	.982514	.62	.461770	7.87	.538230	51
.444720	7.25	.982477	.60	.462242	7.88	.537758	50
9.445155	7.25	9.982441	.62	9.462715	7.85	10.537285	49
.445590	7.25	.982404	.62	.463186	7.87	.536814	48
.446025	7.23	.982367	.60	.463658	7.83	.536342	47
.446459	7.23	.982331	.62	.464128	7.85	.535872	46
.446893	7.22	.982294	.62	.464599	7.83	.535401	45
.447326	7.22	.982257	.62	.465069	7.83	.534931	44
.447759	7.20	.982220	.62	.465539	7.82	.534461	43
.448191	7.20	.982183	.62	.466008	7.82	.533992	42
.448623	7.18	.982146	.62	.466477	7.80	.533523	41
.449054	7.18	.982109	.62	.466945	7.80	.533055	40
9.449485	7.17	9.982072	.62	9.467413	7.78	10.532587	39
.449915	7.17	.982035	.62	.467880	7.78	.532120	38
.450345	7.17	.981998	.62	.468347	7.78	.531653	37
.450775	7.15	.981961	.62	.468814	7.77	.531186	36
.451204	7.13	.981924	.63	.469280	7.77	.530720	35
.451632	7.13	.981886	.62	.469746	7.75	.530254	34
.452060	7.13	.981849	.62	.470211	7.75	.529789	33
.452488	7.12	.981812	.63	.470676	7.75	.529324	32
.452915	7.12	.981774	.62	.471141	7.73	.528859	31
.453342	7.10	.981737	.62	.471605	7.73	.528395	30
9.453768	7.10	9.981700	.63	9.472069	7.72	10.527931	29
.454194	7.08	.981662	.62	.472532	7.72	.527468	28
.454619	7.08	.981625	.63	.472995	7.70	.527005	27
.455044	7.08	.981587	.63	.473457	7.70	.526543	26
.455469	7.07	.981549	.62	.473919	7.70	.526081	25
.455893	7.05	.981512	.63	.474381	7.68	.525619	24
.456316	7.05	.981474	.63	.474842	7.68	.525158	23
.456739	7.05	.981436	.62	.475303	7.67	.524697	22
.457162	7.03	.981399	.63	.475763	7.67	.524237	21
.457584	7.03	.981361	.63	.476223	7.67	.523777	20
9.458006	7.02	9.981323	.63	9.476683	7.65	10.523317	19
.458427	7.02	.981285	.63	.477142	7.65	.522858	18
.458848	7.00	.981247	.63	.477601	7.63	.522399	17
.459268	7.00	.981209	.63	.478059	7.63	.521941	16
.459688	7.00	.981171	.63	.478517	7.63	.521483	15
.460108	6.98	.981133	.63	.478975	7.62	.521025	14
.460527	6.98	.981095	.63	.479432	7.62	.520568	13
.460946	6.97	.981057	.63	.479889	7.60	.520111	12
.461364	6.97	.981019	.63	.480345	7.60	.519655	11
.461782	6.95	.980981	.65	.480801	7.60	.519199	10
9.462199	6.95	9.980942	.63	9.481257	7.58	10.518743	9
.462616	6.93	.980904	.63	.481712	7.58	.518288	8
.463032	6.93	.980866	.65	.482167	7.57	.517833	7
.463448	6.93	.980827	.63	.482621	7.57	.517379	6
.463864	6.92	.980789	.65	.483075	7.57	.516925	5
.464279	6.92	.980750	.63	.483529	7.55	.516471	4
.464694	6.90	.980712	.65	.483982	7.55	.516018	3
.465108	6.90	.980673	.63	.484435	7.53	.515565	2
.465522	6.88	.980635	.65	.484887	7.53	.515113	1
9.465935		9.980596		9.485339		10.514661	0
osine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.465935	6.88	9.980596	.63	9.485339	7.53	10.514661	60
1	.466348	6.88	.980558	.65	.485791	7.52	.514209	59
2	.466761	6.87	.980519	.65	.486242	7.52	.513758	58
3	.467173	6.87	.980480	.63	.486693	7.50	.513307	57
4	.467585	6.85	.980442	.65	.487143	7.50	.512857	56
5	.467996	6.85	.980403	.65	.487593	7.50	.512407	55
6	.468407	6.83	.980364	.65	.488043	7.48	.511957	54
7	.468817	6.83	.980325	.65	.488492	7.48	.511508	53
8	.469227	6.83	.980286	.65	.488941	7.48	.511059	52
9	.469637	6.82	.980247	.65	.489390	7.47	.510610	51
10	.470046	6.82	.980208	.65	.489838	7.47	.510162	50
11	9.470455	6.80	9.980169	.65	9.490286	7.45	10.509714	49
12	.470863	6.80	.980130	.65	.490733	7.45	.509267	48
13	.471271	6.80	.980091	.65	.491180	7.45	.508820	47
14	.471679	6.78	.980052	.67	.491627	7.43	.508373	46
15	.472086	6.77	.980012	.65	.492073	7.43	.507927	45
16	.472492	6.77	.979973	.65	.492519	7.43	.507481	44
17	.472898	6.77	.979934	.65	.492965	7.42	.507035	43
18	.473304	6.77	.979895	.67	.493410	7.40	.506590	42
19	.473710	6.75	.979855	.65	.493854	7.42	.506146	41
20	.474115	6.73	.979816	.67	.494299	7.40	.505701	40
21	9.474519	6.73	9.979776	.65	9.494743	7.38	10.505257	39
22	.474923	6.73	.979737	.67	.495186	7.40	.504814	38
23	.475327	6.72	.979697	.65	.495630	7.38	.504370	37
24	.475730	6.72	.979658	.67	.496073	7.37	.503927	36
25	.476133	6.72	.979618	.65	.496515	7.37	.503485	35
26	.476536	6.70	.979579	.67	.496957	7.37	.503043	34
27	.476938	6.70	.979539	.67	.497399	7.37	.502601	33
28	.477340	6.68	.979499	.67	.497841	7.35	.502159	32
29	.477741	6.68	.979459	.65	.498282	7.33	.501718	31
30	.478142	6.67	.979420	.67	.498722	7.35	.501278	30
31	9.478542	6.67	9.979380	.67	9.499163	7.33	10.500837	29
32	.478942	6.67	.979340	.67	.499603	7.32	.500397	28
33	.479342	6.65	.979300	.67	.500042	7.32	.499958	27
34	.479741	6.65	.979260	.67	.500481	7.32	.499519	26
35	.480140	6.65	.979220	.67	.500920	7.32	.499080	25
36	.480539	6.63	.979180	.67	.501359	7.30	.498641	24
37	.480937	6.62	.979140	.67	.501797	7.30	.498203	23
38	.481334	6.62	.979100	.68	.502235	7.28	.497765	22
39	.481731	6.62	.979059	.67	.502672	7.28	.497328	21
40	.482128	6.62	.979019	.67	.503109	7.28	.496891	20
41	9.482525	6.60	9.978979	.67	9.503546	7.27	10.496454	19
42	.482921	6.58	.978939	.68	.503982	7.27	.496018	18
43	.483316	6.60	.978898	.67	.504418	7.27	.495582	17
44	.483712	6.58	.978858	.68	.504854	7.25	.495146	16
45	.484107	6.57	.978817	.67	.505289	7.25	.494711	15
46	.484501	6.57	.978777	.67	.505724	7.25	.494276	14
47	.484895	6.57	.978737	.68	.506159	7.23	.493841	13
48	.485289	6.55	.978696	.68	.506593	7.23	.493407	12
49	.485682	6.55	.978655	.67	.507027	7.23	.492973	11
50	.486075	6.53	.978615	.68	.507460	7.22	.492540	10
51	9.486467	6.55	9.978574	.68	9.507893	7.22	10.492107	9
52	.486860	6.52	.978533	.67	.508326	7.22	.491674	8
53	.487251	6.53	.978493	.68	.508759	7.20	.491241	7
54	.487643	6.52	.978452	.68	.509191	7.18	.490809	6
55	.488034	6.50	.978411	.68	.509622	7.20	.490378	5
56	.488424	6.50	.978370	.68	.510054	7.18	.489946	4
57	.488814	6.50	.978329	.68	.510485	7.18	.489515	3
58	.489204	6.48	.978288	.68	.510916	7.17	.489084	2
59	.489593	6.48	.978247	.68	.511346	7.17	.488654	1
60	9.489982		9.978206		9.511776		10.488224	0
′	Cosine.	D 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.317879	9.90	9.990404	.43	9.327475	10.33	10.672525	60
1	.318473	9.88	.990378	.45	.328095	10.33	.671905	59
2	.319066	9.87	.990351	.45	.328715	10.32	.671285	58
3	.319658	9.85	.990324	.45	.329334	10.32	.670666	57
4	.320249	9.85	.990297	.45	.329953	10.28	.670047	56
5	.320840	9.83	.990270	.45	.330570	10.28	.669430	55
6	.321430	9.82	.990243	.47	.331187	10.27	.668813	54
7	.322019	9.80	.990215	.45	.331803	10.25	.668197	53
8	.322607	9.78	.990188	.45	.332418	10.25	.667582	52
9	.323194	9.77	.990161	.45	.333033	10.22	.666967	51
10	.323780	9.77	.990134	.45	.333646	10.22	.666354	50
11	9.324366	9.73	9.990107	.47	9.334259	10.20	10.665741	49
12	.324950	9.73	.990079	.45	.334871	10.18	.665129	48
13	.325534	9.72	.990052	.45	.335482	10.18	.664518	47
14	.326117	9.72	.990025	.47	.336093	10.15	.663907	46
15	.326700	9.68	.989997	.45	.336702	10.15	.663298	45
16	.327281	9.68	.989970	.47	.337311	10.13	.662689	44
17	.327862	9.67	.989942	.45	.337919	10.13	.662081	43
18	.328442	9.65	.989915	.47	.338527	10.10	.661473	42
19	.329021	9.63	.989887	.45	.339133	10.10	.660867	41
20	.329599	9.62	.989860	.47	.339739	10.08	.660261	40
21	9.330176	9.62	9.989832	.47	9.340344	10.07	10.659656	39
22	.330753	9.60	.989804	.45	.340948	10.07	.659052	38
23	.331329	9.57	.989777	.47	.341552	10.05	.658448	37
24	.331903	9.58	.989749	.47	.342155	10.03	.657845	36
25	.332478	9.55	.989721	.47	.342757	10.02	.657243	35
26	.333051	9.55	.989693	.47	.343358	10.00	.656642	34
27	.333624	9.52	.989665	.47	.343958	10.00	.656042	33
28	.334195	9.53	.989637	.45	.344558	9.98	.655442	32
29	.334767	9.50	.989610	.47	.345157	9.97	.654843	31
30	.335337	9.48	.989582	.48	.345755	9.97	.654245	30
31	9.335906	9.48	9.989553	.47	9.346353	9.93	10.653647	29
32	.336475	9.47	.989525	.47	.346949	9.93	.653051	28
33	.337043	9.45	.989497	.47	.347545	9.93	.652455	27
34	.337610	9.43	.989469	.47	.348141	9.90	.651859	26
35	.338176	9.43	.989441	.47	.348735	9.90	.651265	25
36	.338742	9.42	.989413	.47	.349329	9.88	.650671	24
37	.339307	9.40	.989385	.48	.349922	9.87	.650078	23
38	.339871	9.38	.989356	.47	.350514	9.87	.649486	22
39	.340434	9.37	.989328	.47	.351106	9.85	.648894	21
40	.340996	9.37	.989300	.48	.351697	9.83	.648303	20
41	9.341558	9.35	9.989271	.47	9.352287	9.82	10.647713	19
42	.342119	9.33	.989243	.48	.352876	9.82	.647124	18
43	.342679	9.33	.989214	.47	.353465	9.80	.646535	17
44	.343239	9.30	.989186	.48	.354053	9.78	.645947	16
45	.343797	9.30	.989157	.48	.354640	9.78	.645360	15
46	.344355	9.28	.989128	.47	.355227	9.77	.644773	14
47	.344912	9.28	.989100	.48	.355813	9.75	.644187	13
48	.345469	9.25	.989071	.48	.356398	9.73	.643602	12
49	.346024	9.25	.989042	.47	.356982	9.73	.643018	11
50	.346579	9.25	.989014	.48	.357566	9.72	.642434	10
51	9.347134	9.22	9.988985	.48	9.358149	9.70	10.641851	9
52	.347687	9.22	.988956	.48	.358731	9.70	.641269	8
53	.348240	9.20	.988927	.48	.359313	9.67	.640687	7
54	.348792	9.18	.988898	.48	.359893	9.68	.640107	6
55	.349343	9.17	.988869	.48	.360474	9.65	.639526	5
56	.349893	9.17	.988840	.48	.361053	9.65	.638947	4
57	.350443	9.15	.988811	.48	.361632	9.63	.638368	3
58	.350992	9.13	.988782	.48	.362210	9.62	.637790	2
59	.351540	9.13	.988753	.48	.362787	9.62	.637213	1
60	9.352088		9.988724		9.363364		10.636636	0
′	Cosine.	D. 1″.	Sine,	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.352088	9.12	9.988724	.48	9.363364	9.60	10.636636	60
1	.352635	9.10	.988695	.48	.363940	9.58	.636060	59
2	.353181	9.08	.988666	.50	.364515	9.58	.635485	58
3	.353726	9.08	.988636	.48	.365090	9.57	.634910	57
4	.354271	9.07	.988607	.48	.365664	9.55	.634336	56
5	.354815	9.05	.988578	.50	.366237	9.55	.633763	55
6	.355358	9.05	.988548	.48	.366810	9.53	.633190	54
7	.355901	9.03	.988519	.50	.367382	9.52	.632618	53
8	.356443	9.02	.988489	.48	.367953	9.52	.632047	52
9	.356984	9.00	.988460	.50	.368524	9.50	.631476	51
10	.357524	9.00	.988430	.48	.369094	9.48	.630906	50
11	9.358064	8.98	9.988401	.50	9.369663	9.48	10.630337	49
12	.358603	8.97	.988371	.48	.370232	9.45	.629768	48
13	.359141	8.95	.988342	.50	.370799	9.47	.629201	47
14	.359678	8.95	.988312	.50	.371367	9.43	.628633	46
15	.360215	8.95	.988282	.50	.371933	9.43	.628067	45
16	.360752	8.92	.988252	.48	.372499	9.42	.627501	44
17	.361287	8.92	.988223	.50	.373064	9.42	.626936	43
18	.361822	8.90	.988193	.50	.373629	9.40	.626371	42
19	.362356	8.88	.988163	.50	.374193	9.38	.625807	41
20	.362889	8.88	.988133	.50	.374756	9.38	.625244	40
21	9.363422	8.87	9.988103	.50	9.375319	9.37	10.624681	39
22	.363954	8.85	.988073	.50	.375881	9.35	.624119	38
23	.364485	8.85	.988043	.50	.376442	9.35	.623558	37
24	.365016	8.83	.988013	.50	.377003	9.33	.622997	36
25	.365546	8.82	.987983	.50	.377563	9.32	.622437	35
26	.366075	8.82	.987953	.52	.378122	9.32	.621878	34
27	.366604	8.78	.987922	.50	.378681	9.30	.621319	33
28	.367131	8.80	.987892	.50	.379239	9.30	.620761	32
29	.367659	8.77	.987862	.50	.379797	9.28	.620203	31
30	.368185	8.77	.987832	.52	.380354	9.27	.619646	30
31	9.368711	8.75	9.987801	.50	9.380910	9.27	10.619090	29
32	.369236	8.75	.987771	.52	.381466	9.23	.618534	28
33	.369761	8.72	.987740	.50	.382020	9.25	.617980	27
34	.370285	8.72	.987710	.52	.382575	9.23	.617425	26
35	.370808	8.70	.987679	.50	.383129	9.22	.616871	25
36	.371330	8.70	.987649	.52	.383682	9.20	.616318	24
37	.371852	8.68	.987618	.50	.384234	9.20	.615766	23
38	.372373	8.68	.987588	.52	.384786	9.18	.615214	22
39	.372894	8.67	.987557	.52	.385337	9.18	.614663	21
40	.373414	8.65	.987526	.50	.385888	9.17	.614112	20
41	9.373933	8.65	9.987496	.52	9.386438	9.15	10.613562	19
42	.374452	8.63	.987465	.52	.386987	9.15	.613013	18
43	.374970	8.62	.987434	.52	.387536	9.13	.612464	17
44	.375487	8.60	.987403	.52	.388084	9.12	.611916	16
45	.376003	8.60	.987372	.52	.388631	9.12	.611369	15
46	.376519	8.60	.987341	.52	.389178	9.10	.610822	14
47	.377035	8.57	.987310	.52	.389724	9.10	.610276	13
48	.377549	8.57	.987279	.52	.390270	9.08	.609730	12
49	.378063	8.57	.987248	.52	.390815	9.08	.609185	11
	.378577	8.53	.987217	.52	.391360	9.05	.608640	10
	9·379089	8.53	9.987186	.52	9.391903	9.07	10.608097	9
	.379601	8.53	.987155	.52	.392447	9.03	.607553	8
	.380113	8.52	.987124	.53	.392989	9.03	.607011	7
	.380624	8.50	.987092	.52	.393531	9.03	.606469	6
	.381134	8.48	.987061	.52	.394073	9.02	.605927	5
	.381643	8.48	.987030	.53	.394614	9.00	.605386	4
	.382152	8.48	.986998	.52	.395154	9.00	.604846	3
	.382661	8.45	.986967	.52	.395694	8.98	.604306	2
	.383168	8.45	.986936	.53	.396233	8.97	.603767	1
	9.383675		9.986904		9.396771		10.603229	0
	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	′
0	9.383675	8.45	9.986904	.52	9.396771	8.97	10.603229	60
1	.384182	8.42	.986873	.53	.397309	8.95	.602691	59
2	.384687	8.42	.986841	.53	.397846	8.95	.602154	58
3	.385192	8.42	.986809	.52	.398383	8.93	.601617	57
4	.385697	8.40	.986778	.53	.398919	8.93	.601081	56
5	.386201	8.38	.986746	.53	.399455	8.92	.600545	55
6	.386704	8.38	.986714	.52	.399990	8.90	.600010	54
7	.387207	8.37	.986683	.53	.400524	8.90	.599476	53
8	.387709	8.35	.986651	.53	.401058	8.88	.598942	52
9	.388210	8.35	.986619	.53	.401591	8.88	.598409	51
10	.388711	8.33	.986587	.53	.402124	8.87	.597876	50
11	9.389211	8.33	9.986555	.53	9.402656	8.85	10.597344	49
12	.389711	8.32	.986523	.53	.403187	8.85	.596813	48
13	.390210	8.30	.986491	.53	.403718	8.85	.596282	47
14	.390708	8.30	.986459	.53	.404249	8.82	.595751	46
15	.391206	8.28	.986427	.53	.404778	8.83	.595222	45
16	.391703	8.27	.986395	.53	.405308	8.80	.594692	44
17	.392199	8.27	.986363	.53	.405836	8.80	.594164	43
18	.392695	8.27	.986331	.53	.406364	8.80	.593636	42
19	.393191	8.23	.986299	.55	.406892	8.78	.593108	41
20	.393685	8.23	.986266	.53	.407419	8.77	.592581	40
21	9.394179	8.23	9.986234	.53	9.407945	8.77	10.592055	39
22	.394673	8.22	.986202	.55	.408471	8.75	.591529	38
23	.395166	8.20	.986169	.53	.408996	8.75	.591004	37
24	.395658	8.20	.986137	.55	.409521	8.73	.590479	36
25	.396150	8.18	.986104	.53	.410045	8.73	.589955	35
26	.396641	8.18	.986072	.55	.410569	8.72	.589431	34
27	.397132	8.15	.986039	.53	.411092	8.72	.588908	33
28	.397621	8.17	.986007	.55	.411615	8.70	.588385	32
29	.398111	8.15	.985974	.53	.412137	8.68	.587863	31
30	.398600	8.13	.985942	.55	.412658	8.68	.587342	30
31	9.399088	8.12	9.985909	.55	9.413179	8.67	10.586821	29
32	.399575	8.12	.985876	.55	.413699	8.67	.586301	28
33	.400062	8.12	.985843	.53	.414219	8.65	.585781	27
34	.400549	8.10	.985811	.55	.414738	8.65	.585262	26
35	.401035	8.08	.985778	.55	.415257	8.63	.584743	25
36	.401520	8.08	.985745	.55	.415775	8.63	.584225	24
37	.402005	8.07	.985712	.55	.416293	8.62	.583707	23
38	.402489	8.05	.985679	.55	.416810	8.60	.583190	22
39	.402972	8.05	.985646	.55	.417326	8.60	.582674	21
40	.403455	8.05	.985613	.55	.417842	8.60	.582158	20
41	9.403938	8.03	9.985580	.55	9.418358	8.58	10.581642	19
42	.404420	8.02	.985547	.55	.418873	8.57	.581127	18
43	.404901	8.02	.985514	.57	.419387	8.57	.580613	17
44	.405382	8.00	.985480	.55	.419901	8.57	.580099	16
45	.405862	7.98	.985447	.55	.420415	8.55	.579585	15
46	.406341	7.98	.985414	.55	.420927	8.55	.579073	14
47	.406820	7.98	.985381	.57	.421440	8.53	.578560	13
48	.407299	7.97	.985347	.55	.421952	8.52	.578048	12
49	.407777	7.95	.985314	.57	.422463	8.52	.577537	11
50	.408254	7.95	.985280	.55	.422974	8.50	.577026	10
51	9.408731	7.93	9.985247	.57	9.423484	8.48	10.576516	9
52	.409207	7.93	.985213	.55	.423993	8.50	.576007	8
53	.409682	7.92	.985180	.57	.424503	8.47	.575497	7
54	.410157	7.92	.985146	.55	.425011	8.47	.574989	6
55	.410632	7.90	.985113	.57	.425519	8.47	.574481	5
56	.411106	7.88	.985079	.57	.426027	8.45	.573973	4
57	.411579	7.88	.985045	.57	.426534	8.45	.573466	3
58	.412052	7.87	.985011	.55	.427041	8.43	.572959	2
59	.412524	7.87	.984978	.57	.427547	8.42	.572453	1
60	9.412996		9.984944		9.428052		10.571948	0
′	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.554329	5.48	9.970152	.82	9.584177	6.30	10.415823	60
1	.554658	5.48	.970103	.80	.584555	6.28	.415445	59
2	.554987	5.47	.970055	.82	.584932	6.28	.415068	58
3	.555315	5.47	.970006	.82	.585309	6.28	.414691	57
4	.555643	5.47	.969957	.80	.585686	6.27	.414314	56
5	.555971	5.47	.969909	.82	.586062	6.28	.413938	55
6	.556299	5.45	.969860	.82	.586439	6.27	.413561	54
7	.556626	5.45	.969811	.82	.586815	6.25	.413185	53
8	.556953	5.45	.969762	.80	.587190	6.27	.412810	52
9	.557280	5.43	.969714	.82	.587566	6.25	.412434	51
10	.557606	5.43	.969665	.82	.587941	6.25	.412059	50
11	9.557932	5.43	9.969616	.82	9.588316	6.25	10.411684	49
12	.558258	5.42	.969567	.82	.588691	6.25	.411309	48
13	.558583	5.43	.969518	.82	.589066	6.23	.410934	47
14	.558909	5.42	.969469	.82	.589440	6.23	.410560	46
15	.559234	5.40	.969420	.83	.589814	6.23	.410186	45
16	.559558	5.42	.969370	.82	.590188	6.23	.409812	44
17	.559883	5.40	.969321	.82	.590562	6.22	.409438	43
18	.560207	5.40	.969272	.82	.590935	6.22	.409065	42
19	.560531	5.40	.969223	.83	.591308	6.22	.408692	41
20	.560855	5.38	.969173	.82	.591681	6.22	.408319	40
21	9.561178	5.38	9.969124	.82	9.592054	6.20	10.407946	39
22	.561501	5.38	.969075	.83	.592426	6.22	.407574	38
23	.561824	5.37	.969025	.82	.592799	6.20	.407201	37
24	.562146	5.37	.968976	.83	.593171	6.18	.406829	36
25	.562468	5.37	.968926	.82	.593542	6.20	.406458	35
26	.562790	5.37	.968877	.83	.593914	6.18	.406086	34
27	.563112	5.35	.968827	.83	.594285	6.18	.405715	33
28	.563433	5.37	.968777	.82	.594656	6.18	.405344	32
29	.563755	5.33	.968728	.83	.595027	6.18	.404973	31
30	.564075	5.35	.968678	.83	.595398	6.17	.404602	30
31	9.564396	5.33	9.968628	.83	9.595768	6.17	10.404232	29
32	.564716	5.33	.968578	.83	.596138	6.17	.403862	28
33	.565036	5.33	.968528	.82	.596508	6.17	.403492	27
34	.565356	5.33	.968479	.83	.596878	6.15	.403122	26
35	.565676	5.32	.968429	.83	.597247	6.15	.402753	25
36	.565995	5.32	.968379	.83	.597616	6.15	.402384	24
37	.566314	5.30	.968329	.85	.597985	6.15	.402015	23
38	.566632	5.32	.968278	.83	.598354	6.13	.401646	22
39	.566951	5.30	.968228	.83	.598722	6.15	.401278	21
40	.567269	5.30	.968178	.83	.599091	6.13	.400909	20
41	9.567587	5.28	9.968128	.83	9.599459	6.13	10.400541	19
42	.567904	5.30	.968078	.85	.599827	6.12	.400173	18
43	.568222	5.28	.968027	.83	.600194	6.13	.399806	17
44	.568539	5.28	.967977	.83	.600562	6.12	.399438	16
45	.568856	5.27	.967927	.85	.600929	6.12	.399071	15
46	.569172	5.27	.967876	.83	.601296	6.12	.398704	14
47	.569488	5.27	.967826	.85	.601663	6.10	.398337	13
48	.569804	5.27	.967775	.83	.602029	6.10	.397971	12
49	.570120	5.25	.967725	.85	.602395	6.10	.397605	11
50	.570435	5.27	.967674	.83	.602761	6.10	.397239	10
51	9.570751	5.25	9.967624	.85	9.603127	6.10	10.396873	9
52	.571066	5.23	.967573	.85	.603493	6.08	.396507	8
53	.571380	5.25	.967522	.85	.603858	6.08	.396142	7
54	.571695	5.23	.967471	.83	.604223	6.08	.395777	6
55	.572009	5.23	.967421	.85	.604588	6.08	.395412	5
56	.572323	5.22	.967370	.85	.604953	6.07	.395047	4
57	.572636	5.23	.967319	.85	.605317	6.08	.394683	3
58	.572950	5.22	.967268	.85	.605682	6.07	.394318	2
59	.573263	5.20	.967217	.85	.606046	6.07	.393954	1
60	9.573575		9.967166		9.606410		10.393590	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.440338	7.33	9.982842	.62	9.457496	7.95	10.542504	60
1	.440778	7.33	.982805	.60	.457973	7.93	.542027	59
2	.441218	7.33	.982769	.60	.458449	7.93	.541551	58
3	.441658	7.30	.982733	.62	.458925	7.92	.541075	57
4	.442096	7.32	.982696	.60	.459400	7.92	.540600	56
5	.442535	7.30	.982660	.60	.459875	7.90	.540125	55
6	.442973	7.28	.982624	.62	.460349	7.90	.539651	54
7	.443410	7.28	.982587	.60	.460823	7.90	.539177	53
8	.443847	7.28	.982551	.62	.461297	7.88	.538703	52
9	.444284	7.27	.982514	.62	.461770	7.87	.538230	51
10	.444720	7.25	.982477	.60	.462242	7.88	.537758	50
11	9.445155	7.25	9.982441	.62	9.462715	7.85	10.537285	49
12	.445590	7.25	.982404	.62	.463186	7.87	.536814	48
13	.446025	7.23	.982367	.60	.463658	7.83	.536342	47
14	.446459	7.23	.982331	.62	.464128	7.85	.535872	46
15	.446893	7.22	.982294	.62	.464599	7.83	.535401	45
16	.447326	7.22	.982257	.62	.465069	7.83	.534931	44
17	.447759	7.20	.982220	.62	.465539	7.82	.534461	43
18	.448191	7.20	.982183	.62	.466008	7.82	.533992	42
19	.448623	7.18	.982146	.62	.466477	7.80	.533523	41
20	.449054	7.18	.982109	.62	.466945	7.80	.533055	40
21	9.449485	7.17	9.982072	.62	9.467413	7.78	10.532587	39
22	.449915	7.17	.982035	.62	.467880	7.78	.532120	38
23	.450345	7.17	.981998	.62	.468347	7.78	.531653	37
24	.450775	7.15	.981961	.62	.468814	7.77	.531186	36
25	.451204	7.13	.981924	.63	.469280	7.77	.530720	35
26	.451632	7.13	.981886	.62	.469746	7.75	.530254	34
27	.452060	7.13	.981849	.62	.470211	7.75	.529789	33
28	.452488	7.12	.981812	.63	.470676	7.75	.529324	32
29	.452915	7.12	.981774	.62	.471141	7.73	.528859	31
30	.453342	7.10	.981737	.62	.471605	7.73	.528395	30
31	9.453768	7.10	9.981700	.63	9.472069	7.72	10.527931	29
32	.454194	7.08	.981662	.62	.472532	7.72	.527468	28
33	.454619	7.08	.981625	.63	.472995	7.70	.527005	27
34	.455044	7.08	.981587	.63	.473457	7.70	.526543	26
35	.455469	7.07	.981549	.62	.473919	7.70	.526081	25
36	.455893	7.05	.981512	.63	.474381	7.68	.525619	24
37	.456316	7.05	.981474	.63	.474842	7.68	.525158	23
38	.456739	7.05	.981436	.62	.475303	7.67	.524697	22
39	.457162	7.03	.981399	.63	.475763	7.67	.524237	21
40	.457584	7.03	.981361	.63	.476223	7.67	.523777	20
41	9.458006	7.02	9.981323	.63	9.476683	7.65	10.523317	19
42	.458427	7.02	.981285	.63	.477142	7.65	.522858	18
43	.458848	7.00	.981247	.63	.477601	7.63	.522399	17
44	.459268	7.00	.981209	.63	.478059	7.63	.521941	16
45	.459688	7.00	.981171	.63	.478517	7.63	.521483	15
46	.460108	6.98	.981133	.63	.478975	7.62	.521025	14
47	.460527	6.98	.981095	.63	.479432	7.62	.520568	13
48	.460946	6.97	.981057	.63	.479889	7.60	.520111	12
49	.461364	6.97	.981019	.63	.480345	7.60	.519655	11
50	.461782	6.95	.980981	.65	.480801	7.60	.519199	10
51	9.462199	6.95	9.980942	.63	9.481257	7.58	10.518743	9
52	.462616	6.93	.980904	.63	.481712	7.58	.518288	8
53	.463032	6.93	.980866	.65	.482167	7.57	.517833	7
54	.463448	6.93	.980827	.63	.482621	7.57	.517379	6
55	.463864	6.92	.980789	.65	.483075	7.57	.516925	5
56	.464279	6.92	.980750	.63	.483529	7.55	.516471	4
57	.464694	6.90	.980712	.65	.483982	7.55	.516018	3
58	.465108	6.90	.980673	.63	.484435	7.53	.515565	2
59	.465522	6.88	.980635	.65	.484887	7.53	.515113	1
60	9.465935		9.980596		9.485339		10.514661	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	′
0	9.465935	6.88	9.980596	.63	9.485339	7.53	10.514661	60
1	.466348	6.88	.980558	.65	.485791	7.52	.514209	59
2	.466761	6.87	.980519	.65	.486242	7.52	.513758	58
3	.467173	6.87	.980480	.63	.486693	7.50	.513307	57
4	.467585	6.85	.980442	.65	.487143	7.50	.512857	56
5	.467996	6.85	.980403	.65	.487593	7.50	.512407	55
6	.468407	6.83	.980364	.65	.488043	7.48	.511957	54
7	.468817	6.83	.980325	.65	.488492	7.48	.511508	53
8	.469227	6.83	.980286	.65	.488941	7.48	.511059	52
9	.469637	6.82	.980247	.65	.489390	7.47	.510610	51
10	.470046	6.82	.980208	.65	.489838	7.47	.510162	50
11	9.470455	6.80	9.980169	.65	9.490286	7.45	10.509714	49
12	.470863	6.80	.980130	.65	.490733	7.45	.509267	48
13	.471271	6.80	.980091	.65	.491180	7.45	.508820	47
14	.471679	6.78	.980052	.67	.491627	7.43	.508373	46
15	.472086	6.77	.980012	.65	.492073	7.43	.507927	45
16	.472492	6.77	.979973	.65	.492519	7.43	.507481	44
17	.472898	6.77	.979934	.65	.492965	7.42	.507035	43
18	.473304	6.77	.979895	.67	.493410	7.40	.506590	42
19	.473710	6.75	.979855	.65	.493854	7.42	.506146	41
20	.474115	6.73	.979816	.67	.494299	7.40	.505701	40
21	9.474519	6.73	9.979776	.65	9.494743	7.38	10.505257	39
22	.474923	6.73	.979737	.67	.495186	7.40	.504814	38
23	.475327	6.72	.979697	.65	.495630	7.38	.504370	37
24	.475730	6.72	.979658	.67	.496073	7.37	.503927	36
25	.476133	6.72	.979618	.65	.496515	7.37	.503485	35
26	.476536	6.70	.979579	.67	.496957	7.37	.503043	34
27	.476938	6.70	.979539	.67	.497399	7.37	.502601	33
28	.477340	6.68	.979499	.67	.497841	7.35	.502159	32
29	.477741	6.68	.979459	.65	.498282	7.33	.501718	31
30	.478142	6.67	.979420	.67	.498722	7.35	.501278	30
31	9.478542	6.67	9.979380	.67	9.499163	7.33	10.500837	29
32	.478942	6.67	.979340	.67	.499603	7.32	.500397	28
33	.479342	6.65	.979300	.67	.500042	7.32	.499958	27
34	.479741	6.65	.979260	.67	.500481	7.32	.499519	26
35	.480140	6.65	.979220	.67	.500920	7.32	.499080	25
36	.480539	6.63	.979180	.67	.501359	7.30	.498641	24
37	.480937	6.62	.979140	.67	.501797	7.30	.498203	23
38	.481334	6.62	.979100	.68	.502235	7.28	.497765	22
39	.481731	6.62	.979059	.67	.502672	7.28	.497328	21
40	.482128	6.62	.979019	.67	.503109	7.28	.496891	20
41	9.482525	6.60	9.978979	.67	9.503546	7.27	10.496454	19
42	.482921	6.58	.978939	.68	.503982	7.27	.496018	18
43	.483316	6.60	.978898	.67	.504418	7.27	.495582	17
44	.483712	6.58	.978858	.68	.504854	7.25	.495146	16
45	.484107	6.57	.978817	.67	.505289	7.25	.494711	15
46	.484501	6.57	.978777	.67	.505724	7.25	.494276	14
47	.484895	6.57	.978737	.68	.506159	7.23	.493841	13
48	.485289	6.55	.978696	.68	.506593	7.23	.493407	12
	.485682	6.55	.978655	.67	.507027	7.23	.492973	11
	.486075	6.53	.978615	.68	.507460	7.22	.492540	10
	9.486467	6.55	9.978574	.68	9.507893	7.22	10.492107	9
	.486860	6.52	.978533	.67	.508326	7.22	.491674	8
	.487251	6.53	.978493	.68	.508759	7.20	.491241	7
	.487643	6.52	.978452	.68	.509191	7.18	.490809	6
	.488034	6.50	.978411	.68	.509622	7.20	.490378	5
	.488424	6.50	.978370	.68	.510054	7.18	.489946	4
	.488814	6.50	.978329	.68	.510485	7.18	.489515	3
	.489204	6.48	.978288	.68	.510916	7.17	.489084	2
	.489593	6.48	.978247	.68	.511346	7.17	.488654	1
	9.489982		9.978206		9.511776		10.488224	0
′	Cosine.	D 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.489982	6.48	9.978206	.68	9.511776	7.17	10.488224	60
1	.490371	6.47	.978165	.68	.512206	7.15	.487794	59
2	.490759	6.47	.978124	.68	.512635	7.15	.487365	58
3	.491147	6.47	.978083	.68	.513064	7.15	.486936	57
4	.491535	6.45	.978042	.68	.513493	7.13	.486507	56
5	.491922	6.43	.978001	.70	.513921	7.13	.486079	55
6	.492308	6.45	.977959	.68	.514349	7.13	.485651	54
7	.492695	6.43	.977918	.68	.514777	7.12	.485223	53
8	.493081	6.42	.977877	.70	.515204	7.12	.484796	52
9	.493466	6.42	.977835	.68	.515631	7.10	.484369	51
10	.493851	6.42	.977794	.70	.516057	7.12	.483943	50
11	9.494236	6.42	9.977752	.68	9.516484	7.10	10.483516	49
12	.494621	6.40	.977711	.70	.516910	7.08	.483090	48
13	.495005	6.38	.977669	.68	.517335	7.10	.482665	47
14	.495388	6.40	.977628	.70	.517761	7.08	.482239	46
15	.495772	6.37	.977586	.70	.518186	7.07	.481814	45
16	.496154	6.38	.977544	.68	.518610	7.07	.481390	44
17	.496537	6.37	.977503	.70	.519034	7.07	.480966	43
18	.496919	6.37	.977461	.70	.519458	7.07	.480542	42
19	.497301	6.35	.977419	.70	.519882	7.05	.480118	41
20	.497682	6.35	.977377	.70	.520305	7.05	.479695	40
21	9.498064	6.33	9.977335	.70	9.520728	7.05	10.479272	39
22	.498444	6.35	.977293	.70	.521151	7.03	.478849	38
23	.498825	6.32	.977251	.70	.521573	7.03	.478427	37
24	.499204	6.33	.977209	.70	.521995	7.03	.478005	36
25	.499584	6.32	.977167	.70	.522417	7.02	.477583	35
26	.499963	6.32	.977125	.70	.522838	7.02	.477162	34
27	.500342	6.32	.977083	.70	.523259	7.02	.476741	33
28	.500721	6.30	.977041	.70	.523680	7.00	.476320	32
29	.501099	6.28	.976999	.70	.524100	7.00	.475900	31
30	.501476	6.30	.976957	.72	.524520	7.00	.475480	30
31	9.501854	6.28	9.976914	.70	9.524940	6.98	10.475060	29
32	.502231	6.27	.976872	.70	.525359	6.98	.474641	28
33	.502607	6.28	.976830	.72	.525778	6.98	.474222	27
34	.502984	6.27	.976787	.70	.526197	6.97	.473803	26
35	.503360	6.25	.976745	.72	.526615	6.97	.473385	25
36	.503735	6.25	.976702	.70	.527033	6.97	.472967	24
37	.504110	6.25	.976660	.72	.527451	6.95	.472549	23
38	.504485	6.25	.976617	.72	.527868	6.95	.472132	22
39	.504860	6.23	.976574	.70	.528285	6.95	.471715	21
40	.505234	6.23	.976532	.72	.528702	6.95	.471298	20
41	9.505608	6.22	9.976489	.72	9.529119	6.93	10.470881	19
42	.505981	6.22	.976446	.70	.529535	6.93	.470465	18
43	.506354	6.22	.976404	.72	.529951	6.92	.470049	17
44	.506727	6.20	.976361	.72	.530366	6.92	.469634	16
45	.507099	6.20	.976318	.72	.530781	6.92	.469219	15
46	.507471	6.20	.976275	.72	.531196	6.92	.468804	14
47	.507843	6.18	.976232	.72	.531611	6.90	.468389	13
48	.508214	6.18	.976189	.72	.532025	6.90	.467975	12
49	.508585	6.18	.976146	.72	.532439	6.90	.467561	11
50	.508956	6.17	.976103	.72	.532853	6.88	.467147	10
51	9.509326	6.17	9.976060	.72	9.533266	6.88	10.466734	9
52	.509696	6.15	.976017	.72	.533679	6.88	.466321	8
53	.510065	6.15	.975974	.73	.534092	6.87	.465908	7
54	.510434	6.15	.975930	.72	.534504	6.87	.465496	6
55	.510803	6.15	.975887	.72	.534916	6.87	.465084	5
56	.511172	6.13	.975844	.73	.535328	6.85	.464672	4
57	.511540	6.12	.975800	.72	.535739	6.85	.464261	3
58	.511907	6.13	.975757	.72	.536150	6.85	.463850	2
59	.512275	6.12	.975714	.73	.536561	6.85	.463439	1
60	9.512642		9.975670		9.536972		10.463028	
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.512642	6.12	9.975670	.72	9.536972	6.83	10.463028	60
1	.513009	6.10	.975627	.73	.537382	6.83	.462618	59
2	.513375	6.10	.975583	.73	.537792	6.83	.462208	58
3	.513741	6.10	.975539	.72	.538202	6.82	.461798	57
4	.514107	6.08	.975496	.73	.538611	6.82	.461389	56
5	.514472	6.08	.975452	.73	.539020	6.82	.460980	55
6	.514837	6.08	.975408	.72	.539429	6.80	.460571	54
7	.515202	6.07	.975365	.73	.539837	6.80	.460163	53
8	.515566	6.07	.975321	.73	.540245	6.80	.459755	52
9	.515930	6.07	.975277	.73	.540653	6.80	.459347	51
10	.516294	6.05	.975233	.73	.541061	6.78	.458939	50
11	9.516657	6.05	9.975189	.73	9.541468	6.78	10.458532	49
12	.517020	6.03	.975145	.73	.541875	6.77	.458125	48
13	.517382	6.05	.975101	.73	.542281	6.78	.457719	47
14	.517745	6.03	.975057	.73	.542688	6.77	.457312	46
15	.518107	6.02	.975013	.73	.543094	6.75	.456906	45
16	.518468	6.02	.974969	.73	.543499	6.77	.456501	44
17	.518829	6.02	.974925	.75	.543905	6.75	.456095	43
18	.519190	6.02	.974880	.73	.544310	6.75	.455690	42
19	.519551	6.00	.974836	.73	.544715	6.73	.455285	41
20	.519911	6.00	.974792	.73	.545119	6.75	.454881	40
21	9.520271	6.00	9.974748	.75	9.545524	6.73	10.454476	39
22	.520631	5.98	.974703	.73	.545928	6.72	.454072	38
23	.520990	5.98	.974659	.75	.546331	6.73	.453669	37
24	.521349	5.97	.974614	.73	.546735	6.72	.453265	36
25	.521707	5.98	.974570	.75	.547138	6.70	.452862	35
26	.522066	5.97	.974525	.73	.547540	6.72	.452460	34
27	.522424	5.95	.974481	.75	.547943	6.70	.452057	33
28	.522781	5.95	.974436	.75	.548345	6.70	.451655	32
29	.523138	5.95	.974391	.73	.548747	6.70	.451253	31
30	.523495	5.95	.974347	.75	.549149	6.68	.450851	30
31	9.523852	5.93	9.974302	.75	9.549550	6.68	10.450450	29
32	.524208	5.93	.974257	.75	.549951	6.68	.450049	28
33	.524564	5.93	.974212	.75	.550352	6.67	.449648	27
34	.524920	5.92	.974167	.75	.550752	6.68	.449248	26
35	.525275	5.92	.974122	.75	.551153	6.65	.448847	25
36	.525630	5.90	.974077	.75	.551552	6.67	.448448	24
37	.525984	5.92	.974032	.75	.551952	6.65	.448048	23
38	.526339	5.90	.973987	.75	.552351	6.65	.447649	22
39	.526693	5.88	.973942	.75	.552750	6.65	.447250	21
40	.527046	5.90	.973897	.75	.553149	6.65	.446851	20
41	9.527400	5.88	9.973852	.75	9.553548	6.63	10.446452	19
42	.527753	5.87	.973807	.77	.553946	6.63	.446054	18
43	.528105	5.88	.973761	.75	.554344	6.62	.445656	17
44	.528458	5.87	.973716	.75	.554741	6.63	.445259	16
45	.528810	5.85	.973671	.77	.555139	6.62	.444861	15
46	.529161	5.87	.973625	.75	.555536	6.62	.444464	14
47	.529513	5.85	.973580	.75	.555933	6.60	.444067	13
48	.529864	5.85	.973535	.77	.556329	6.60	.443671	12
49	.530215	5.83	.973489	.75	.556725	6.60	.443275	11
[illegible]	.530565	5.83	.973444	.77	.557121	6.60	.442879	10
[illegible]	530915	5.83	9.973398	.77	9.557517	6.60	10.442483	9
[illegible]	531265	5.82	.973352	.75	.557913	6.58	.442087	8
[illegible]	531614	5.82	.973307	.77	.558308	6.58	.441692	7
[illegible]	531963	5.82	.973261	.77	.558703	6.57	.441297	6
[illegible]	532312	5.82	.973215	.77	.559097	6.57	.440903	5
[illegible]	532661	5.80	.973169	.75	.559491	6.57	.440509	4
[illegible]	533009	5.80	.973124	.77	.559885	6.57	.440115	3
[illegible]	533357	5.78	.973078	.77	.560279	6.57	.439721	2
[illegible]	.533704	5.80	.973032	.77	.560673	6.55	.439327	1
[illegible]	.534052		9.972986		9.561066		10.438934	0
	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.534052	5.78	9.972986	.77	9.561066	6.55	10.438934	60
1	.534399	5.77	.972940	.77	.561459	6.53	.438541	59
2	.534745	5.78	.972894	.77	.561851	6.55	.438149	58
3	.535092	5.77	.972848	.77	.562244	6.53	.437756	57
4	.535438	5.75	.972802	.78	.562636	6.53	.437364	56
5	.535783	5.77	.972755	.77	.563028	6.52	.436972	55
6	.536129	5.75	.972709	.77	.563419	6.53	.436581	54
7	.536474	5.73	.972663	.77	.563811	6.52	.436189	53
8	.536818	5.75	.972617	.78	.564202	6.52	.435798	52
9	.537163	5.73	.972570	.77	.564593	6.50	.435407	51
10	.537507	5.73	.972524	.77	.564983	6.50	.435017	50
11	9.537851	5.72	9.972478	.78	9.565373	6.50	10.434627	49
12	.538194	5.70	.972431	.77	.565763	6.50	.434237	48
13	.538538	5.70	.972385	.78	.566153	6.48	.433847	47
14	.538880	5.72	.972338	.78	.566542	6.50	.433458	46
15	.539223	5.70	.972291	.77	.566932	6.47	.433068	45
16	.539565	5.70	.972245	.78	.567320	6.48	.432680	44
17	.539907	5.68	.972198	.78	.567709	6.48	.432291	43
18	.540249	5.68	.972151	.77	.568098	6.47	.431902	42
19	.540590	5.68	.972105	.78	.568486	6.45	.431514	41
20	.540931	5.68	.972058	.78	.568873	6.47	.431127	40
21	9.541272	5.68	9.972011	.78	9.569261	6.45	10.430739	39
22	.541613	5.67	.971964	.78	.569648	6.45	.430352	38
23	.541953	5.67	.971917	.78	.570035	6.45	.429965	37
24	.542293	5.65	.971870	.78	.570422	6.45	.429578	36
25	.542632	5.65	.971823	.78	.570809	6.43	.429191	35
26	.542971	5.65	.971776	.78	.571195	6.43	.428805	34
27	.543310	5.65	.971729	.78	.571581	6.43	.428419	33
28	.543649	5.63	.971682	.78	.571967	6.42	.428033	32
29	.543987	5.63	.971635	.78	.572352	6.43	.427648	31
30	.544325	5.63	.971588	.80	.572738	6.42	.427262	30
31	9.544663	5.62	9.971540	.78	9.573123	6.40	10.426877	29
32	.545000	5.63	.971493	.78	.573507	6.42	.426493	28
33	.545338	5.60	.971446	.80	.573892	6.40	.426108	27
34	.545674	5.62	.971398	.78	.574276	6.40	.425724	26
35	.546011	5.60	.971351	.80	.574660	6.40	.425340	25
36	.546347	5.60	.971303	.78	.575044	6.38	.424956	24
37	.546683	5.60	.971256	.80	.575427	6.38	.424573	23
38	.547019	5.58	.971208	.78	.575810	6.38	.424190	22
39	.547354	5.58	.971161	.80	.576193	6.38	.423807	21
40	.547689	5.58	.971113	.78	.576576	6.38	.423424	20
41	9.548024	5.58	9.971066	.80	9.576959	6.37	10.423041	19
42	.548359	5.57	.971018	.80	.577341	6.37	.422659	18
43	.548693	5.57	.970970	.80	.577723	6.35	.422277	17
44	.549027	5.55	.970922	.80	.578104	6.37	.421896	16
45	.549360	5.55	.970874	.78	.578486	6.35	.421514	15
46	.549693	5.55	.970827	.80	.578867	6.35	.421133	14
47	.550026	5.55	.970779	.80	.579248	6.35	.420752	13
48	.550359	5.55	.970731	.80	.579629	6.33	.420371	12
49	.550692	5.53	.970683	.80	.580009	6.33	.419991	11
50	.551024	5.53	.970635	.82	.580389	6.33	.419611	10
51	9.551356	5.52	9.970586	.80	9.580769	6.33	10.419231	9
52	.551687	5.52	.970538	.80	.581149	6.32	.418851	8
53	.552018	5.52	.970490	.80	.581528	6.32	.418472	7
54	.552349	5.52	.970442	.80	.581907	6.32	.418093	6
55	.552680	5.50	.970394	.82	.582286	6.32	.417714	5
56	.553010	5.52	.970345	.80	.582665	6.32	.417335	4
57	.553341	5.48	.970297	.80	.583044	6.30	.416956	3
58	.553670	5.50	.970249	.82	.583422	6.30	.416578	2
59	.554000	5.48	.970200	.80	.583800	6.28	.416200	1
60	9.554329		9.970152		9.584177		10.415823	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

21° TABLE XXV.—LOGARITHMIC SINES, 158°

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.554329	5.48	9.970152	.82	9.584177	6.30	10.415823	60
1	.554658	5.48	.970103	.80	.584555	6.28	.415445	59
2	.554987	5.47	.970055	.82	.584932	6.28	.415068	58
3	.555315	5.47	.970006	.82	.585309	6.28	.414691	57
4	.555643	5.47	.969957	.80	.585686	6.27	.414314	56
5	.555971	5.47	.969909	.82	.586062	6.28	.413938	55
6	.556299	5.45	.969860	.82	.586439	6.27	.413561	54
7	.556626	5.45	.969811	.82	.586815	6.25	.413185	53
8	.556953	5.45	.969762	.80	.587190	6.27	.412810	52
9	.557280	5.43	.969714	.82	.587566	6.25	.412434	51
10	.557606	5.43	.969665	.82	.587941	6.25	.412059	50
11	9.557932	5.43	9.969616	.82	9.588316	6.25	10.411684	49
12	.558258	5.42	.969567	.82	.588691	6.25	.411309	48
13	.558583	5.43	.969518	.82	.589066	6.23	.410934	47
14	.558909	5.42	.969469	.82	.589440	6.23	.410560	46
15	.559234	5.40	.969420	.83	.589814	6.23	.410186	45
16	.559558	5.42	.969370	.82	.590188	6.23	.409812	44
17	.559883	5.40	.969321	.82	.590562	6.22	.409438	43
18	.560207	5.40	.969272	.82	.590935	6.22	.409065	42
19	.560531	5.40	.969223	.83	.591308	6.22	.408692	41
20	.560855	5.38	.969173	.82	.591681	6.22	.408319	40
21	9.561178	5.38	9.969124	.82	9.592054	6.20	10.407946	39
22	.561501	5.38	.969075	.83	.592426	6.22	.407574	38
23	.561824	5.37	.969025	.82	.592799	6.20	.407201	37
24	.562146	5.37	.968976	.83	.593171	6.18	.406829	36
25	.562468	5.37	.968926	.82	.593542	6.20	.406458	35
26	.562790	5.37	.968877	.83	.593914	6.18	.406086	34
27	.563112	5.35	.968827	.83	.594285	6.18	.405715	33
28	.563433	5.37	.968777	.82	.594656	6.18	.405344	32
29	.563755	5.33	.968728	.83	.595027	6.18	.404973	31
30	.564075	5.35	.968678	.83	.595398	6.17	.404602	30
31	9.564396	5.33	9.968628	.83	9.595768	6.17	10.404232	29
32	.564716	5.33	.968578	.83	.596138	6.17	.403862	28
33	.565036	5.33	.968528	.82	.596508	6.17	.403492	27
34	.565356	5.33	.968479	.83	.596878	6.15	.403122	26
35	.565676	5.32	.968429	.83	.597247	6.15	.402753	25
36	.565995	5.32	.968379	.83	.597616	6.15	.402384	24
37	.566314	5.30	.968329	.85	.597985	6.15	.402015	23
38	.566632	5.32	.968278	.83	.598354	6.13	.401646	22
39	.566951	5.30	.968228	.83	.598722	6.15	.401278	21
40	.567269	5.30	.968178	.83	.599091	6.13	.400909	20
41	9.567587	5.28	9.968128	.83	9.599459	6.13	10.400541	19
42	.567904	5.30	.968078	.85	.599827	6.12	.400173	18
43	.568222	5.28	.968027	.83	.600194	6.13	.399806	17
44	.568539	5.28	.967977	.83	.600562	6.12	.399438	16
45	.568856	5.27	.967927	.85	.600929	6.12	.399071	15
46	.569172	5.27	.967876	.83	.601296	6.12	.398704	14
47	.569488	5.27	.967826	.85	.601663	6.10	.398337	13
48	.569804	5.27	.967775	.83	.602029	6.10	.397971	12
49	.570120	5.25	.967725	.85	.602395	6.10	.397605	11
50	.570435	5.27	.967674	.83	.602761	6.10	.397239	10
	751	5.25	9.967624	.85	9.603127	6.10	10.396873	9
	6	5.23	.967573	.85	.603493	6.08	.396507	8
	0	5.25	.967522	.85	.603858	6.08	.396142	7
	5	5.23	.967471	.83	.604223	6.08	.395777	6
	9	5.23	.967421	.85	.604588	6.08	.395412	5
	3	5.22	.967370	.85	.604953	6.07	.395047	4
	5	5.23	.967319	.85	.605317	6.08	.394683	3
	0	5.22	.967268	.85	.605682	6.07	.394318	2
	3	5.20	.967217	.85	.606046	6.07	.393954	1
	75		9.967166		9.606410		10.393590	0
	ne.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

'	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	'
0	9.573575	5.22	9.967166	.85	9.606410	6.05	10.393590	60
1	.573888	5.20	.967115	.85	.606773	6.07	.393227	59
2	.574200	5.20	.967064	.85	.607137	6.05	.392863	58
3	.574512	5.20	.967013	.87	.607500	6.05	.392500	57
4	.574824	5.20	.966961	.85	.607863	6.03	.392137	56
5	.575136	5.18	.966910	.85	.608225	6.05	.391775	55
6	.575447	5.18	.966859	.85	.608588	6.03	.391412	54
7	.575758	5.18	.966808	.87	.608950	6.03	.391050	53
8	.576069	5.17	.966756	.85	.609312	6.03	.390688	52
9	.576379	5.17	.966705	.87	.609674	6.03	.390326	51
10	.576689	5.17	.966653	.85	.610036	6.02	.389964	50
11	9.576999	5.17	9.966602	.87	9.610397	6.03	10.389603	49
12	.577309	5.15	.966550	.85	.610759	6.02	.389241	48
13	.577618	5.15	.966499	.87	.611120	6.00	.388880	47
14	.577927	5.15	.966447	.87	.611480	6.02	.388520	46
15	.578236	5.15	.966395	.85	.611841	6.00	.388159	45
16	.578545	5.13	.966344	.87	.612201	6.00	.387799	44
17	.578853	5.15	.966292	.87	.612561	6.00	.387439	43
18	.579162	5.13	.966240	.87	.612921	6.00	.387079	42
19	.579470	5.12	.966188	.87	.613281	6.00	.386719	41
20	.579777	5.13	.966136	.85	.613641	5.98	.386359	40
21	9.580085	5.12	9.966085	.87	9.614000	5.98	10.386000	39
22	.580392	5.12	.966033	.87	.614359	5.98	.385641	38
23	.580699	5.10	.965981	.87	.614718	5.98	.385282	37
24	.581005	5.12	.965929	.88	.615077	5.97	.384923	36
25	.581312	5.10	.965876	.87	.615435	5.97	.384565	35
26	.581618	5.10	.965824	.87	.615793	5.97	.384207	34
27	.581924	5.08	.965772	.87	.616151	5.97	.383849	33
28	.582229	5.10	.965720	.87	.616509	5.97	.383491	32
29	.582535	5.08	.965668	.88	.616867	5.95	.383133	31
30	.582840	5.08	.965615	.87	.617224	5.97	.382776	30
31	9.583145	5.07	9.965563	.87	9.617582	5.95	10.382418	29
32	.583449	5.08	.965511	.88	.617939	5.93	.382061	28
33	.583754	5.07	.965458	.87	.618295	5.95	.381705	27
34	.584058	5.05	.965406	.88	.918652	5.93	.381348	26
35	.584361	5.07	.965353	.87	.619008	5.93	.380992	25
36	.584665	5.05	.965301	.88	.619364	5.93	.380636	24
37	.584968	5.07	.965248	.88	.619720	5.93	.380280	23
38	.585272	5.03	.965195	.87	.620076	5.93	.379924	22
39	.585574	5.05	.965143	.88	.620432	5.92	.379568	21
40	.585877	5.03	.965090	.88	.620787	5.92	.379213	20
41	9.586179	5.05	9.965037	.88	9.621142	5.92	10.378858	19
42	.586482	5.02	.964984	.88	.621497	5.92	.378503	18
43	.586783	5.03	.964931	.87	.621852	5.92	.378148	17
44	.587085	5.02	.964879	.88	.622207	5.90	.377793	16
45	.587386	5.03	.964826	.88	.622561	5.90	.377439	15
46	.587688	5.02	.964773	.88	.622915	5.90	.377085	14
47	.587989	5.00	.964720	.90	.623269	5.90	.376731	13
48	.588289	5.02	.964666	.88	.623623	5.88	.376377	12
49	.588590	5.00	.964613	.88	.623976	5.90	.376024	11
50	.588890	5.00	.964560	.88	.624330	5.88	.375670	10
51	9.589190	4.98	9.964507	.88	9.624683	5.88	10.375317	9
52	.589489	5.00	.964454	.90	.625036	5.87	.374964	8
53	.589789	4.98	.964400	.88	.625388	5.88	.374612	7
54	.590088	4.98	.964347	.88	.625741	5.87	.374259	6
55	.590387	4.98	.964294	.90	.626093	5.87	.373907	5
56	.590686	4.97	.964240	.88	.626445	5.87	.373555	4
57	.590984	4.97	.964187	.90	.626797	5.87	.373203	3
58	.591282	4.97	.964133	.88	.627149	5.87	.372851	2
59	*.591580*	*4.97*	.964080	.90	.627501	5.85	.372499	1
60	*9.591878*		9.964026		9.627852		10.372148	
'	*Cosine.*	*D. 1".*	Sine.	D. 1".	Cotang.	D. 1".	Tang.	

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.591873	4.97	9.964026	.90	9.627852	5.85	10.372148	60
1	.592176	4.95	.963972	.88	.628203	5.85	.371797	59
2	.592473	4.95	.963919	.90	.628554	5.85	.371446	58
3	.592770	4.95	.963865	.90	.628905	5.83	.371095	57
4	.593067	4.93	.963811	.90	.629255	5.85	.370745	56
5	.593364	4.93	.963757	.88	.629606	5.83	.370394	55
6	.593659	4.93	.963704	.90	.629956	5.83	.370044	54
7	.593955	4.93	.963650	.90	.630306	5.83	.369694	53
8	.594251	4.93	.963596	.90	.630656	5.82	.369344	52
9	.594547	4.92	.963542	.90	.631005	5.83	.368995	51
10	.594842	4.92	.963488	.90	.631355	5.82	.368645	50
11	9.595137	4.92	9.963434	.92	9.631704	5.82	10.368296	49
12	.595432	4.92	.963379	.90	.632053	5.82	.367947	48
13	.595727	4.90	.963325	.90	.632402	5.80	.367598	47
14	.596021	4.90	.963271	.90	.632750	5.82	.367250	46
15	.596315	4.90	.963217	.90	.633099	5.80	.366901	45
16	.596609	4.90	.963163	.92	.633447	5.80	.366553	44
17	.596903	4.88	.963108	.90	.633795	5.80	.366205	43
18	.597196	4.90	.963054	.92	.634143	5.78	.365857	42
19	.597490	4.88	.962999	.90	.634490	5.80	.365510	41
20	.597783	4.87	.962945	.92	.634838	5.78	.365162	40
21	9.598075	4.88	9.962890	.90	9.635185	5.78	10.364815	39
22	.598368	4.87	.962836	.92	.635532	5.78	.364468	38
23	.598660	4.87	.962781	.90	.635879	5.78	.364121	37
24	.598952	4.87	.962727	.92	.636226	5.77	.363774	36
25	.599244	4.87	.962672	.92	.636572	5.78	.363428	35
26	.599536	4.85	.962617	.92	.636919	5.77	.363081	34
27	.599827	4.85	.962562	.90	.637265	5.77	.362735	33
28	.600118	4.85	.962508	.92	.637611	5.75	.362389	32
29	.600409	4.85	.962453	.92	.637956	5.77	.362044	31
30	.600700	4.83	.962398	.92	.638302	5.75	.361698	30
31	9.600990	4.83	9.962343	.92	9.638647	5.75	10.361353	29
32	.601280	4.83	.962288	.92	.638992	5.75	.361008	28
33	.601570	4.83	.962233	.92	.639337	5.75	.360663	27
34	.601860	4.83	.962178	.92	.639682	5.75	.360318	26
35	.602150	4.82	.962123	.93	.640027	5.73	.359973	25
36	.602439	4.82	.962067	.92	.640371	5.75	.359629	24
37	.602728	4.82	.962012	.92	.640716	5.73	.359284	23
38	.603017	4.80	.961957	.92	.641060	5.73	.358940	22
39	.603305	4.82	.961902	.93	.641404	5.72	.358596	21
40	.603594	4.80	.961846	.92	.641747	5.73	.358253	20
41	9.603882	4.80	9.961791	.93	9.642091	5.72	10.357909	19
42	.604170	4.78	.961735	.92	.642434	5.72	.357566	18
43	.604457	4.80	.961680	.93	.642777	5.72	.357223	17
44	.604745	4.78	.961624	.92	.643120	5.72	.356880	16
45	.605032	4.78	.961569	.93	.643463	5.72	.356537	15
46	.605319	4.78	.961513	.92	.643806	5.70	.356194	14
47	.605606	4.77	.961458	.93	.644148	5.70	.355852	13
48	.605892	4.78	.961402	.93	.644490	5.70	.355510	12
49	.606179	4.77	.961346	.93	.644832	5.70	.355168	11
50	.606465	4.77	.961290	.92	.645174	5.70	.354826	10
	606751	4.75	9.961235	.93	9.645516	5.68	10.354484	9
	036	4.77	.961179	.93	.645857	5.70	.354143	8
	322	4.75	.961123	.93	.646199	5.68	.353801	7
	607	4.75	.961067	.93	.646540	5.68	.353460	6
	892	4.75	.961011	.93	.646881	5.68	.353119	5
	177	4.73	.960955	.93	.647222	5.67	.352778	4
	461	4.73	.960899	.93	.647562	5.68	.352438	3
	745	4.73	.960843	.95	.647903	5.67	.352097	2
	029	4.73	.960786	.93	.648243	5.67	.351757	1
	313		9.960730		9.648583		10.351417	0
	sine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.609313	4.73	9.960730	.93	9.648583	5.67	10.351417	60
1	.609597	4.72	.960674	.93	.648923	5.67	.351077	59
2	.609880	4.73	.960618	.95	.649263	5.65	.350737	58
3	.610164	4.72	.960561	.93	.649602	5.67	.350398	57
4	.610447	4.70	.960505	.95	.649942	5.65	.350058	56
5	.610729	4.72	.960448	.93	.650281	5.65	.349719	55
6	.611012	4.70	.960392	.95	.650620	5.65	.349380	54
7	.611294	4.70	.960335	.93	.650959	5.63	.349041	53
8	.611576	4.70	.960279	.95	.651297	5.65	.348703	52
9	.611858	4.70	.960222	.95	.651636	5.63	.348364	51
10	.612140	4.68	.960165	.93	.651974	5.63	.348026	50
11	9.612421	4.68	9.960109	.95	9.652312	5.63	10.347688	49
12	.612702	4.68	.960052	.95	.652650	5.63	.347350	48
13	.612983	4.68	.959995	.95	.652988	5.63	.347012	47
14	.613264	4.68	.959938	.93	.653326	5.62	.346674	46
15	.613545	4.67	.959882	.95	.653663	5.62	.346337	45
16	.613825	4.67	.959825	.95	.654000	5.62	.346000	44
17	.614105	4.67	.959768	.95	.654337	5.62	.345663	43
18	.614385	4.67	.959711	.95	.654674	5.62	.345326	42
19	.614665	4.65	.959654	.97	.655011	5.62	.344989	41
20	.614944	4.65	.959596	.95	.655348	5.60	.344652	40
21	9.615223	4.65	9.959539	.95	9.655684	5.60	10.344316	39
22	.615502	4.65	.959482	.95	.656020	5.60	.343980	38
23	.615781	4.65	.959425	.95	.656356	5.60	.343644	37
24	.616060	4.63	.959368	.97	.656692	5.60	.343308	36
25	.616338	4.63	.959310	.95	.657028	5.60	.342972	35
26	.616616	4.63	.959253	.97	.657364	5.58	.342636	34
27	.616894	4.63	.959195	.95	.657699	5.58	.342301	33
28	.617172	4.63	.959138	.97	.658034	5.58	.341966	32
29	.617450	4.62	.959080	.95	.658369	5.58	.341631	31
30	.617727	4.62	.959023	.97	.658704	5.58	.341296	30
31	9.618004	4.62	9.958965	.95	9.659039	5.57	10.340961	29
32	.618281	4.62	.958908	.97	.659373	5.58	.340627	28
33	.618558	4.60	.958850	.97	.659708	5.57	.340292	27
34	.618834	4.60	.958792	.97	.660042	5.57	.339958	26
35	.619110	4.60	.958734	.95	.660376	5.57	.339624	25
36	.619386	4.60	.958677	.97	.660710	5.55	.339290	24
37	.619662	4.60	.958619	.97	.661043	5.57	.338957	23
38	.619938	4.58	.958561	.97	.661377	5.55	.338623	22
39	.620213	4.58	.958503	.97	.661710	5.55	.338290	21
40	.620488	4.58	.958445	.97	.662043	5.55	.337957	20
41	9.620763	4.58	9.958387	.97	9.662376	5.55	10.337624	19
42	.621038	4.58	.958329	.97	.662709	5.55	.337291	18
43	.621313	4.57	.958271	.97	.663042	5.55	.336958	17
44	.621587	4.57	.958213	.98	.663375	5.53	.336625	16
45	.621861	4.57	.958154	.97	.663707	5.53	.366293	15
46	.622135	4.57	.958096	.97	.664039	5.53	.335961	14
47	.622409	4.55	.958038	.98	.664371	5.53	.335629	13
48	.622682	4.57	.957979	.97	.664703	5.53	.335297	12
49	.622956	4.55	.957921	.97	.665035	5.52	.334965	11
50	.623229	4.55	.957863	.98	.665366	5.53	.334634	10
51	9.623502	4.53	9.957804	.97	9.665698	5.52	10.334302	9
52	.623774	4.55	.957746	.98	.666029	5.52	.333971	8
53	.624047	4.53	.957687	.98	.666360	5.52	.333640	7
54	.624319	4.53	.957628	.97	.666691	5.50	.333309	6
55	.624591	4.53	.957570	.98	.667021	5.52	.332979	
56	.624863	4.53	.957511	.98	.667352	5.50	.332648	
57	.625135	4.52	.957452	.98	.667682	5.52	.332318	
58	.625406	4.52	.957393	.97	.668013	5.50	.331987	
59	.625677	4.52	.957335	.98	.668343	5.50	.3316	
60	9.625948		9.957276		9.668673		10.33	
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Ta	

TABLE XXV.—LOGARITHMIC SINES,

'	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	'
0	9.625948	4.52	9.957276	.98	9.668673	5.48	10.331327	60
1	.626219	4.52	.957217	.98	.669002	5.50	.330998	59
2	.626490	4.50	.957158	.98	.669332	5.48	.330668	58
3	.626760	4.50	.957099	.98	.669661	5.50	.330339	57
4	.627030	4.50	.957040	.98	.669991	5.48	.330009	56
5	.627300	4.50	.956981	1.00	.670320	5.48	.329680	55
6	.627570	4.50	.956921	.98	.670649	5.47	.329351	54
7	.627840	4.48	.956862	.98	.670977	5.48	.329023	53
8	.628109	4.48	.956803	.98	.671306	5.48	.328694	52
9	.628378	4.48	.956744	1.00	.671635	5.47	.328365	51
10	.628647	4.48	.956684	.98	.671963	5.47	.328037	50
11	9.628916	4.48	9.956625	.98	9.672291	5.47	10.327709	49
12	.629185	4.47	.956566	1.00	.672619	5.47	.327381	48
13	.629453	4.47	.956506	.98	.672947	5.45	.327053	47
14	.629721	4.47	.956447	1.00	.673274	5.47	.326726	46
15	.629989	4.47	.956387	1.00	.673602	5.45	.326398	45
16	.630257	4.45	.956327	.98	.673929	5.47	.326071	44
17	.630524	4.47	.956268	1.00	.674257	5.45	.325743	43
18	.630792	4.45	.956208	1.00	.674584	5.45	.325416	42
19	.631059	4.45	.956148	.98	.674911	5.43	.325089	41
20	.631326	4.45	.956089	1.00	.675237	5.45	.324763	40
21	9.631593	4.43	9.956029	1.00	9.675564	5.43	10.324436	39
22	.631859	4.43	.955969	1.00	.675890	5.45	.324110	38
23	.632125	4.45	.955909	1.00	.676217	5.43	.323783	37
24	.632392	4.43	.955849	1.00	.676543	5.43	.323457	36
25	.632658	4.42	.955789	1.00	.676869	5.42	.323131	35
26	.632923	4.43	.955729	1.00	.677194	5.43	.322806	34
27	.633189	4.42	.955669	1.00	.677520	5.43	.322480	33
28	.633454	4.42	.955609	.98	.677846	5.42	.322154	32
29	.633719	4.42	.955548	1.00	.678171	5.42	.321829	31
30	.633984	4.42	.955488	1.00	.678496	5.42	.321504	30
31	9.634249	4.42	9.955428	1.00	9.678821	5.42	10.321179	29
32	.634514	4.40	.955368	1.02	.679146	5.42	.320854	28
33	.634778	4.40	.955307	1.00	.679471	5.40	.320529	27
34	.635042	4.40	.955247	1.02	.679795	5.42	.320205	26
35	.635306	4.40	.955186	1.00	.680120	5.40	.319880	25
36	.635570	4.40	.955126	1.02	.680444	5.40	.319556	24
37	.635834	4.38	.955065	1.00	.680768	5.40	.319232	23
38	.636097	4 38	.955005	1.02	.681092	5.40	.318908	22
39	.636360	4.38	.954944	1.02	.681416	5.40	.318584	21
40	.636623	4.38	.954883	1.00	.681740	5.38	.318260	20
41	9.636886	4.37	9.954823	1.02	9.682063	5.40	10.317937	19
42	.637148	4.38	.954762	1.02	.682387	5.38	.317613	18
43	.637411	4.37	.954701	1.02	.682710	5.38	.317290	17
44	.637673	4.37	.954640	1.02	.683033	5.38	.316967	16
45	.637935	4.37	.954579	1.02	.683356	5.38	.316644	15
46	.638197	4.35	.954518	1.02	.683679	5.37	.316321	14
47	.638458	4.37	.954457	1.02	.684001	5.38	.315999	13
48	.638720	4.35	.954396	1.02	.684324	5.37	.315676	12
49	.638981	4.35	.954335	1.02	.684646	5.37	.315354	11
50	.639242	4.35	.954274	1.02	.684968	5.37	.315032	10
	9.639503	4.35	9.954213	1.02	9.685290	5.37	10.314710	9
	.639764	4.33	.954152	1.03	.685612	5.37	.314388	8
	.640024	4.33	.954090	1.02	.685934	5.35	.314066	7
	.640284	4.33	.954029	1.02	.686255	5.37	.313745	6
	.640544	4.33	.953968	1.03	.686577	5.35	.313423	5
	.640804	4.33	.953906	1.02	.686898	5.35	.313102	4
	.641064	4.33	.953845	1.03	.687219	5.35	.312781	3
	.641324	4.32	.953783	1.02	.687540	5.35	.312460	2
	.641583	4.32	.953722	1.03	.687861	5.35	.312139	1
	9.641842		9.953660		9.688182		10.311818	0
	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	'

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.641842	4.32	9.953660	1.02	9.688182	5.33	10.311818	60
1	.642101	4.32	.953599	1.03	.688502	5.32	.311498	59
2	.642360	4.30	.953537	1.03	.688823	5.33	.311177	58
3	.642618	4.32	.953475	1.03	.689143	5.33	.310857	57
4	.642877	4.30	.953413	1.02	.689463	5.33	.310537	56
5	.643135	4.30	.953352	1.03	.689783	5.33	.310217	55
6	.643393	4.28	.953290	1.03	.690103	5.33	.309897	54
7	.643650	4.30	.953228	1.03	.690423	5.32	.309577	53
8	.643908	4.28	.953166	1.03	.690742	5.33	.309258	52
9	.644165	4.30	.953104	1.03	.691062	5.32	.308938	51
10	.644423	4.28	.953042	1.03	.691381	5.32	.308619	50
11	9.644680	4.27	9.952980	1.03	9.691700	5.32	10.308300	49
12	.644936	4.28	.952918	1.05	.692019	5.32	.307981	48
13	.645193	4.28	.952855	1.03	.692338	5.30	.307662	47
14	.645450	4.27	.952793	1.03	.692656	5.32	.307344	46
15	.645706	4.27	.952731	1.03	.692975	5.30	.307025	45
16	.645962	4.27	.952669	1.05	.693293	5.32	.306707	44
17	.646218	4.27	.952606	1.03	.693612	5.30	.306388	43
18	.646474	4.25	.952544	1.05	.693930	5.30	.306070	42
19	.646729	4.25	.952481	1.03	.694248	5.30	.305752	41
20	.646984	4.27	.952419	1.05	.694566	5.28	.305434	40
21	9.647240	4.23	9.952356	1.03	9.694883	5.30	10.305117	39
22	.647494	4.25	.952294	1.05	.695201	5.28	.304799	38
23	.647749	4.25	.952231	1.05	.695518	5.30	.304482	37
24	.648004	4.23	.952168	1.03	.695836	5.28	.304164	36
25	.648258	4.23	.952106	1.05	.696153	5.28	.303847	35
26	.648512	4.23	.952043	1.05	.696470	5.28	.303530	34
27	.648766	4.23	.951980	1.05	.696787	5.27	.303213	33
28	.649020	4.23	.951917	1.05	.697103	5.28	.302897	32
29	.649274	4.22	.951854	1.05	.697420	5.27	.302580	31
30	.649527	4.23	.951791	1.05	.697736	5.28	.302264	30
31	9.649781	4.22	9.951728	1.05	9.698053	5.27	10.301947	29
32	.650034	4.22	.951665	1.05	.698369	5.27	.301631	28
33	.650287	4.20	.951602	1.05	.698685	5.27	.301315	27
34	.650539	4.22	.951539	1.05	.699001	5.25	.300999	26
35	.650792	4.20	.951476	1.07	.699316	5.27	.300684	25
36	.651044	4.22	.951412	1.05	.699632	5.25	.300368	24
37	.651297	4.20	.951349	1.05	.699947	5.27	.300053	23
38	.651549	4.18	.951286	1.07	.700263	5.25	.299737	22
39	.651800	4.20	.951222	1.05	.700578	5.25	.299422	21
40	.652052	4.20	.951159	1.05	.700893	5.25	.299107	20
41	9.652304	4.18	9.951096	1.07	9.701208	5.25	10.298792	19
42	.652555	4.18	.951032	1.07	.701523	5.23	.298477	18
43	.652806	4.18	.950968	1.05	.701837	5.25	.298163	17
44	.653057	4.18	.950905	1.07	.702152	5.23	.297848	16
45	.653308	4.17	.950841	1.05	.702466	5.25	.297534	15
46	.653558	4.17	.950778	1.07	.702781	5.23	.297219	14
47	.653808	4.18	.950714	1.07	.703095	5.23	.296905	13
48	.654059	4.17	.950650	1.07	.703409	5.22	.296591	12
49	.654309	4.15	.950586	1.07	.703722	5.23	.296278	11
50	.654558	4.17	.950522	1.07	.704036	5.23	.295964	10
51	9.654808	4.17	9.950458	1.07	9.704350	5.22	10.295650	9
52	.655058	4.15	.950394	1.07	.704663	5.22	.295337	8
53	.655307	4.15	.950330	1.07	.704976	5.23	.295024	7
54	.655556	4.15	.950266	1.07	.705290	5.22	.294710	6
55	.655805	4.15	.950202	1.07	.705603	5.22	.294397	5
56	.656054	4.13	.950138	1.07	.705916	5.20	.294084	4
57	.656302	4.15	.950074	1.07	.706228	5.22	.293772	3
58	.656551	4.13	.950010	1.08	.706541	5.22	.293459	2
59	.656799	4.13	.949945	1.07	.706854	5.20	.293146	1
60	9.657047		9.949881		9.707166		10.292834	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.657047	4.13	9.949881	1.08	9.707166	5.20	10.292834	60
1	.657295	4.12	.949816	1.07	.707478	5.20	.292522	59
2	.657542	4.13	.949752	1.07	.707790	5.20	.292210	58
3	.657790	4.12	.949688	1.08	.708102	5.20	.291898	57
4	.658037	4.12	.949623	1.08	.708414	5.20	.291586	56
5	.658284	4.12	.949558	1.07	.708726	5.18	.291274	55
6	.658531	4.12	.949494	1.08	.709037	5.20	.290963	54
7	.658778	4.12	.949429	1.08	.709349	5.18	.290651	53
8	.659025	4.10	.949364	1.07	.709660	5.18	.290340	52
9	.659271	4.10	.949300	1.08	.709971	5.18	.290029	51
10	.659517	4.10	.949235	1.08	.710282	5.18	.289718	50
11	9.659763	4.10	9.949170	1.08	9.710593	5.18	10.289407	49
12	.660009	4.10	.949105	1.08	.710904	5.18	.289096	48
13	.660255	4.10	.949040	1.08	.711215	5.17	.288785	47
14	.660501	4.08	.948975	1.08	.711525	5.18	.288475	46
15	.660746	4.08	.948910	1.08	.711836	5.17	.288164	45
16	.660991	4.08	.948845	1.08	.712146	5.17	.287854	44
17	.661236	4.08	.948780	1.08	.712456	5.17	.287544	43
18	.661481	4.08	.948715	1.08	.712766	5.17	.287234	42
19	.661726	4.07	.948650	1.10	.713076	5.17	.286924	41
20	.661970	4.07	.948584	1.08	.713386	5.17	.286614	40
21	9.662214	4.08	9.948519	1.08	9.713696	5.15	10.286304	39
22	.662459	4.07	.948454	1.10	.714005	5.15	.285995	38
23	.662703	4.05	.948388	1.08	.714314	5.17	.285686	37
24	.662946	4.07	.948323	1.10	.714624	5.15	.285376	36
25	.663190	4.05	.948257	1.08	.714933	5.15	.285067	35
26	.663433	4.07	.948192	1.10	.715242	5.15	.284758	34
27	.663677	4.05	.948126	1.10	.715551	5.15	.284449	33
28	.663920	4.05	.948060	1.08	.715860	5.13	.284140	32
29	.664163	4.05	.947995	1.10	.716168	5.15	.283832	31
30	.664406	4.03	.947929	1.10	.716477	5.13	.283523	30
31	9.664648	4.05	9.947863	1.10	9.716785	5.13	10.283215	29
32	.664891	4.03	.947797	1.10	.717093	5.13	.282907	28
33	.665133	4.03	.947731	1.10	.717401	5.13	.282599	27
34	.665375	4.03	.947665	1.08	.717709	5.13	.282291	26
35	.665617	4.03	.947600	1.12	.718017	5.13	.281983	25
36	.665859	4.02	.947533	1.10	.718325	5.13	.281675	24
37	.666100	4.03	.947467	1.10	.718633	5.12	.281367	23
38	.666342	4.02	.947401	1.10	.718940	5.13	.281060	22
39	.666583	4.02	.947335	1.10	.719248	5.12	.280752	21
40	.666824	4.02	.947269	1.10	.719555	5.12	.280445	20
41	9.667065	4.00	9.947203	1.12	9.719862	5.12	10.280138	19
42	.667305	4.02	.947136	1.10	.720169	5.12	.279831	18
43	.667546	4.00	.947070	1.10	.720476	5.12	.279524	17
44	.667786	4.02	.947004	1.12	.720783	5.10	.279217	16
45	.668027	4.00	.946937	1.10	.721089	5.12	.278911	15
46	.668267	3.98	.946871	1.12	.721396	5.10	.278604	14
47	.668506	4.00	.946804	1.10	.721702	5.12	.278298	13
48	.668746	4.00	.946738	1.12	.722009	5.10	.277991	12
49	.668986	3.98	.946671	1.12	.722315	5.10	.277685	11
50	.669225	3.98	.946604	1.10	.722621	5.10	.277379	10
51	9.669464	3.98	9.946538	1.12	9.722927	5.08	10.277073	9
52	.669703	3.98	.946471	1.12	.723232	5.10	.276768	8
53	.669942	3.98	.946404	1.12	.723538	5.10	.276462	7
54	.670181	3.97	.946337	1.12	.723844	5.08	.276156	6
55	.670419	3.98	.946270	1.12	.724149	5.08	.275851	5
56	.670658	3.97	.946203	1.12	.724454	5.10	.275546	4
57	.670896	3.97	.946136	1.12	.724760	5.08	.275240	3
58	.671134	3.97	.946069	1.12	.725065	5.08	.274935	2
59	.671372	3.95	.946002	1.12	.725370	5.07	.274630	1
60	9.671609		9.945935		9.725674		10.274326	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	′
0	9.671609	3.97	9.945935	1.12	9.725674	5.08	10.274326	60
1	.671847	3.95	.945868	1.13	.725979	5.08	.274021	59
2	.672084	3.95	.945800	1.12	.726284	5.07	.273716	58
3	.672321	3.95	.945733	1.12	.726588	5.07	.273412	57
4	.672558	3.95	.945666	1.13	.726892	5.05	.273108	56
5	.672795	3.95	.945598	1.12	.727197	5.07	.272803	55
6	.673032	3.93	.945531	1.12	.727501	5.07	.272499	54
7	.673268	3.95	.945464	1.13	.727805	5.07	.272195	53
8	.673505	3.93	.945396	1.13	.728109	5.05	.271891	52
9	.673741	3.93	.945328	1.12	.728412	5.07	.271588	51
10	.673977	3.93	.945261	1.13	.728716	5.07	.271284	50
11	9.674213	3.92	9.945193	1.13	9.729020	5.05	10.270980	49
12	.674448	3.93	.945125	1.12	.729323	5.05	.270677	48
13	.674684	3.92	.945058	1.13	.729626	5.05	.270374	47
14	.674919	3.93	.944990	1.13	.729929	5.07	.270071	46
15	.675155	3.92	.944922	1.13	.730233	5.03	.269767	45
16	.675390	3.90	.944854	1.13	.730535	5.05	.269465	44
17	.675624	3.92	.944786	1.13	.730838	5.05	.269162	43
18	.675859	3.92	.944718	1.13	.731141	5.05	.268859	42
19	.676094	3.90	.944650	1.13	.731444	5.03	.268556	41
20	.676328	3.90	.944582	1.13	.731746	5.03	.268254	40
21	9.676562	3.90	9.944514	1.13	9.732048	5.05	10.267952	39
22	.676796	3.90	.944446	1.15	.732351	5.03	.267649	38
23	.677030	3.90	.944377	1.13	.732653	5.03	.267347	37
24	.677264	3.90	.944309	1.13	.732955	5.03	.267045	36
25	.677498	3.88	.944241	1.15	.733257	5.02	.266743	35
26	.677731	3.88	.944172	1.13	.733558	5.03	.266442	34
27	.677964	3.88	.944104	1.13	.733860	5.03	.266140	33
28	.678197	3.88	.944036	1.15	.734162	5.02	.265838	32
29	.678430	3.88	.943967	1.13	.734463	5.02	.265537	31
30	.678663	3.87	.943899	1.15	.734764	5.03	.265236	30
31	9.678895	3.88	9.943830	1.15	9.735066	5.02	10.264934	29
32	.679128	3.87	.943761	1.13	.735367	5.02	.264633	28
33	.679360	3.87	.943693	1.15	.735668	5.02	.264332	27
34	.679592	3.87	.943624	1.15	.735969	5.00	.264031	26
35	.679824	3.87	.943555	1.15	.736269	5.02	.263731	25
36	.680056	3.87	.943486	1.15	.736570	5.00	.263430	24
37	.680288	3.85	.943417	1.15	.736870	5.02	.263130	23
38	.680519	3.85	.943348	1.15	.737171	5.00	.262829	22
39	.680750	3.87	.943279	1.15	.737471	5.00	.262529	21
40	.680982	3.85	.943210	1.15	.737771	5.00	.262229	20
41	9.681213	3.83	9.943141	1.15	9.738071	5.00	10.261929	19
42	.681443	3.85	.943072	1.15	.738371	5.00	.261629	18
43	.681674	3.85	.943003	1.15	.738671	5.00	.261329	17
44	.681905	3.83	.942934	1.17	.738971	5.00	.261029	16
45	.682135	3.83	.942864	1.15	.739271	4.98	.260729	15
46	.682365	3.83	.942795	1.15	.739570	5.00	.260430	14
47	.682595	3.83	.942726	1.17	.739870	4.98	.260130	13
48	.682825	3.83	.942656	1.15	.740169	4.98	.259831	12
49	.683055	3.82	.942587	1.17	.740468	4.98	.259532	11
50	.683284	3.83	.942517	1.15	.740767	4.98	.259233	10
51	9.683514	3.82	9.942448	1.17	9.741066	4.98	10.258934	9
52	.683743	3.82	.942378	1.17	.741365	4.98	.258635	8
53	.683972	3.82	.942308	1.15	.741664	4.97	.258336	7
54	.684201	3.82	.942239	1.17	.741962	4.98	.258038	6
55	.684430	3.80	.942169	1.17	.742261	4.97	.257739	5
56	.684658	3.82	.942099	1.17	.742559	4.98	.257441	4
57	.684887	3.80	.942029	1.17	.742858	4.97	.257142	3
58	.685115	3.80	.941959	1.17	.743156	4.97	.256844	2
59	.685343	3.80	.941889	1.17	.743454	4.97	.256546	1
60	9.685571		9.941819		9.743752		10.256248	0
′	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	′

'	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.
0	9.685571	3.80	9.941819	1.17	9.743752	4.97	10.25624
1	.685799	3.80	.941749	1.17	.744050	4.97	.25595
2	.686027	3.78	.941679	1.17	.744348	4.95	.25565
3	.686254	3.80	.941609	1.17	.744645	4.97	.25535
4	.686482	3.78	.941539	1.17	.744943	4.95	.25505
5	.686709	3.78	.941469	1.18	.745240	4.97	.25476
6	.686936	3.78	.941398	1.17	.745538	4.95	.25446
7	.687163	3.77	.941328	1.17	.745835	4.95	.25416
8	.687389	3.78	.941258	1.18	.746132	4.95	.25386
9	.687616	3.78	.941187	1.17	.746429	4.95	.253571
10	.687843	3.77	.941117	1.18	.746726	4.95	.25327
11	9.688069	3.77	9.941046	1.18	9.747023	4.93	10.252977
12	.688295	3.77	.940975	1.17	.747319	4.95	.252681
13	.688521	3.77	.940905	1.18	.747616	4.95	.252384
14	.688747	3.75	.940834	1.18	.747913	4.93	.252087
15	.688972	3.77	.940763	1.17	.748209	4.93	.251791
16	.689198	3.75	.940693	1.18	.748505	4.93	.251495
17	.689423	3.75	.940622	1.18	.748801	4.93	.251199
18	.689648	3.75	.940551	1.18	.749097	4.93	.250903
19	.689873	3.75	.940480	1.18	.749393	4.93	.250607
20	.690098	3.75	.940409	1.18	.749689	4.93	.250311
21	9.690323	3.75	9.940338	1.18	9.749985	4.93	10.250015
22	.690548	3.73	.940267	1.18	.750281	4.92	.249719
23	.690772	3.73	.940196	1.18	.750576	4.93	.249424
24	.690996	3.73	.940125	1.18	.750872	4.92	.249128
25	.691220	3.73	.940054	1.20	.751167	4.92	.248833
26	.691444	3.73	.939982	1.18	.751462	4.92	.248538
27	.691668	3.73	.939911	1.18	.751757	4.92	.248243
28	.691892	3.72	.939840	1.20	.752052	4.92	.247948
29	.692115	3.73	.939768	1.18	.752347	4.92	.247653
30	.692339	3.72	.939697	1.20	.752642	4.92	.247358
31	9.692562	3.72	9.939625	1.18	9.752937	4.90	10.247063
32	.692785	3.72	.939554	1.20	.753231	4.92	.246769
33	.693008	3.72	.939482	1.20	.753526	4.90	.246474
34	.693231	3.70	.939410	1.18	.753820	4.92	.246180
35	.693453	3.72	.939339	1.20	.754115	4.90	.245885
36	.693676	3.70	.939267	1.20	.754409	4.90	.245591
37	.693898	3.70	.939195	1.20	.754703	4.90	.245297
38	.694120	3.70	.939123	1.18	.754997	4.90	.245003
39	.694342	3.70	.939052	1.20	.755291	4.90	.244709
40	.694564	3.70	.938980	1.20	.755585	4.88	.244415
41	9.694786	3.68	9.938908	1.20	9.755878	4.90	10.244122
42	.695007	3.70	.938836	1.22	.756172	4.88	.243828
43	.695229	3.68	.938763	1.20	.756465	4.90	.243535
44	.695450	3.68	.938691	1.20	.756759	4.88	.243241
45	.695671	3.68	.938619	1.20	.757052	4.88	.242948
46	.695892	3.68	.938547	1.20	.757345	4.88	.242655
47	.696113	3.68	.938475	1.22	.757638	4.88	.242362
48	.696334	3.67	.938402	1.20	.757931	4.88	.242069
49	.696554	3.68	.938330	1.20	.758224	4.88	.241776
50	.696775	3.67	.938258	1.22	.758517	4.88	.241483
51	9.696995	3.67	9.938185	1.20	9.758810	4.87	10.241190
52	.697215	3.67	.938113	1.22	.759102	4.88	.240898
53	.697435	3.65	.938040	1.22	.759395	4.87	.240605
54	.697654	3.67	.937967	1.20	.759687	4.87	.240313
55	.697874	3.67	.937895	1.22	.759979	4.88	.240021
56	.698094	3.65	.937822	1.22	.760272	4.87	.239728
57	.698313	3.65	.937749	1.22	.760564	4.87	.239436
58	.698532	3.65	.937676	1.20	.760856	4.87	.239144
59	.698751	3.65	.937604	1.22	.761148	4.85	.238852
60	9.698970		9.937531		9.761439		10.23856
'	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang

119°

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.769219	2.90	9.907958	1.53	9.861261	4.43	10.138739	60
1	.769393	2.88	.907866	1.53	.861527	4.42	.138473	59
2	.769566	2.90	.907774	1.53	.861792	4.43	.138208	58
3	.769740	2.88	.907682	1.53	.862058	4.42	.137942	57
4	.769913	2.90	.907590	1.53	.862323	4.43	.137677	56
5	.770087	2.88	.907498	1.53	.862589	4.42	.137411	55
6	.770260	2.88	.907406	1.53	.862854	4.42	.137146	54
7	.770433	2.88	.907314	1.53	.863119	4.43	.136881	53
8	.770606	2.88	.907222	1.55	.863385	4.42	.136615	52
9	.770779	2.88	.907129	1.53	.863650	4.42	.136350	51
10	.770952	2.88	.907037	1.53	.863915	4.42	.136085	50
11	9.771125	2.88	9.906945	1.55	9.864180	4.42	10.135820	49
12	.771298	2.87	.906852	1.53	.864445	4.42	.135555	48
13	.771470	2.88	.906760	1.55	.864710	4.42	.135290	47
14	.771643	2.87	.906667	1.53	.864975	4.42	.135025	46
15	.771815	2.87	.906575	1.55	.865240	4.42	.134760	45
16	.771987	2.87	.906482	1.55	.865505	4.42	.134495	44
17	.772159	2.87	.906389	1.55	.865770	4.42	.134230	43
18	.772331	2.87	.906296	1.53	.866035	4.42	.133965	42
19	.772503	2.87	.906204	1.55	.866300	4.40	.133700	41
20	.772675	2.87	.906111	1.55	.866564	4.42	.133436	40
21	9.772847	2.85	9.906018	1.55	9.866829	4.42	10.133171	39
22	.773018	2.87	.905925	1.55	.867094	4.40	.132906	38
23	.773190	2.85	.905832	1.55	.867358	4.42	.132642	37
24	.773361	2.87	.905739	1.57	.867623	4.40	.132377	36
25	.773533	2.85	.905645	1.55	.867887	4.42	.132113	35
26	.773704	2.85	.905552	1.55	.868152	4.40	.131848	34
27	.773875	2.85	.905459	1.55	.868416	4.40	.131584	33
28	.774046	2.85	.905366	1.57	.868680	4.42	.131320	32
29	.774217	2.85	.905272	1.55	.868945	4.40	.131055	31
30	.774388	2.83	.905179	1.57	.869209	4.40	.130791	30
31	9.774558	2.85	9.905085	1.55	9.869473	4.40	10.130527	29
32	.774729	2.83	.904992	1.57	.869737	4.40	.130263	28
33	.774899	2.85	.904898	1.57	.870001	4.40	.129999	27
34	.775070	2.83	.904804	1.55	.870265	4.40	.129735	26
35	.775240	2.83	.904711	1.57	.870529	4.40	.129471	25
36	.775410	2.83	.904617	1.57	.870793	4.40	.129207	24
37	.775580	2.83	.904523	1.57	.871057	4.40	.128943	23
38	.775750	2.83	.904429	1.57	.871321	4.40	.128679	22
39	.775920	2.83	.904335	1.57	.871585	4.40	.128415	21
40	.776090	2.82	.904241	1.57	.871849	4.38	.128151	20
41	9.776259	2.83	9.904147	1.57	9.872112	4.40	10.127888	19
42	.776429	2.82	.904053	1.57	.872376	4.40	.127624	18
43	.776598	2.83	.903959	1.58	.872640	4.38	.127360	17
44	.776768	2.82	.903864	1.57	.872903	4.40	.127097	16
45	.776937	2.82	.903770	1.57	.873167	4.38	.126833	15
46	.777106	2.82	.903676	1.58	.873430	4.40	.126570	14
47	.777275	2.82	.903581	1.57	.873694	4.38	.126306	13
48	.777444	2.82	.903487	1.58	.873957	4.38	.126043	12
49	.777613	2.80	.903392	1.57	.874220	4.40	.125780	11
50	.777781	2.82	.903298	1.58	.874484	4.38	.125516	10
51	9.777950	2.82	9.903203	1.58	9.874747	4.38	10.125253	9
52	.778119	2.80	.903108	1.57	.875010	4.38	.124990	8
53	.778287	2.80	.903014	1.58	.875273	4.40	.124727	7
54	.778455	2.82	.902919	1.58	.875537	4.38	.124463	6
55	.778624	2.80	.902824	1.58	.875800	4.38	.124200	5
56	.778792	2.80	.902729	1.58	.876063	4.38	.123937	4
57	.778960	2.80	.902634	1.58	.876326	4.38	.123674	3
58	.779128	2.78	.902539	1.58	.876589	4.38	.123411	2
59	.779295	2.80	.902444	1.58	.876852	4.37	.123148	1
60	9.779463		9.902349		9.877114		10.122886	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.711839	3.52	9.933066	1.27	9.778774	4.77	10.221226	60
1	.712050	3.50	.932990	1.27	.779060	4.77	.220940	59
2	.712260	3.48	.932914	1.27	.779346	4.77	.220654	58
3	.712469	3.50	.932838	1.27	.779632	4.77	.220368	57
4	.712679	3.50	.932762	1.28	.779918	4.75	.220082	56
5	.712889	3.48	.932685	1.27	.780203	4.77	.219797	55
6	.713098	3.50	.932609	1.27	.780489	4.77	.219511	54
7	.713308	3.48	.932533	1.27	.780775	4.75	.219225	53
8	.713517	3.48	.932457	1.28	.781060	4.77	.218940	52
9	.713726	3.48	.932380	1.27	.781346	4.75	.218654	51
10	.713935	3.48	.932304	1.27	.781631	4.75	.218369	50
11	9.714144	3.47	9.932228	1.28	9.781916	4.75	10.218084	49
12	.714352	3.48	.932151	1.27	.782201	4.75	.217799	48
13	.714561	3.47	.932075	1.28	.782486	4.75	.217514	47
14	.714769	3.48	.931998	1.28	.782771	4.75	.217229	46
15	.714978	3.47	.931921	1.27	.783056	4.75	.216944	45
16	.715186	3.47	.931845	1.28	.783341	4.75	.216659	44
17	.715394	3.47	.931768	1.28	.783626	4.73	.216374	43
18	.715602	3.45	.931691	1.28	.783910	4.75	.216090	42
19	.715809	3.47	.931614	1.28	.784195	4.73	.215805	41
20	.716017	3.45	.931537	1.28	.784479	4.75	.215521	40
21	9.716224	3.47	9.931460	1.28	9.784764	4.73	10.215236	39
22	.716432	3.45	.931383	1.28	.785048	4.73	.214952	38
23	.716639	3.45	.931306	1.28	.785332	4.73	.214668	37
24	.716846	3.45	.931229	1.28	.785616	4.73	.214384	36
25	.717053	3.43	.931152	1.28	.785900	4.73	.214100	35
26	.717259	3.45	.931075	1.28	.786184	4.73	.213816	34
27	.717466	3.45	.930998	1.28	.786468	4.73	.213532	33
28	.717673	3.43	.930921	1.30	.786752	4.73	.213248	32
29	.717879	3.43	.930843	1.28	.787036	4.72	.212964	31
30	.718085	3.43	.930766	1.30	.787319	4.73	.212681	30
31	9.718291	3.43	9.930688	1.28	9.787603	4.72	10.212397	29
32	.718497	3.43	.930611	1.30	.787886	4.73	.212114	28
33	.718703	3.43	.930533	1.28	.788170	4.72	.211830	27
34	.718909	3.42	.930456	1.30	.788453	4.72	.211547	26
35	.719114	3.43	.930378	1.30	.788736	4.72	.211264	25
36	.719320	3.42	.930300	1.28	.789019	4.72	.210981	24
37	.719525	3.42	.930223	1.30	.789302	4.72	.210698	23
38	.719730	3.42	.930145	1.30	.789585	4.72	.210415	22
39	.719935	3.42	.930067	1.30	.789868	4.72	.210132	21
40	.720140	3.42	.929989	1.30	.790151	4.72	.209849	20
41	9.720345	3.40	9.929911	1.30	9.790434	4.70	10.209566	19
42	.720549	3.42	.929833	1.30	.790716	4.72	.209284	18
43	.720754	3.40	.929755	1.30	.790999	4.70	.209001	17
44	.720958	3.40	.929677	1.30	.791281	4.70	.208719	16
45	.721162	3.40	.929599	1.30	.791563	4.72	.208437	15
46	.721366	3.40	.929521	1.32	.791846	4.70	.208154	14
47	.721570	3.40	.929442	1.30	.792128	4.70	.207872	13
48	.721774	3.40	.929364	1.30	.792410	4.70	.207590	12
49	.721978	3.38	.929286	1.32	.792692	4.70	.207308	11
50	.722181	3.40	.929207	1.30	.792974	4.70	.207026	10
	9.722385	3.38	9.929129	1.32	9.793256	4.70	10.206744	9
	.722588	3.38	.929050	1.30	.793538	4.68	.206462	8
	.722791	3.38	.928972	1.32	.793819	4.70	.206181	7
	.722994	3.38	.928893	1.30	.794101	4.70	.205899	6
	.723197	3.38	.928815	1.32	.794383	4.68	.205617	5
	.723400	3.38	.928736	1.32	.794664	4.70	.205336	4
	.723603	3.37	.928657	1.32	.794946	4.68	.205054	3
	.723805	3.37	.928578	1.32	.795227	4.68	.204773	2
	.724007	3.38	.928499	1.32	.795508	4.68	.204492	1
	9.724210		9.928420		9.795789		10.204211	0
	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.724210	3.37	9.928420	1.30	9.795789	4.68	10.204211	60
1	.724412	3.37	.928342	1.32	.796070	4.68	.203930	59
2	.724614	3.37	.928263	1.33	.796351	4.68	.203649	58
3	.724816	3.35	.928183	1.32	.796632	4.68	.203368	57
4	.725017	3.37	.928104	1.32	.796913	4.68	.203087	56
5	.725219	3.35	.928025	1.32	.797194	4.67	.202806	55
6	.725420	3.37	.927946	1.32	.797474	4.68	.202526	54
7	.725622	3.35	.927867	1.33	.797755	4.68	.202245	53
8	.725823	3.35	.927787	1.32	.798036	4.67	.201964	52
9	.726024	3.35	.927708	1.32	.798316	4.67	.201684	51
10	.726225	3.35	.927629	1.33	.798596	4.68	.201404	50
11	9.726426	3.33	9.927549	1.32	9.798877	4.67	10.201123	49
12	.726626	3.35	.927470	1.33	.799157	4.67	.200843	48
13	.726827	3.33	.927390	1.33	.799437	4.67	.200563	47
14	.727027	3.35	.927310	1.32	.799717	4.67	.200283	46
15	.727228	3.33	.927231	1.33	.799997	4.67	.200003	45
16	.727428	3.33	.927151	1.33	.800277	4.67	.199723	44
17	.727628	3.33	.927071	1.33	.800557	4.65	.199443	43
18	.727828	3.32	.926991	1.33	.800836	4.67	.199164	42
19	.728027	3.33	.926911	1.33	.801116	4.67	.198884	41
20	.728227	3.33	.926831	1.33	.801396	4.65	.198604	40
21	9.728427	3.32	9.926751	1.33	9.801675	4.67	10.198325	39
22	.728626	3.32	.926671	1.33	.801955	4.65	.198045	38
23	.728825	3.32	.926591	1.33	.802234	4.65	.197766	37
24	.729024	3.32	.926511	1.33	.802513	4.65	.197487	36
25	.729223	3.32	.926431	1.33	.802792	4.67	.197208	35
26	.729422	3.32	.926351	1.35	.803072	4.65	.196928	34
27	.729621	3.32	.926270	1.33	.803351	4.65	.196649	33
28	.729820	3.30	.926190	1.33	.803630	4.65	.196370	32
29	.730018	3.32	.926110	1.35	.803909	4.63	.196091	31
30	.730217	3.30	.926029	1.33	.804187	4.65	.195813	30
31	9.730415	3.30	9.925949	1.35	9.804466	4.65	10.195534	29
32	.730613	3.30	.925868	1.33	.804745	4.63	.195255	28
33	.730811	3.30	.925788	1.35	.805023	4.65	.194977	27
34	.731009	3.28	.925707	1.35	.805302	4.63	.194698	26
35	.731206	3.30	.925626	1.35	.805580	4.65	.194420	25
36	.731404	3.30	.925545	1.33	.805859	4.63	.194141	24
37	.731602	3.28	.925465	1.35	.806137	4.63	.193863	23
38	.731799	3.28	.925384	1.35	.806415	4.63	.193585	22
39	.731996	3.28	.925303	1.35	.806693	4.63	.193307	21
40	.732193	3.28	.925222	1.35	.806971	4.63	.193029	20
41	9.732390	3.28	9.925141	1.35	9.807249	4.63	10.192751	19
42	.732587	3.28	.925060	1.35	.807527	4.63	.192473	18
43	.732784	3.27	.924979	1.37	.807805	4.63	.192195	17
44	.732980	3.28	.924897	1.35	.808083	4.63	.191917	16
45	.733177	3.27	.924816	1.35	.808361	4.62	.191639	15
46	.733373	3.27	.924735	1.35	.808638	4.63	.191362	14
47	.733569	3.27	.924654	1.37	.808916	4.62	.191084	13
48	.733765	3.27	.924572	1.35	.809193	4.63	.190807	12
49	.733961	3.27	.924491	1.37	.809471	4.62	.190529	11
50	.734157	3.27	.924409	1.35	.809748	4.62	.190252	10
51	9.734353	3.27	9.924328	1.37	9.810025	4.62	10.189975	9
52	.734549	3.25	.924246	1.37	.810302	4.63	.189698	8
53	.734744	3.25	.924164	1.35	.810580	4.62	.189420	7
54	.734939	3.27	.924083	1.37	.810857	4.62	.189143	6
55	.735135	3.25	.924001	1.37	.811134	4.60	.188866	5
56	.735330	3.25	.923919	1.37	.811410	4.62	.188590	4
57	.735525	3.23	.923837	1.37	.811687	4.62	.188313	3
58	.735719	3.25	.923755	1.37	.811964	4.62	.188036	2
59	.735914	3.25	.923673	1.37	.812241	4.60	.187759	1
60	9.736109		9.923591		9.812517		10.187483	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.
0	9.779463	2.80	9.902349	1.60	9.877114	4.38	10.122886
1	.779631	2.78	.902253	1.58	.877377	4.38	.122623
2	.779798	2.80	.902158	1.58	.877640	4.38	.122360
3	.779966	2.78	.902063	1.60	.877903	4.37	.122097
4	.780133	2.78	.901967	1.58	.878165	4.38	.121835
5	.780300	2.78	.901872	1.60	.878428	4.38	.121572
6	.780467	2.78	.901776	1.58	.878691	4.37	.121309
7	.780634	2.78	.901681	1.60	.878953	4.38	.121047
8	.780801	2.78	.901585	1.58	.879216	4.37	.120784
9	.780968	2.77	.901490	1.60	.879478	4.38	.120522
10	.781134	2.78	.901394	1.60	.879741	4.37	.120259
11	9.781301	2.78	9.901298	1.60	9.880003	4.37	10.119997
12	.781468	2.77	.901202	1.60	.880265	4.38	.119735
13	.781634	2.77	.901106	1.60	.880528	4.37	.119472
14	.781800	2.77	.901010	1.60	.880790	4.37	.119210
15	.781966	2.77	.900914	1.60	.881052	4.37	.118948
16	.782132	2.77	.900818	1.60	.881314	4.38	.118686
17	.782298	2.77	.900722	1.60	.881577	4.37	.118423
18	.782464	2.77	.900626	1.62	.881839	4.37	.118161
19	.782630	2.77	.900529	1.60	.882101	4.37	.117899
20	.782796	2.75	.900433	1.60	.882363	4.37	.117637
21	9.782961	2.77	9.900337	1.62	9.882625	4.37	10.117375
22	.783127	2.75	.900240	1.60	.882887	4.35	.117113
23	.783292	2.77	.900144	1.62	.883148	4.37	.116852
24	.783458	2.75	.900047	1.60	.883410	4.37	.116590
25	.783623	2.75	.899951	1.62	.883672	4.37	.116328
26	.783788	2.75	.899854	1.62	.883934	4.37	.116066
27	.783953	2.75	.899757	1.62	.884196	4.35	.115804
28	.784118	2.73	.899660	1.60	.884457	4.37	.115543
29	.784282	2.75	.899564	1.62	.884719	4.35	.115281
30	.784447	2.75	.899467	1.62	.884980	4.37	.115020
31	9.784612	2.73	9.899370	1.62	9.885242	4.37	10.114758
32	.784776	2.75	.899273	1.62	.885504	4.35	.114496
33	.784941	2.73	.899176	1.63	.885765	4.35	.114235
34	.785105	2.73	.899078	1.62	.886026	4.37	.113974
35	.785269	2.73	.898981	1.62	.886288	4.35	.113712
36	.785433	2.73	.898884	1.62	.886549	4.37	.113451
37	.785597	2.73	.898787	1.63	.886811	4.35	.113189
38	.785761	2.73	.898689	1.62	.887072	4.35	.112928
39	.785925	2.73	.898592	1.63	.887333	4.35	.112667
40	.786089	2.72	.898494	1.62	.887594	4.35	.112406
41	9.786252	2.73	9.898397	1.63	9.887855	4.35	10.112145
42	.786416	2.72	.898299	1.62	.888116	4.37	.111884
43	.786579	2.72	.898202	1.63	.888378	4.35	.111622
44	.786742	2.73	.898104	1.63	.888639	4.35	.111361
45	.786906	2.72	.898006	1.63	.888900	4.35	.111100
46	.787069	2.72	.897908	1.63	.889161	4.33	.110839
47	.787232	2.72	.897810	1.63	.889421	4.35	.110579
48	.787395	2.70	.897712	1.63	.889682	4.35	.110318
49	.787557	2.72	.897614	1.63	.889943	4.35	.110057
50	.787720	2.72	.897516	1.63	.890204	4.35	.109796
51	9.787883	2.70	9.897418	1.63	9.890465	4.33	10.109535
52	.788045	2.72	.897320	1.63	.890725	4.35	.109275
53	.788208	2.70	.897222	1.65	.890986	4.35	.109014
4	.788370	2.70	.897123	1.63	.891247	4.33	.108753
	.788532	2.70	.897025	1.65	.891507	4.35	.108493
	.788694	2.70	.896926	1.63	.891768	4.33	.108232
	.788856	2.70	.896828	1.65	.892028	4.35	.107972
	.789018	2.70	.896729	1.63	.892289	4.33	.107711
	.789180	2.70	.896631	1.65	.892549	4.35	.107451
	9.789342		9.896532		9.892810		10.10719
	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.808067	2.52	9.884254	1.77	9.923814	4.27	10.076186	60
1	.808218	2.50	.884148	1.77	.924070	4.28	.075930	59
2	.808368	2.52	.884042	1.77	.924327	4.27	.075673	58
3	.808519	2.50	.883936	1.78	.924583	4.28	.075417	57
4	.808669	2.50	.883829	1.77	.924840	4.27	.075160	56
5	.808819	2.50	.883723	1.77	.925096	4.27	.074904	55
6	.808969	2.50	.883617	1.78	.925352	4.28	.074648	54
7	.809119	2.50	.883510	1.77	.925609	4.27	.074391	53
8	.809269	2.50	.883404	1.78	.925865	4.28	.074135	52
9	.809419	2.50	.883297	1.77	.926122	4.27	.073878	51
10	.809569	2.48	.883191	1.78	.926378	4.27	.073622	50
11	9.809718	2.50	9.883084	1.78	9.926634	4.27	10.073366	49
12	.809868	2.48	.882977	1.77	.926890	4.28	.073110	48
13	.810017	2.50	.882871	1.78	.927147	4.27	.072853	47
14	.810167	2.48	.882764	1.78	.927403	4.27	.072597	46
15	.810316	2.48	.882657	1.78	.927659	4.27	.072341	45
16	.810465	2.48	.882550	1.78	.927915	4.27	.072085	44
17	.810614	2.48	.882443	1.78	.928171	4.27	.071829	43
18	.810763	2.48	.882336	1.78	.928427	4.28	.071573	42
19	.810912	2.48	.882229	1.80	.928684	4.27	.071316	41
20	.811061	2.48	.882121	1.78	.928940	4.27	.071060	40
21	9.811210	2.47	9.882014	1.78	9.929196	4.27	10.070804	39
22	.811358	2.48	.881907	1.80	.929452	4.27	.070548	38
23	.811507	2.47	.881799	1.78	.929708	4.27	.070292	37
24	.811655	2.48	.881692	1.80	.929964	4.27	.070036	36
25	.811804	2.47	.881584	1.78	.930220	4.25	.069780	35
26	.811952	2.47	.881477	1.80	.930475	4.27	.069525	34
27	.812100	2.47	.881369	1.80	.930731	4.27	.069269	33
28	.812248	2.47	.881261	1.80	.930987	4.27	.069013	32
29	.812396	2.47	.881153	1.78	.931243	4.27	.068757	31
30	.812544	2.47	.881046	1.80	.931499	4.27	.068501	30
31	9.812692	2.47	9.880938	1.80	9.931755	4.25	10.068245	29
32	.812840	2.47	.880830	1.80	.932010	4.27	.067990	28
33	.812988	2.45	.880722	1.82	.932266	4.27	.067734	27
34	.813135	2.47	.880613	1.80	.932522	4.27	.067478	26
35	.813283	2.45	.880505	1.80	.932778	4.25	.067222	25
36	.813430	2.47	.880397	1.80	.933033	4.27	.066967	24
37	.813578	2.45	.880289	1.82	.933289	4.27	.066711	23
38	.813725	2.45	.880180	1.80	.933545	4.25	.066455	22
39	.813872	2.45	.880072	1.82	.933800	4.27	.066200	21
40	.814019	2.45	.879963	1.80	.934056	4.25	.065944	20
41	9.814166	2.45	9.879855	1.82	9.934311	4.27	10.065689	19
42	.814313	2.45	.879746	1.82	.934567	4.25	.065433	18
43	.814460	2.45	.879637	1.80	.934822	4.27	.065178	17
44	.814607	2.43	.879529	1.82	.935078	4.25	.064922	16
45	.814753	2.45	.879420	1.82	.935333	4.27	.064667	15
46	.814900	2.43	.879311	1.82	.935589	4.25	.064411	14
47	.815046	2.45	.879202	1.82	.935844	4.27	.064156	13
48	.815193	2.43	.879093	1.82	.936100	4.25	.063900	12
49	.815339	2.43	.878984	1.82	.936355	4.27	.063645	11
50	.815485	2.45	.878875	1.82	.936611	4.25	.063389	10
51	9.815632	2.43	9.878766	1.83	9.936866	4.25	10.063134	9
52	.815778	2.43	.878656	1.82	.937121	4.27	.062879	8
53	.815924	2.42	.878547	1.82	.937377	4.25	.062623	7
54	.816069	2.43	.878438	1.83	.937632	4.25	.062368	6
55	.816215	2.43	.878328	1.82	.937887	4.25	.062113	5
56	.816361	2.43	.878219	1.83	.938142	4.27	.061858	4
57	.816507	2.42	.878109	1.83	.938398	4.25	.061602	3
58	.816652	2.43	.877999	1.82	.938653	4.25	.061347	2
59	.816798	2.42	.877890	1.83	.938908	4.25	.061092	1
60	9.816943		9.877780		9.939163		10.060837	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

'	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	'
0	9.816943	2.42	9.877780	1.83	9.939163	4.25	10.060837	60
1	.817088	2.42	.877670	1.83	.939418	4.25	.060582	59
2	.817233	2.43	.877560	1.83	.939673	4.25	.060327	58
3	.817379	2.42	.877450	1.83	.939928	4.25	.060072	57
4	.817524	2.40	.877340	1.83	.940183	4.27	.059817	56
5	.817668	2.42	.877230	1.83	.940439	4.25	.059561	55
6	.817813	2.42	.877120	1.83	.940694	4.25	.059306	54
7	.817958	2.42	.877010	1.85	.940949	4.25	.059051	53
8	.818103	2.40	.876899	1.83	.941204	4.25	.058796	52
9	.818247	2.42	.876789	1.85	.941459	4.23	.058541	51
10	.818392	2.40	.876678	1.83	.941713	4.25	.058287	50
11	9.818536	2.42	9.876568	1.85	9.941968	4.25	10.058032	49
12	.818681	2.40	.876457	1.83	.942223	4.25	.057777	48
13	.818825	2.40	.876347	1.85	.942478	4.25	.057522	47
14	.818969	2.40	.876236	1.85	.942733	4.25	.057267	46
15	.819113	2.40	.876125	1.85	.942988	4.25	.057012	45
16	.819257	2.40	.876014	1.83	.943243	4.25	.056757	44
17	.819401	2.40	.875904	1.85	.943498	4.23	.056502	43
18	.819545	2.40	.875793	1.85	.943752	4.25	.056248	42
19	.819689	2.38	.875682	1.85	.944007	4.25	.055993	41
20	.819832	2.40	.875571	1.87	.944262	4.25	.055738	40
21	9.819976	2.40	9.875459	1.85	9.944517	4.23	10.055483	39
22	.820120	2.38	.875348	1.85	.944771	4.25	.055229	38
23	.820263	2.38	.875237	1.85	.945026	4.25	.054974	37
24	.820406	2.40	.875126	1.87	.945281	4.23	.054719	36
25	.820550	2.38	.875014	1.85	.945535	4.25	.054465	35
26	.820693	2.38	.874903	1.87	.945790	4.25	.054210	34
27	.820836	2.38	.874791	1.85	.946045	4.23	.053955	33
28	.820979	2.38	.874680	1.87	.946299	4.25	.053701	32
29	.821122	2.38	.874568	1.87	.946554	4.23	.053446	31
30	.821265	2.37	.874456	1.87	.946808	4.25	.053192	30
31	9.821407	2.38	9.874344	1.87	9.947063	4.25	10.052937	29
32	.821550	2.38	.874232	1.85	.947318	4.23	.052682	28
33	.821693	2.37	.874121	1.87	.947572	4.25	.052428	27
34	.821835	2.37	.874009	1.88	.947827	4.23	.052173	26
35	.821977	2.38	.873896	1.87	.948081	4.23	.051919	25
36	.822120	2.37	.873784	1.87	.948335	4.25	.051665	24
37	.822262	2.37	.873672	1.87	.948590	4.23	.051410	23
38	.822404	2.37	.873560	1.87	.948844	4.25	.051156	22
39	.822546	2.37	.873448	1.88	.949099	4.23	.050901	21
40	.822688	2.37	.873335	1.87	.949353	4.25	.050647	20
41	9.822830	2.37	9.873223	1.88	9.949608	4.23	10.050392	19
42	.822972	2.37	.873110	1.87	.949862	4.23	.050138	18
43	.823114	2.35	.872998	1.88	.950116	4.25	.049884	17
44	.823255	2.37	.872885	1.88	.950371	4.23	.049629	16
45	.823397	2.37	.872772	1.88	.950625	4.23	.049375	15
46	.823539	2.35	.872659	1.87	.950879	4.23	.049121	14
47	.823680	2.35	.872547	1.88	.951133	4.25	.048867	13
48	.823821	2.37	.872434	1.88	.951388	4.23	.048612	12
49	.823963	2.35	.872321	1.88	.951642	4.23	.048358	11
50	.824104	2.35	.872208	1.88	.951896	4.23	.048104	10
51	9.824245	2.35	9.872095	1.90	9.952150	4.25	10.047850	9
52	.824386	2.35	.871981	1.88	.952405	4.23	.047595	8
53	.824527	2.35	.871868	1.88	.952659	4.23	.047341	7
54	.824668	2.33	.871755	1.90	.952913	4.23	.047087	6
55	.824808	2.35	.871641	1.88	.953167	4.23	.046833	5
56	.824949	2.35	.871528	1.90	.953421	4.23	.046579	4
57	.825090	2.33	.871414	1.88	.953675	4.23	.046325	3
58	.825230	2.35	.871301	1.90	.953929	4.23	.046071	2
59	.825371	2.33	.871187	1.90	.954183	4.23	.045817	1
60	9.825511		9.871073		9.954437		10.045563	0
'	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	'

Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	'
9.825511	2.33	9.871073	1.88	9.954437	4.23	10.045563	60
.825651	2.33	.870960	1.90	.954691	4.25	.045309	59
.825791	2.33	.870846	1.90	.954946	4.23	.045054	58
.825931	2.33	.870732	1.90	.955200	4.23	.044800	57
.826071	2.33	.870618	1.90	.955454	4.23	.044546	56
.826211	2.33	.870504	1.90	.955708	4.22	.044292	55
.826351	2.33	.870390	1.90	.955961	4.23	.044039	54
.826491	2.33	.870276	1.92	.956215	4.23	.043785	53
.826631	2.32	.870161	1.90	.956469	4.23	.043531	52
.826770	2.33	.870047	1.90	.956723	4.23	.043277	51
.826910	2.32	.869933	1.92	.956977	4.23	.043023	50
9.827049	2.33	9.869818	1.90	9.957231	4.23	10.042769	49
.827189	2.32	.869704	1.92	.957485	4.23	.042515	48
.827328	2.32	.869589	1.92	.957739	4.23	.042261	47
.827467	2.32	.869474	1.90	.957993	4.23	.042007	46
.827606	2.32	.869360	1.92	.958247	4.22	.041753	45
.827745	2.32	.869245	1.92	.958500	4.23	.041500	44
.827884	2.32	.869130	1.92	.958754	4.23	.041246	43
.828023	2.32	.869015	1.92	.959008	4.23	.040992	42
.828162	2.32	.868900	1.92	.959262	4.23	.0407 8	41
.828301	2.30	.868785	1.92	.959516	4.22	.040484	40
9.828439	2.32	9.868670	1.92	9.959769	4.23	10.040231	39
.828578	2.30	.868555	1.92	.960023	4.23	.039977	38
.828716	2.32	.868440	1.93	.960277	4.22	.039723	37
.828855	2.30	.868324	1.92	.960530	4.23	.039470	36
.828993	2.30	.868209	1.93	.960784	4.23	.039216	35
.829131	2.30	.868093	1.92	.961038	4.23	.038962	34
.829269	2.30	.867978	1.93	.961292	4.22	.038708	33
.829407	2.30	.867862	1.92	.961545	4.23	.038455	32
.829545	2.30	.867747	1.93	.961799	4.22	.038201	31
.829683	2.30	.867631	1.93	.962052	4.23	.037948	30
9.829821	2.30	9.867515	1.93	9.962306	4.23	10.037694	29
.829959	2.30	.867399	1.93	.962560	4.22	.037440	28
.830097	2.28	.867283	1.93	.962813	4.23	.037187	27
.830234	2.30	.867167	1.93	.963067	4.22	.036933	26
.830372	2.28	.867051	1.93	.963320	4.23	.036680	25
.830509	2.28	.866935	1.93	.963574	4.23	.036426	24
.830646	2.30	.866819	1.93	.963828	4.22	.036172	23
.830784	2.28	.866703	1.95	.964081	4.23	.035919	22
.830921	2.28	.866586	1.93	.964335	4.22	.035665	21
.831058	2.28	.866470	1.95	.964588	4.23	.035412	20
9.831195	2.28	9.866353	1.93	9.964842	4.22	10.035158	19
.831332	2.28	.866237	1.95	.965095	4.23	.034905	18
.831469	2.28	.866120	1.93	.965349	4.22	.034651	17
.831606	2.27	.866004	1.95	.965602	4.22	.034398	16
.831742	2.28	.865887	1.95	.965855	4.23	.034145	15
.831879	2.27	.865770	1.95	.966109	4.22	.033891	14
.832015	2.28	.865653	1.95	.966362	4.23	.033638	13
.832152	2.27	.865536	1.95	.966616	4.22	.033384	12
.832288	2.28	.865419	1.95	.966869	4.23	.033131	11
.832425	2.27	.865302	1.95	.967123	4.22	.032877	10
9·832561	2.27	9.865185	1.95	9.967376	4.22	10.032624	9
.832697	2.27	.865068	1.97	.967629	4.23	.032371	8
.832833	2.27	.864950	1.95	.967883	4.22	.032117	7
.832969	2.27	.864833	1.95	.968136	4.22	.031864	6
.833105	2.27	.864716	1.97	.968389	4.23	.031611	5
.833241	2.27	.864598	1.95	.968643	4.22	.031357	4
.833377	2.25	.864481	1.97	.968896	4.22	.031104	3
.833512	2.27	.864363	1.97	.969149	4.23	.030851	2
.833648	2.25	.864245	1.97	.969403	4.22	.030597	1
9.833783		9.864127		9.969656		10.030344	0
Cosine.	*D. 1".*	Sine.	D. 1".	Cotang.	D. 1".	Tang.	'

′	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	′
0	9.833783	2.27	9.864127	1.95	9.969656	4.22	10.030344	60
1	.833919	2.25	.864010	1.97	.969909	4.22	.030091	59
2	.834054	2.25	.863892	1.97	.970162	4.23	.029838	58
3	.834189	2.27	.863774	1.97	.970416	4.22	.029584	57
4	.834325	2.25	.863656	1.97	.970669	4.22	.029331	56
5	.834460	2.25	.863538	1.98	.970922	4.22	.029078	55
6	.834595	2.25	.863419	1.97	.971175	4.23	.028825	54
7	.834730	2.25	.863301	1.97	.971429	4.22	.028571	53
8	.834865	2.23	.863183	1.98	.971682	4.22	.028318	52
9	.834999	2.25	.863064	1.97	.971935	4.22	.028065	51
10	.835134	2.25	.862946	1.98	.972188	4.22	.027812	50
11	9.835269	2.23	9.862827	1.97	9.972441	4.23	10.027559	49
12	.835403	2.25	.862709	1.98	.972695	4.22	.027305	48
13	.835538	2.23	.862590	1.98	.972948	4.22	.027052	47
14	.835672	2.25	.862471	1.97	.973201	4.22	.026799	46
15	.835807	2.23	.862353	1.98	.973454	4.22	.026546	45
16	.835941	2.23	.862234	1.98	.973707	4.22	.026293	44
17	.836075	2.23	.862115	1.98	.973960	4.22	.026040	43
18	.836209	2.23	.861996	1.98	.974213	4.22	.025787	42
19	.836343	2.23	.861877	1.98	.974466	4.23	.025534	41
20	.836477	2.23	.861758	2.00	.974720	4.22	.025280	40
21	9.836611	2.23	9.861638	1.98	9.974973	4.22	10.025027	39
22	.836745	2.22	.861519	1.98	.975226	4.22	.024774	38
23	.836878	2.23	.861400	2.00	.975479	4.22	.024521	37
24	.837012	2.23	.861280	1.98	.975732	4.22	.024268	36
25	.837146	2.22	.861161	2.00	.975985	4.22	.024015	35
26	.837279	2.22	.861041	1.98	.976238	4.22	.023762	34
27	.837412	2.23	.860922	2.00	.976491	4.22	.023509	33
28	.837546	2.22	.860802	2.00	.976744	4.22	.023256	32
29	.837679	2.22	.860682	2.00	.976997	4.22	.023003	31
30	.837812	2.22	.860562	2.00	.977250	4.22	.022750	30
31	9.837945	2.22	9.860442	2.00	9.977503	4.22	10.022497	29
32	.838078	2.22	.860322	2.00	.977756	4.22	.022244	28
33	.838211	2.22	.860202	2.00	.978009	4.22	.021991	27
34	.838344	2.22	.860082	2.00	.978262	4.22	.021738	26
35	.838477	2.22	.859962	2.00	.978515	4.22	.021485	25
36	.838610	2.20	.859842	2.02	.978768	4.22	.021232	24
37	.838742	2.22	.859721	2.00	.979021	4.22	.020979	23
38	.838875	2.20	.859601	2.02	.979274	4.22	.020726	22
39	.839007	2.22	.859480	2.00	.979527	4.22	.020473	21
40	.839140	2.20	.859360	2.02	.979780	4.22	.020220	20
41	9.839272	2.20	9.859239	2.00	9.980033	4.22	10.019967	19
42	.839404	2.20	.859119	2.02	.980286	4.20	.019714	18
43	.839536	2.20	.858998	2.02	.980538	4.22	.019462	17
44	.839668	2.20	.858877	2.02	.980791	4.22	.019209	16
45	.839800	2.20	.858756	2.02	.981044	4.22	.018956	15
46	.839932	2.20	.858635	2.02	.981297	4.22	.018703	14
47	.840064	2.20	.858514	2.02	.981550	4.22	.018450	13
48	.840196	2.20	.858393	2.02	.981803	4.22	.018197	12
49	.840328	2.18	.858272	2.02	.982056	4.22	.017944	11
50	.840459	2.20	.858151	2.03	.982309	4.22	.017691	10
51	9.840591	2.18	9.858029	2.02	9.982562	4.20	10.017438	9
52	.840722	2.20	.857908	2.03	.982814	4.22	.017186	8
53	.840854	2.18	.857786	2.02	.983067	4.22	.016933	7
54	.840985	2.18	.857665	2.03	.983320	4.22	.016680	6
55	.841116	2.18	.857543	2.02	.983573	4.22	.016427	5
56	.841247	2.18	.857422	2.03	.983826	4.22	.016174	4
57	.841378	2.18	.857300	2.03	.984079	4.22	.015921	3
58	.841509	2.18	.857178	2.03	.984332	4.20	.015668	2
59	.841640	2.18	.857056	2.03	.984584	4.22	.015416	1
60	9.841771		9.856934		9.984837		10.015163	0
′	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	′

44° **135°**

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.841771	2.18	9.856934	2.03	9.984837	4.22	10.015163	60
1	.841902	2.18	.856812	2.03	.985090	4.22	.014910	59
2	.842033	2.17	.856690	2.03	.985343	4.22	.014657	58
3	.842163	2.18	.856568	2.03	.985596	4.20	.014404	57
4	.842294	2.17	.856446	2.05	.985848	4.22	.014152	56
5	.842424	2.18	.856323	2.03	.986101	4.22	.013899	55
6	.842555	2.17	.856201	2.05	.986354	4.22	.013646	54
7	.842685	2.17	.856078	2.03	.986607	4.22	.013393	53
8	.842815	2.18	.855956	2.05	.986860	4.20	.013140	52
9	.842946	2.17	.855833	2.03	.987112	4.22	.012888	51
10	.843076	2.17	.855711	2.05	.987365	4.22	.012635	50
11	9.843206	2.17	9.855588	2.05	9.987618	4.22	10.012382	49
12	.843336	2.17	.855465	2.05	.987871	4.20	.012129	48
13	.843466	2.15	.855342	2.05	.988123	4.22	.011877	47
14	.843595	2.17	.855219	2.05	.988376	4.22	.011624	46
15	.843725	2.17	.855096	2.05	.988629	4.22	.011371	45
16	.843855	2.15	.854973	2.05	.988882	4.20	.011118	44
17	.843984	2.17	.854850	2.05	.989134	4.22	.010866	43
18	.844114	2.15	.854727	2.07	.989387	4.22	.010613	42
19	.844243	2.15	.854603	2.05	.989640	4.22	.010360	41
20	.844372	2.17	.854480	2.07	.989893	4.20	.010107	40
21	9.844502	2.15	9.854356	2.05	9.990145	4.22	10.009855	39
22	.844631	2.15	.854233	2.07	.990398	4.22	.009602	38
23	.844760	2.15	.854109	2.05	.990651	4.20	.009349	37
24	.844889	2.15	.853986	2.07	.990903	4.22	.009097	36
25	.845018	2.15	.853862	2.07	.991156	4.22	.008844	35
26	.845147	2.15	.853738	2.07	.991409	4.22	.008591	34
27	.845276	2.15	.853614	2.07	.991662	4.20	.008338	33
28	.845405	2.13	.853490	2.07	.991914	4.22	.008086	32
29	.845533	2.15	.853366	2.07	.992167	4.22	.007833	31
30	.845662	2.13	.853242	2.07	.992420	4.20	.007580	30
31	9.845790	2.15	9.853118	2.07	9.992672	4.22	10.007328	29
32	.845919	2.13	.852994	2.08	.992925	4.22	.007075	28
33	.846047	2.13	.852869	2.07	.993178	4.22	.006822	27
34	.846175	2.15	.852745	2.08	.993431	4.20	.006569	26
35	.846304	2.13	.852620	2.07	.993683	4.22	.006317	25
36	.846432	2.13	.852496	2.08	.993936	4.22	.006064	24
37	.846560	2.13	.852371	2.07	.994189	4.20	.005811	23
38	.846688	2.13	.852247	2.08	.994441	4.22	.005559	22
39	.846816	2.13	.852122	2.08	.994694	4.22	.005306	21
40	.846944	2.12	.851997	2.08	.994947	4.20	.005053	20
41	9.847071	2.13	9.851872	2.08	9.995199	4.22	10.004801	19
42	.847199	2.13	.851747	2.08	.995452	4.22	.004548	18
43	.847327	2.12	.851622	2.08	.995705	4.20	.004295	17
44	.847454	2.13	.851497	2.08	.995957	4.22	.004043	16
45	.847582	2.12	.851372	2.10	.996210	4.22	.003790	15
46	.847709	2.12	.851246	2.08	.996463	4.20	.003537	14
47	.847836	2.13	.851121	2.08	.996715	4.22	.003285	13
48	.847964	2.12	.850996	2.10	.996968	4.22	.003032	12
49	.848091	2.12	.850870	2.08	.997221	4.20	.002779	11
50	.848218	2.12	.850745	2.10	.997473	4.22	.002527	10
51	9.848345	2.12	9.850619	2.10	9.997726	4.22	10.002274	9
52	.848472	2.12	.850493	2.08	.997979	4.20	.002021	8
53	.848599	2.12	.850368	2.10	.998231	4.22	.001769	7
54	.848726	2.10	.850242	2.10	.998484	4.22	.001516	6
55	.848852	2.12	.850116	2.10	.998737	4.20	.001263	5
56	.848979	2.12	.849990	2.10	.998989	4.22	.001011	4
57	.849106	2.10	.849864	2.10	.999242	4.22	.000758	3
58	.849232	2.12	.849738	2.12	.999495	4.20	.000505	2
59	.849359	2.10	.849611	2.10	.999747	4.22	.000253	1
60	9.849485		9.849485		10.000000		10.000000	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	

34°

TABLE XXVI.—LOGARITHMIC VERSED SINES.

TABLE XXVI.—LOGARITHMIC VERSED SINES

0°						1°					
″	′	Vers.	q − 2l		Ex. sec.	″	′	Vers.	q − 2l		Ex. sec.
			9.070						9.070		
0	0	Inf. neg.	120	120	Inf. neg.	3600	0	6.182714	109	175	6.182780
60	1	2.626422	120	120	2.626422	3660	1	.197071	108	177	.197139
120	2	3.228482	120	120	3.228482	3720	2	.211194	108	179	.211264
180	3	.580665	120	120	.580665	3780	3	.225091	108	181	.225163
240	4	3.830542	120	120	3.830542	3840	4	.238770	107	182	.238845
300	5	4.024362	120	120	4.024363	3900	5	.252236	107	184	.252314
360	6	.182725	120	120	.182725	3960	6	.265497	106	186	.265577
420	7	.316618	120	120	.316619	4020	7	.278558	106	188	.278641
480	8	.432602	120	121	.432603	4080	8	.291426	106	191	.291511
540	9	.534907	119	121	.534908	4140	9	.304106	105	193	.304193
600	10	.626422	119	121	.626424	4200	10	.316603	105	195	.316693
660	11	4.709207	119	122	4.709209	4260	11	6.328923	104	197	6.329016
720	12	.784784	119	122	.784787	4320	12	.341071	104	199	.341167
780	13	.854308	119	122	.854312	4380	13	.353052	103	201	.353150
840	14	.918678	119	123	.918681	4440	14	.364869	103	204	.364970
900	15	4.978604	119	123	4.978608	4500	15	.376528	103	206	.376631
960	16	5.034661	119	124	5.034666	4560	16	.388032	102	208	.388138
1020	17	.087319	119	124	.087325	4620	17	.399386	102	211	.399494
1080	18	.136966	119	125	.136972	4680	18	.410593	101	213	.410705
1140	19	.183928	119	125	.183935	4740	19	.421657	101	215	.421772
1200	20	.228481	119	126	.228488	4800	20	.432583	100	218	.432700
1260	21	5.270859	118	126	5.270868	4860	21	6.443372	100	220	6.443493
1320	22	.311266	118	127	.311275	4920	22	.454029	099	223	.454153
1380	23	.349876	118	128	.349886	4980	23	.464557	099	225	.464684
1440	24	.386843	118	129	.386854	5040	24	.474959	098	228	.475089
1500	25	.422300	118	129	.422312	5100	25	.485238	098	230	.485371
1560	26	.456367	118	130	.456379	5160	26	.495396	097	233	.495532
1620	27	.489148	118	131	.489161	5220	27	.505438	097	236	.505577
1680	28	.520736	117	132	.520750	5280	28	.515364	096	238	.515506
1740	29	.551216	117	133	.551231	5340	29	.525178	095	241	.525324
1800	30	.580662	117	134	.580679	5400	30	.534882	095	244	.535031
1860	31	5.609143	117	134	5.609160	5460	31	6.544480	094	247	6.544632
1920	32	.636719	117	135	.636738	5520	32	.553972	094	249	.554128
1980	33	.663447	116	136	.663467	5580	33	.563362	093	252	.563521
2040	34	.689377	116	137	.689398	5640	34	.572651	093	255	.572813
2100	35	.714555	116	138	.714577	5700	35	.581842	092	258	.582008
2160	36	.739023	116	140	.739047	5760	36	.590936	092	261	.591106
2220	37	.762821	116	141	.762847	5820	37	.599937	091	264	.600110
2280	38	.785985	115	142	.786012	5880	38	.608845	090	267	.609021
2340	39	.808547	115	143	.808575	5940	39	.617663	090	270	.617843
2400	40	.830537	115	144	.830567	6000	40	.626392	089	273	.626575
2460	41	5.851985	115	145	5.852016	6060	41	6.635034	089	276	6.635221
2520	42	.872915	114	147	.872948	6120	42	.643591	088	279	.643782
2580	43	.893353	114	148	.893387	6180	43	.652064	087	282	.652259
2640	44	.913322	114	149	.913357	6240	44	.660456	087	285	.660655
2700	45	.932841	114	151	.932878	6300	45	.668767	086	289	.668970
2760	46	.951931	113	152	.951970	6360	46	.677000	085	292	.677206
2820	47	.970611	113	154	.970652	6420	47	.685155	085	295	.685365
2880	48	5.988898	113	155	5.988940	6480	48	.693234	084	298	.693448
2940	49	6.006807	112	157	6.006851	6540	49	.701239	083	302	.701457
3000	50	.024355	112	158	.024401	6600	50	.709171	083	305	.709393
3060	51	6.041555	112	160	6.041602	6660	51	6.717080	082	308	6.717257
3120	52	.058421	111	161	.058470	6720	52	.724820	081	312	.725050
3180	53	.074965	111	163	.075017	6780	53	.732540	081	315	.732775
3240	54	.091201	111	164	.091254	6840	54	.740192	080	319	.740431
3300	55	.107138	110	166	.107194	6900	55	.747777	079	322	.748020
3360	56	.122789	110	168	.122846	6960	56	.755297	079	326	.755544
3420	57	.138162	110	169	.138222	7020	57	.762752	078	329	.763004
3480	58	.153268	109	171	.153330	7080	58	.770144	077	333	.770400
3540	59	.168116	109	173	.168180	7140	59	.777473	076	337	.777733
3600	60	6.182714	109	175	6.182780	7200	60	6.784741	076	340	6.785005
″	′					″	′				

2°						3°					
″	′	Vers.	$q - 2l$		Ex. sec.	″	′	Vers.	$q - 2l$		Ex. sec.
			9.070						9.070*		
7200	0	6.784741	076	340	6.785005	10800	0	7.136868	021	616	7.137464
7260	1	.791948	075	344	.792217	10860	1	.141679	019	622	.142281
7320	2	.799096	074	348	.799370	10920	2	.146464	018	627	.147072
7380	3	.806186	073	351	.806464	10980	3	.151222	017	633	.151837
7440	4	.813219	073	355	.813501	11040	4	.155954	016	638	.156577
7500	5	.820194	072	359	.820482	11100	5	.160661	015	644	.161290
7560	6	.827115	071	363	.827406	11160	6	.165342	014	650	.165978
7620	7	.833980	070	367	.834277	11220	7	.169998	013	655	.170641
7680	8	.840792	070	371	.841093	11280	8	.174630	011	661	.175279
7740	9	.847551	069	375	.847857	11340	9	.179236	010	667	.179893
7800	10	.854257	068	379	.854568	11400	10	.183819	009	673	.184483
7860	11	6.860912	067	383	6.861228	11460	11	7.188377	008	679	7.189048
7920	12	.867517	066	387	.867837	11520	12	.192912	007	685	.193589
7980	13	.874071	066	391	.874396	11580	13	.197423	006	690	.198108
8040	14	.880577	065	395	.880907	11640	14	.201910	004	696	.202602
8100	15	.887034	064	399	.887369	11700	15	.206375	003	702	.207074
8160	16	.893443	063	403	.893783	11760	16	.210817	002	708	.211523
8220	17	.899806	062	407	.900151	11820	17	.215236	001	714	.215949
8280	18	.906122	061	411	.906472	11880	18	219633	◆00	720	.220353
8340	19	.912393	061	416	.912748	11940	19	.224007	998	727	.224735
8400	20	.918618	060	420	.918979	12000	20	.228360	997	733	.229095
8460	21	6.924800	059	424	6.925165	12060	21	7.232691	996	739	7.233433
8520	22	.930937	058	429	.931308	12120	22	.237000	995	745	.237750
8580	33	.937032	057	433	.937408	12180	23	.241288	994	751	.242046
8640	24	.943084	056	437	.943465	12240	24	.245555	992	757	.246320
8700	25	.949094	055	442	.949480	12300	25	.249801	991	764	.250574
8760	26	.955063	054	446	.955455	12360	26	.254027	990	770	.254807
8820	27	.960991	054	451	.961388	12420	27	.258232	989	776	.259019
8880	28	.966879	053	455	.967281	12480	28	.262416	987	783	.263212
8940	29	.972727	052	460	.973135	12540	29	.266581	986	789	.267384
9000	30	.978536	051	464	.978949	12600	30	.270726	985	795	.271537
9060	31	6.984306	050	469	6.984725	12660	31	7.274851	983	802	7.275669
9120	32	.990039	049	474	.990463	12720	32	.278956	982	808	.279783
9180	33	6.995733	048	478	6.996164	13780	33	.283043	981	815	.283877
9240	34	7.001391	047	483	7.001827	12840	34	.287110	980	821	.287952
9300	35	.007012	046	488	.007454	12900	35	.291158	978	828	.292007
9360	36	.012597	045	493	.013044	12960	36	.295187	977	835	.296045
9420	37	.018146	044	497	.018599	13020	37	.299197	976	841	.300063
9480	38	.023660	043	502	.024119	13080	38	.303190	974	848	.304063
9540	39	.029139	042	507	.029604	13140	39	.307164	973	855	.308045
9600	40	.034584	041	512	.035054	13200	40	.311119	972	861	.312009
9660	41	7.039995	040	517	7.040471	13260	41	7.315057	970	868	7.315955
9720	42	.045372	039	522	.045854	13320	42	.318977	969	875	.319883
9780	43	.050716	038	527	.051204	13380	43	.322880	967	882	.323794
9840	44	.056028	037	532	.056522	13440	44	.326765	966	889	.327687
9900	45	.061307	036	537	.061807	13500	45	.330632	965	896	.331563
9960	46	.066554	035	542	.067061	13560	46	.334483	963	902	.335422
10020	47	.071770	034	547	.072282	13620	47	.338316	962	908	.339263
10080	48	.076954	033	552	.077473	13680	48	.342133	961	916	.343089
10140	49	.082108	032	557	.082633	13740	49	.345933	959	923	.346897
10200	50	.087232	031	562	.087763	13800	50	.349716	958	930	.350689
10260	51	7.092325	030	568	7.092862	13860	51	7.353483	956	938	7.354464
10320	52	.097389	029	573	.097932	13920	52	.357233	955	945	.358223
10380	53	.102423	028	578	.102973	13980	53	.360968	953	952	.361966
10440	54	.107428	027	584	.107985	14040	54	.364686	952	959	.365693
10500	55	.112405	026	589	.112968	14100	55	.368389	951	966	.369404
10560	56	.117353	025	594	.117922	14160	56	.372076	949	973	.3731
10620	57	.122273	024	600	.122849	14220	57	.375747	948	981	.378
10680	58	.127165	023	605	.127748	14280	58	.379403	946	988	.38
10740	59	.132030	022	611	.132619	14340	59	.383043	945	995	.3
10800	60	7.136868	021	616	7.137464	14400	60	7.386668	943	◆03	7
″	′					″	′		9.069*	9.071*	

TABLE XXVI.—LOGARITHMIC VERSED SINES.

TABLE XXVI.—LOGARITHMIC VERSED SINES

8°					9°			
′	Vers.	D. 1".	Ex. sec.	D. 1".	′	Vers.	D. 1".	Ex. sec.
0	7.988199	30.08	7.992446	30.38	0	8.090317	26.72	8.095697
1	.990004	30.02	.994269	30.32	1	.091920	26.68	.097320
2	.991805	29.95	.996088	30.25	2	.093521	26.63	.098941
3	.993602	29.88	.997903	30.18	3	.095119	26.58	.100559
4	.995395	29.83	7.999714	30.13	4	.096714	26.52	.102174
5	.997185	29.77	8.001522	30.07	5	.098305	26.48	.103786
6	7.998971	29.72	.003326	30.00	6	.099894	26.43	.105395
7	8.000754	29.63	.005126	29.95	7	.101480	26.40	.107001
8	.002532	29.60	.006923	29.88	8	.103064	26.33	.108605
9	.004308	29.52	.008716	29.83	9	.104644	26.28	.110205
10	.006079	29.47	.010506	29.73	10	.106221	26.25	.111803
11	8.007847	29.40	8.012292	29.70	11	8.107796	26.18	8.113398
12	.009611	29.35	.014074	29.65	12	.109367	26.15	.114990
13	.011372	29.28	.015853	29.58	13	.110936	26.10	.116579
14	.013129	29.22	.017628	29.53	14	.112502	26.05	.118166
15	.014882	29.17	.019400	29.47	15	.114065	26.00	.119749
16	.016632	29.10	.021168	29.42	16	.115625	25.95	.121330
17	.018378	29.05	.022933	29.35	17	.117182	25.92	.122908
18	.020121	29.00	.024694	29.30	18	.118737	25.87	.124483
19	.021861	28.93	.026452	29.23	19	.120289	25.82	.126056
20	.023597	28.87	.028206	29.18	20	.121838	25.77	.127626
21	8.025329	28.82	8.029957	29.13	21	8.123384	25.72	8.129193
22	.027058	28.75	.031705	29.07	22	.124927	25.68	.130757
23	.028783	28.70	.033449	29.00	23	.126468	25.65	.132318
24	.030505	28.65	.035189	28.97	24	.128006	25.58	.133877
25	.032224	28.58	.036927	28.90	25	.129541	25.55	.135433
26	.033939	28.53	.038661	28.83	26	.131074	25.50	.136987
27	.035651	28.47	.040391	28.78	27	.132604	25.45	.138538
28	.037359	28.42	.042118	28.73	28	.134131	25.40	.140086
29	.039064	28.37	.043842	28.68	29	.135655	25.37	.141631
30	.040766	28.30	.045563	28.62	30	.137177	25.32	.143174
31	8.042464	28.25	8.047280	28.57	31	8.138696	25.27	8.144714
32	.044159	28.20	.048994	28.50	32	.140212	25.23	.146252
33	.045851	28.13	.050704	28.47	33	.141726	25.18	.147787
34	.047539	28.08	.052412	28.40	34	.143237	25.13	.149319
35	.049224	28.03	.054116	28.35	35	.144745	25.10	.150849
36	.050906	27.98	.055817	28.28	36	.146251	25.05	.152376
37	.052585	27.92	.057514	28.25	37	.147754	25.02	.153900
38	.054260	27.87	.059209	28.18	38	.149255	24.95	.155422
39	.055932	27.82	.060900	28.13	39	.150752	24.93	.156942
40	.057601	27.75	.062588	28.08	40	.152248	24.88	.158458
41	8.059266	27.72	8.064273	28.03	41	8.153741	24.83	8.159973
42	.060929	27.65	.065955	27.97	42	.155231	24.78	.161484
43	.062588	27.60	.067633	27.93	43	.156718	24.75	.162994
44	.064244	27.55	.069309	27.87	44	.158203	24.72	.164500
45	.065897	27.48	.070981	27.82	45	.159686	24.67	.166004
46	.067546	27.45	.072650	27.77	46	.161166	24.62	.167506
47	.069193	27.38	.074316	27.72	47	.162643	24.58	.169005
48	.070836	27.33	.075979	27.67	48	.164118	24.53	.170502
49	.072476	27.30	.077639	27.60	49	.165590	24.50	.171996
50	.074114	27.23	.079295	27.57	50	.167060	24.45	.173488
51	8.075748	27.18	8.080949	27.52	51	8.168527	24.42	8.174977
52	.077379	27.13	.082600	27.45	52	.169992	24.37	.176464
53	.079007	27.07	.084247	27.42	53	.171454	24.33	.177948
54	.080631	27.03	.085892	27.37	54	.172914	24.30	.179430
55	.082253	26.98	.087534	27.30	55	.174372	24.25	.180909
56	.083872	26.93	.089172	27.27	56	.175827	24.20	.182386
57	.085488	26.87	.090808	27.20	57	.177279	24.17	.18398
58	.087100	26.83	.092440	27.17	58	.178729	24.13	.185
59	.088710	26.78	.094070	27.12	59	.180177	24.08	.18
60	8.090317	26.72	8.095697	27.05	60	8.181622	24.05	8.18

2°						3°			
Vers.	$q-2l$		Ex. sec.	"	'	Vers.	$q-2l$		Ex. sec.
	9.070						9.070*		
6.784741	076	340	6.785005	10800	0	7.136868	021	616	7.137464
.791948	075	344	.792217	10860	1	.141679	019	622	.142281
.799096	074	348	.799370	10920	2	.146464	018	627	.147072
.806186	073	351	.806464	10980	3	.151222	017	633	.151837
.813219	073	355	.813501	11040	4	.155954	016	638	.156577
.820194	072	359	.820482	11100	5	.160661	015	644	.161290
.827115	071	363	.827406	11160	6	.165342	014	650	.165978
.833980	070	367	.834277	11220	7	.169998	013	655	.170641
.840792	070	371	.841093	11280	8	.174630	011	661	.175279
.847551	069	375	.847857	11340	9	.179236	010	667	.179893
.854257	068	379	.854568	11400	10	.183819	009	673	.184483
6.860912	067	383	6.861228	11460	11	7.188377	008	679	7.189048
.867517	066	387	.867837	11520	12	.192912	007	685	.193589
.874071	066	391	.874396	11580	13	.197423	006	690	.198108
.880577	065	395	.880907	11640	14	.201910	004	696	.202602
.887034	064	399	.887369	11700	15	.206375	003	702	.207074
.893443	063	403	.893783	11760	16	.210817	002	708	.211523
.899806	062	407	.900151	11820	17	.215236	001	714	.215949
.906122	061	411	.906472	11880	18	219633	◆00	720	.220353
.912393	061	416	.912748	11940	19	.224007	998	727	.224735
.918618	060	420	.918979	12000	20	.228360	997	733	.229095
6.924800	059	424	6.925165	12060	21	7.232691	996	739	7.233433
.930937	058	429	.931308	12120	22	.237000	995	745	.237750
.937032	057	433	.937408	12180	23	.241288	994	751	.242046
.943084	056	437	.943465	12240	24	.245555	992	757	.246320
.949094	055	442	.949480	12300	25	.249801	991	764	.250574
.955063	054	446	.955455	12360	26	.254027	990	770	.254807
.960991	054	451	.961388	12420	27	.258232	989	776	.259019
.966879	053	455	.967281	12480	28	.262416	987	783	.263212
.972727	052	460	.973135	12540	29	.266581	986	789	.267384
.978536	051	464	.978949	12600	30	.270726	985	795	.271537
6.984306	050	469	6.984725	12660	31	7.274851	983	802	7.275669
.990039	049	474	.990463	12720	32	.278956	982	808	.279783
6.995733	048	478	6.996164	13780	33	.283043	981	815	.283877
7.001391	047	483	7.001827	12840	34	.287110	980	821	.287952
.007012	046	488	.007454	12900	35	.291158	978	828	.292007
.012597	045	493	.013044	12960	36	.295187	977	835	.296045
.018146	044	497	.018599	13020	37	.299197	976	841	.300063
.023660	043	502	.024119	13080	38	.303190	974	848	.304063
.029139	042	507	.029604	13140	39	.307164	973	855	.308045
.034584	041	512	.035054	13200	40	.311119	972	861	.312009
7.039995	040	517	7.040471	13260	41	7.315057	970	868	7.315955
.045372	039	522	.045854	13320	42	.318977	969	875	.319883
.050716	038	527	.051204	13380	43	.322880	967	882	.323794
.056028	037	532	.056522	13440	44	.326765	966	889	.327687
.061307	036	537	.061807	13500	45	.330632	965	896	.331563
.066554	035	542	.067061	13560	46	.334483	963	902	.335422
.071770	034	547	.072282	13620	47	.338316	962	908	.339263
.076954	033	552	.077473	13680	48	.342133	961	916	.343089
.082108	032	557	.082633	13740	49	.345933	959	923	.346897
.087232	031	562	.087763	13800	50	.349716	958	930	.350689
7.092325	030	568	7.092862	13860	51	7.353483	956	938	7.354464
.097389	029	573	.097932	13920	52	.357233	955	945	.358223
.102423	028	578	.102973	13980	53	.360968	953	952	.361966
.107428	027	584	.107985	14040	54	.364686	952	959	.365693
.112405	026	589	.112968	14100	55	.368389	951	966	.369404
.117353	025	594	.117922	14160	56	.372076	949	973	.373100
.122273	024	600	.122849	14220	57	.375747	948	981	[illegible]
.127165	023	605	.127748	14280	58	.379403	946	988	[illegible]
.132030	022	611	.132619	14340	59	.383043	945	995	[illegible]
136868	021	616	7.137464	14400	60	7.386668	943	◆00	[illegible]
				"	'		9.069*	9.07	

4°					5°			
′	Vers.	D. 1″.	Ex. sec.	D. 1″.	′	Vers.	D. 1″.	Ex. sec.
0	7.386668	60.17	7.387728	60.32	0	7.580389	48.15	7.582045
1	.390278	59.93	.391347	60.07	1	.583278	47.98	.584945
2	.393874	59.67	.394951	59.82	2	.586157	47.82	.587835
3	.397454	59.43	.398540	59.57	3	.589026	47.67	.590715
4	.401020	59.18	.402114	59.33	4	.591886	47.52	.593587
5	.404571	58.93	.405674	59.10	5	.594737	47.35	.596449
6	.408107	58.70	.409220	58.85	6	.597578	47.20	.599301
7	.411629	58.47	.412751	58.62	7	.600410	47.05	.602144
8	.415137	58.23	.416268	58.38	8	.603233	46.90	.604979
9	.418631	58.00	.419771	58.15	9	.606047	47.73	.607804
10	.422111	57.77	.423260	57.92	10	.608851	46.60	.610619
11	7.425577	57.53	7.426735	57.70	11	7.611647	46.43	7.613426
12	.429029	57.30	.430197	57.45	12	.614433	46.30	.616224
13	.432467	57.08	.433644	57.25	13	.617211	46.15	.619013
14	.435892	56.85	.437079	57.00	14	.619980	45.98	.621794
15	.439303	56.63	.440499	56.80	15	.622739	45.87	.624565
16	.442701	56.42	.443907	56.57	16	.625491	45.70	.627328
17	.446086	56.20	.447301	56.35	17	.628233	45.57	.630082
18	.449458	55.97	.450682	56.13	18	.630967	45.42	.632827
19	.452816	55.77	.454050	55.92	19	.633692	45.28	.635564
20	.456162	55.55	.457405	55.72	20	.636409	45.13	.638293
21	7.459495	55.33	7.460748	55.48	21	7.639117	44.98	7.641013
22	.462815	55.12	.464077	55.28	22	.641816	44.87	.643724
23	.466122	54.92	.467394	55.08	23	.644508	44.72	.646428
24	.469417	54.70	.470699	54.87	24	.647191	44.57	.649122
25	.472699	54.50	.473991	54.65	25	.649865	44.45	.651809
26	.475969	54.28	.477270	54.47	26	.652532	44.30	.654488
27	.479226	54.10	.480538	54.25	27	.655190	44.17	.657158
28	.482472	53.88	.483793	54.05	28	.657840	44.05	.659820
29	.485705	53.70	.487036	53.85	29	.660483	43.90	.662474
30	.488927	53.48	.490267	53.67	30	.663117	43.77	.665121
31	7.492136	53.28	7.493487	53.45	31	7.665743	43.63	7.667759
32	.495333	53.10	.496694	53.27	32	.668361	43.50	.670389
33	.498519	52.90	.499890	53.07	33	.670971	43.38	.673012
34	.501693	52.72	.503074	52.88	34	.673574	43.23	.675626
35	.504856	52.52	.506247	52.68	35	.676168	43.12	.678233
36	.508007	52.33	.509408	52.50	36	.678755	42.98	.680833
37	.511147	52.13	.512558	52.32	37	.681334	42.87	.683424
38	.514275	51.95	.515697	52.12	38	.683906	42.73	.686008
39	.517392	51.77	.518824	51.93	39	.686470	42.60	.688585
40	.520498	51.58	.521940	51.77	40	.689026	42.48	.691154
41	7.523593	51.40	7.525046	51.57	41	7.691575	42.35	7.693715
42	.526677	51.22	.528140	51.38	42	.694116	42.23	.696269
43	.529750	51.03	.531223	51.22	43	.696650	42.12	.698815
44	.532812	50.85	.534296	51.02	44	.699177	41.98	.701355
45	.535863	50.68	.537357	50.85	45	.701696	41.87	.703887
46	.538904	50.50	.540408	50.68	46	.704208	41.73	.706411
47	.541934	50.32	.543449	50.50	47	.706712	41.63	.708929
48	.544953	50.15	.546479	50.33	48	.709210	41.50	.711439
49	.547962	49.98	.549499	50.15	49	.711700	41.38	.713942
50	.550961	49.80	.552508	49.98	50	.714183	41.27	.716438
51	7.553949	49.63	7.555507	49.80	51	7.716659	41.15	7.718927
52	.556927	49.43	.558495	49.65	52	.719128	41.03	.721409
53	.559895	49.28	.561474	49.47	53	.721590	40.92	.723884
54	.562852	49.13	.564442	49.32	54	.724045	40.80	.726352
55	.565800	48.95	.567401	49.13	55	.726493	40.68	.728813
56	.568737	48.80	.570349	48.98	56	.728934	40.57	.731287
	.571665	48.63	.573288	48.82	57	.731368	40.47	.733714
	.574583	48.47	.576217	48.65	58	.733796	40.33	.736155
	577491	48.30	.579136	48.48	59	.736216	40.23	.7385
	580389	48.15	7.582045	48.33	60	7.738630	40.13	7.7410

6°					7°			
Vers.	D. 1".	Ex. sec.	D. 1".	′	Vers.	D. 1".	Ex. sec.	D. 1".
7.738630	40.13	7.741016	40.33	0	7.872381	34.38	7.875630	34.63
.741038	40.00	.743436	40.23	1	.874444	34.30	.877708	34.57
.743438	39.90	.745850	40.13	2	.876502	34.22	.879782	34.48
.745832	39.78	.748258	40.00	3	.878555	34.13	.881851	34.40
.748219	39.68	.750658	39.90	4	.880603	34.07	.883915	34.32
.750600	39.57	.753052	39.80	5	.882647	33.98	.885974	34.25
.752974	39.47	.755440	39.68	6	.884686	33.90	.888029	34.15
.755342	39.35	.757821	39.58	7	.886720	33.82	.890078	34.08
.757703	39.25	.760196	39.48	8	.888749	33.73	.892123	34.02
.760058	39.13	.762565	39.37	9	.890773	33.67	.894164	33.92
.762406	39.05	.764927	39.25	10	.892793	33.58	.896199	33.85
7.764749	38.92	7.767282	39.17	11	7.894808	33.50	7.898230	33.77
.767084	38.83	.769632	39.05	12	.896818	33.43	.900256	33.70
.769414	38.72	.771975	38.95	13	.898824	33.35	.902278	33.62
.771737	38.62	.774312	38.85	14	.900825	33.27	.904295	33.53
.774054	38.52	.776643	38.75	15	.902821	33.20	.906307	33.47
.776365	38.42	.778968	38.63	16	.904813	33.12	.908315	33.40
.778670	38.30	.781286	38.55	17	.906800	33.05	.910319	33.30
.780968	38.22	.783599	38.43	18	.908783	32.97	.912317	33.25
.783261	38.10	.785905	38.35	19	.910761	32.90	.914312	33.17
.785547	38.02	.788206	38.23	20	.912735	32.82	.916302	33.08
7.787828	37.90	7.790500	38.15	21	7.914704	32.73	7.918287	33.02
.790102	37.82	.792789	38.03	22	.916668	32.68	.920268	32.95
.792371	37.70	.795071	37.95	23	.918629	32.58	.922245	32.87
.794633	37.62	.797348	37.85	24	.920584	32.53	.924217	32.78
.796890	37.52	.799619	37.75	25	.922536	32.45	.926184	32.73
.799141	37.40	.801884	37.65	26	.924483	32.37	.928148	32.65
.801385	37.33	.804143	37.57	27	.926425	32.32	.930107	32.58
.803625	37.22	.806397	37.45	28	.928364	32.22	.932062	32.50
.805858	37.13	.808644	37.37	29	.930297	32.17	.934012	32.43
.808086	37.03	.810886	37.28	30	.932227	32.08	.935958	32.37
7.810308	36.93	7.813123	37.17	31	7.934152	32.02	7.937900	32.30
.812524	36.83	.815353	37.08	32	.936073	31.95	.939838	32.23
.814734	36.75	.817578	37.00	33	.937990	31.88	.941772	32.15
.816939	36.67	.819798	36.90	34	.939903	31.80	.943701	32.08
.819139	36.55	.822012	36.80	35	.941811	31.73	.945626	32.02
.821332	36.48	.824220	36.72	36	.943715	31.67	.947547	31.95
.823521	36.37	.826423	36.62	37	.945615	31.60	.949464	31.87
.825708	36.28	.828620	36.53	38	.947511	31.52	.951376	31.82
.827880	36.20	.830812	36.45	39	.949402	31.47	.953285	31.73
.830052	36.10	.832999	36.35	40	.951290	31.38	.955189	31.68
7.832218	36.02	7.835180	36.27	41	7.953173	31.32	7.957090	31.60
.834379	35.93	.837356	36.17	42	.955052	31.27	.958986	31.53
.836535	35.83	.839526	36.08	43	.956928	31.18	.960878	31.48
.838685	35.75	.841691	36.00	44	.958799	31.12	.962767	31.40
.840830	35.65	.843851	35.90	45	.960666	31.05	.964651	31.35
.842969	35.58	.846005	35.83	46	.962529	30.98	.966531	31.28
.845104	35.48	.848155	35.73	47	.964388	30.92	.968408	31.20
.847233	35.40	.850299	35.63	48	.966243	30.85	.970280	31.13
.849357	35.30	.852437	35.57	49	.968094	30.78	.972148	31.08
.851475	35.23	.854571	35.48	50	.969941	30.73	.974013	31.02
7.853589	35.13	7.856700	35.38	51	7.971785	30.65	7.975874	30.93
.855697	35.05	.858823	35.32	52	.973624	30.58	.977730	30.88
.857800	34.97	.860942	35.22	53	.975459	30.53	.979583	30.82
.859898	34.88	.863055	35.13	54	.977291	30.45	.981432	30.75
.861991	34.80	.865163	35.05	55	.979118	30.40	.983277	30.70
.864079	34.72	.867266	34.98	56	.980942	30.33	.985119	30.62
.866162	34.63	.869365	34.88	57	.982762	30.27	.986956	30.57
.868240	34.55	.871458	34.80	58	.984578	30.22	.988790	30.50
.870313	34.47	.873546	34.73	59	.986391	30.13	.990620	30.43
7.872381	34.38	7.875630	34.63	60	7.988193	30.08	7.992446	30.3

8°					9°		
′	Vers.	D. 1″.	Ex. sec.	D. 1″.	′	Vers.	D. 1″.
0	7.988199	30.08	7.992446	30.38	0	8.090317	26.72
1	.990004	30.02	.994269	30.32	1	.091920	[illegible]
2	.991805	29.95	.996088	30.25	2	.093521	[illegible]
3	.993602	29.88	.997903	30.18	3	.095119	[illegible]
4	.995395	29.83	7.999714	30.13	4	.096714	[illegible]
5	.997185	29.77	8.001522	30.07	5	.098305	[illegible]
6	7.998971	29.72	.003326	30.00	6	.099894	[illegible]
7	8.000754	29.63	.005126	29.95	7	.101480	[illegible]
8	.002532	29.60	.006923	29.88	8	.103063	[illegible]
9	.004308	29.52	.008716	29.83	9	.104644	[illegible]
10	.006079	29.47	.010506	29.73	10	.106221	[illegible]
11	8.007847	29.40	8.012292	29.70	11	8.107796	[illegible]
12	.009611	29.35	.014074	29.65	12	.109367	[illegible]
13	.011372	29.28	.015853	29.58	13	.110936	[illegible]
14	.013129	29.22	.017628	29.53	14	.112502	[illegible]
15	.014882	29.17	.019400	29.47	15	.114065	[illegible]
16	.016632	29.10	.021168	29.42	16	.115625	[illegible]
17	.018378	29.05	.022933	29.35	17	.117182	[illegible]
18	.020121	29.00	.024694	29.30	18	.118737	[illegible]
19	.021861	28.93	.026452	29.23	19	.120289	[illegible]
20	.023597	28.87	.028206	29.18	20	.121838	[illegible]
21	8.025329	28.82	8.029957	29.13	21	8.123384	[illegible]
22	.027058	28.75	.031705	29.07	22	.124927	[illegible]
23	.028783	28.70	.033449	29.00	23	[illegible]	[illegible]
24	.030505	28.65	.035189	28.97	24	[illegible]	[illegible]
25	.032224	28.58	.036927	28.90	25	[illegible]	[illegible]
26	.033939	28.53	.038661	28.83	26	[illegible]	[illegible]
27	.035651	28.47	.040391	28.78	27	[illegible]	[illegible]
28	.037359	28.42	.042118	28.73	28	[illegible]	[illegible]
29	.039064	28.37	.043842	28.68	29	[illegible]	[illegible]
30	.040766	28.30	.045563	28.62	30	[illegible]	[illegible]
31	8.042464	28.25	8.047280	28.57	31	[illegible]	[illegible]
32	.044159	28.20	.048994	28.50	32	[illegible]	[illegible]
33	.045851	28.13	.050704	28.47	33	[illegible]	[illegible]
34	.047539	28.08	.052412	28.40	34	[illegible]	[illegible]
35	.049224	28.03	.054116	28.35	35	[illegible]	[illegible]
36	.050906	27.98	.055817	28.28	36	[illegible]	[illegible]
37	.052585	27.92	.057514	28.25	37	[illegible]	[illegible]
38	.054260	27.87	.059209	28.18	38	[illegible]	[illegible]
39	.055932	27.82	.060900	28.13	39	[illegible]	[illegible]
40	.057601	27.75	.062588	28.08	40	[illegible]	[illegible]
41	8.059266	27.72	8.064273	28.03	41	[illegible]	[illegible]
42	.060929	27.65	.065955	27.97	42	[illegible]	[illegible]
43	.062588	27.60	.067633	27.93	43	[illegible]	[illegible]
44	.064244	27.55	.069309	27.87	44	[illegible]	[illegible]
45	.065897	27.48	.070981	27.82	45	[illegible]	[illegible]
46	.067546	27.45	.072650	27.77	46	[illegible]	[illegible]
47	.069193	27.38	.074316	27.72	47	[illegible]	[illegible]
48	.070836	27.33	.075979	27.67	48	[illegible]	[illegible]
49	.072476	27.30	.077639	27.60	49	[illegible]	[illegible]
50	.074114	27.23	.079295	27.57	50	[illegible]	[illegible]
51	8.075748	27.18	8.080949	27.52	51	[illegible]	[illegible]
52	.077379	27.13	.082600	27.45	52	[illegible]	[illegible]
53	.079007	27.07	.084247	27.42	53	[illegible]	[illegible]
54	.080631	27.03	.085892	27.37	54	[illegible]	[illegible]
55	.082253	26.98	.087534	27.30	55	[illegible]	[illegible]
56	.083872	26.93	.089172	27.27	56	[illegible]	[illegible]
57	.085488	26.87	.090808	27.20	57	[illegible]	[illegible]
58	.087100	26.83	.092440	27.17	58	[illegible]	[illegible]
59	.088710	26.78	.094070	27.12	59	[illegible]	[illegible]
60	8.090317	26.72	8.095697	27.05	60	[illegible]	[illegible]

	15°				
D. 1".	′	Vers.	D. 1".	Ex. sec.	D. 1".
17.67	0	8.532425	15.98	8.547482	16.53
17.63	1	.533384	15.97	.548474	16.53
17.63	2	.534342	15.95	.549466	16.52
17.60	3	.535299	15.93	.550457	16.50
17.58	4	.536255	15.92	.551447	16.48
17.57	5	.537210	15.88	.552436	16.47
17.53	6	.538163	15.88	.553424	16.43
17.53	7	.539116	15.87	.554410	16.43
17.50	8	.540068	15.83	.555396	16.42
17.48	9	.541018	15.83	.556381	16.38
17.47	10	.541968	15.80	.557364	16.38
17.45	11	8.542916	15.78	8.558347	16.37
17.43	12	.543863	15.78	.559329	16.33
17.40	13	.544810	15.75	.560309	16.33
17.38	14	.545755	15.73	.561289	16.30
17.38	15	.546699	15.72	.562267	16.30
17.35	16	.547642	15.70	.563245	16.28
17.32	17	.548584	15.68	.564222	16.25
17.32	18	.549525	15.67	.565197	16.25
17.30	19	.550465	15.65	.566172	16.22
17.27	20	.551404	15.63	.567145	16.22
17.25	21	8.552342	15.62	8.568118	16.20
17.25	22	.553279	15.60	.569090	16.17
17.22	23	.554215	15.58	.570060	16.17
17.18	24	.555150	15.57	.571030	16.15
17.18	25	.556084	15.55	.571999	16.12
17.17	26	.557017	15.53	.572966	16.12
17.13	27	.557949	15.50	.573933	16.10
17.13	28	.558879	15.50	.574899	16.08
17.10	29	.559800	15.48	.575864	16.05
17.08	30	.560738	15.47	.576827	16.05
17.07	31	8.561666	15.43	8.577790	16.03
17.05	32	.562592	15.43	.578752	16.02
17.03	33	.563518	15.42	.579713	16.00
17.02	34	.564443	15.40	.580673	15.98
16.98	35	.565367	15.37	.581632	15.97
16.98	36	.566289	15.37	.582590	15.95
16.95	37	.567211	15.35	.583547	15.93
16.95	38	.568132	15.33	.584503	15.92
16.92	39	.569052	15.30	.585458	15.90
16.90	40	.569970	15.30	.586412	15.88
16.88	41	8.570888	15.28	8.587365	15.88
16.87	42	.571805	15.27	.588318	15.85
16.85	43	.572721	15.25	.589269	15.83
16.83	44	.573636	15.22	.590219	15.83
16.82	45	.574549	15.22	.591169	15.80
16.78	46	.575463	15.20	.592117	15.80
16.78	47	.576374	15.18	.593065	15.78
16.75	48	.577285	15.17	.594012	15.75
16.75	49	.578193	[illegible]	.594957	15.75
16.72	50	.579104	[illegible]	[illegible]	15.73
16.72	51	[illegible]	[illegible]	[illegible]	[illegible]
16.68	52	[illegible]	[illegible]	[illegible]	[illegible]
16.67	53	[illegible]	[illegible]	[illegible]	[illegible]
16.65	54	[illegible]	[illegible]	[illegible]	[illegible]
16.63	55	[illegible]	[illegible]	[illegible]	[illegible]
16.63	56	[illegible]	[illegible]	[illegible]	[illegible]
16.60	57	[illegible]	[illegible]	[illegible]	[illegible]
16.58	58	[illegible]	[illegible]	[illegible]	[illegible]
16.57	59	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	60	[illegible]	[illegible]	[illegible]	[illegible]

12°					13°			
′	Vers.	D. 1".	Ex. sec.	D. 1".	′	Vers.	D. 1".	Ex. sec.
0	8.339499	20.02	8.349095	20.47	0	8.408748	18.47	8.420024
1	.340700	20.00	.350323	20.43	1	.409856	18.43	.421161
2	.341900	19.95	.351549	20.42	2	.410962	18.42	.422297
3	.343097	19.95	.352774	20.38	3	.412067	18.40	.423431
4	.344294	19.90	.353997	20.35	4	.413171	18.38	.424564
5	.345488	19.88	.355218	20.33	5	.414274	18.35	.425696
6	.346681	19.85	.356438	20.30	6	.415375	18.32	.426826
7	.347872	19.82	.357656	20.28	7	.416474	18.30	.427955
8	.349061	19.80	.358873	20.25	8	.417572	18.28	.429083
9	.350249	19.77	.360088	20.22	9	.418669	18.25	.430209
10	.351435	19.75	.361301	20.20	10	.419764	18.23	.431334
11	8.352620	19.72	8.362513	20.18	11	8.420858	18.22	8.432458
12	.353803	19.68	.363724	20.13	12	.421951	18.18	.433580
13	.354984	19.67	.264932	20.12	13	.423042	18.17	.434700
14	.356164	19.63	.366139	20.10	14	.424132	18.13	.435820
15	.357342	19.60	.367345	20.07	15	.425220	18.12	.436938
16	.358518	19.58	.368549	20.03	16	.426307	18.10	.438055
17	.359693	19.55	.369751	20.02	17	.427393	18.07	.439170
18	.360866	19.53	.370952	19.98	18	.428477	18.05	.440284
19	.362038	19.50	.372151	19.95	19	.429560	18.02	.441397
20	.363208	19.48	.373348	19.95	20	.430641	18.02	.442509
21	8.364377	19.43	8.374545	19.90	21	8.431722	17.97	8.443619
22	.365543	19.43	.375739	19.88	22	.432800	17.97	.444727
23	.366709	19.38	.376932	19.85	23	.433878	17.93	.445835
24	.367872	19.37	.378123	19.83	24	.434954	17.92	.446941
25	.369034	19.35	.379313	19.82	25	.436029	17.88	.448046
26	.370195	19.32	.380502	19.78	26	.437102	17.87	.449149
27	.371354	19.28	.381689	19.75	27	.438174	17.85	.450252
28	.372511	19.27	.382874	19.73	28	.439245	17.82	.451353
29	.373667	19.25	.384058	19.70	29	.440314	17.80	.452452
30	.374822	19.20	.385240	19.68	30	.441382	17.78	.453551
31	8.375974	19.18	8.386421	19.65	31	8.442449	17.75	8.454648
32	.377125	19.17	.387600	19.63	32	.443514	17.73	.455743
33	.378275	19.13	.388778	19.60	33	.444578	17.72	.456838
34	.379423	19.12	.389954	19.58	34	.445641	17.68	.457931
35	.380570	19.08	.391129	19.55	35	.446702	17.68	.459023
36	.381715	19.05	.392302	19.53	36	.447763	17.63	.460114
37	.382858	19.03	.393474	19.50	37	.448821	17.63	.461203
38	.384000	19.02	.394644	19.48	38	.449879	17.62	.462291
39	.385141	18.98	.395813	19.45	39	.450935	17.58	.463378
40	.386280	18.95	.396980	19.43	40	.451990	17.55	.464464
41	8.387417	18.93	8.398146	19.42	41	8.453043	17.55	8.465548
42	.388553	18.92	.399311	19.38	42	.454096	17.52	.466631
43	.389688	18.88	.400474	19.35	43	.455147	17.48	.467713
44	.390821	18.85	.401635	19.33	44	.456196	17.48	.468793
45	.391952	18.83	.402795	19.32	45	.457245	17.45	.469873
46	.393082	18.82	.403954	19.28	46	.458292	17.43	.470951
47	.394211	18.78	.405111	19.27	47	.459338	17.40	.472028
48	.395338	18.75	.406267	19.23	48	.460382	17.40	.473103
49	.396463	18.73	.407421	19.22	49	.461426	17.37	.474177
50	.397587	18.72	.408574	19.18	50	.462468	17.35	.475251
51	8.398710	18.68	8.409725	19.17	51	8.463509	17.32	8.476322
52	.399831	18.67	.410875	19.13	52	.464548	17.30	.477393
53	.400951	18.63	.412023	19.13	53	.465586	17.28	.478463
54	.402069	18.62	.413171	19.08	54	.466623	17.27	.479531
55	.403186	18.58	.414316	19.08	55	.467659	17.23	.480598
56	.404301	18.57	.415461	19.03	56	.468693	17.23	.481664
57	.405415	18.53	.416603	19.03	57	.469727	17.20	.482728
58	.406527	18.52	.417745	19.00	58	.470759	17.17	.483792
59	.407638	18.50	.418885	18.98	59	.471789	17.17	.48485
60	8.408748	18.47	8.420024	18.95	60	8.472819	17.13	8.4859

14°					15°			
Vers.	D. 1".	Ex. sec.	D. 1".	′	Vers.	D. 1".	Ex. sec.	D. 1".
8.472819	17.13	8.485915	17.67	0	8.532425	15.98	8.547482	16.53
.473847	17.12	.486975	17.63	1	.533384	15.97	.548474	16.53
.474874	17.10	.488033	17.63	2	.534342	15.95	.549466	16.52
.475900	17.08	.489091	17.60	3	.535299	15.93	.550457	16.50
.476925	17.05	.490147	17.58	4	.536255	15.92	.551447	16.48
.477948	17.03	.491202	17.57	5	.537210	15.88	.552436	16.47
.478970	17.02	.492256	17.53	6	.538163	15.88	.553424	16.43
.479991	17.00	.493308	17.53	7	.539116	15.87	.554410	16.43
.481011	16.97	.494360	17.50	8	.540068	15.83	.555396	16.42
.482029	16.95	.495410	17 48	9	.541018	15.83	.556381	16.38
.483046	16.93	.496459	17.47	10	.541968	15.80	.557364	16.38
8.484062	16.92	8.497507	17.45	11	8.542916	15.78	8.558347	16.37
.485077	16.90	.498554	17.43	12	.543863	15.78	.559329	16.33
.486091	16.87	.499600	17.40	13	.544810	15.75	.560309	16.33
.487103	16.87	.500644	17.38	14	.545755	15.73	.561289	16.30
.488115	16.83	.501687	17.38	15	.546699	15.72	.562267	16.30
.489125	16 82	.502730	17.35	16	.547642	15.70	.563245	16.28
.490134	16.78	.503771	17.32	17	.548584	15.68	.564222	16.25
.491141	16.78	.504810	17.32	18	.549525	15.67	.565197	16.25
.492148	16.75	.505849	17.30	19	.550465	15.65	.566172	16.22
.493153	16.73	.506887	17.27	20	.551404	15.63	.567145	16.22
8.494157	16.72	8.507923	17.25	21	8.552342	15.62	8.568118	16.20
.495160	16.70	.508958	17.25	22	.553279	15.60	.569090	16.17
.496162	16.67	.509993	17.22	23	.554215	15.58	.570060	16.17
.497162	16.67	.511026	17.18	24	.555150	15.57	.571030	16.15
.498162	16.63	.512057	17.18	25	.556084	15.55	.571999	16.12
.499160	16.62	.513088	17.17	26	.557017	15.53	.572966	16.12
.500157	16.60	.514118	17.13	27	.557949	15.50	.573933	16.10
.501153	16.58	.515146	17.13	28	.558879	15.50	.574899	16.08
.502148	16.57	.516174	17.10	29	.559809	15.48	.575864	16.05
.503142	16.53	.517200	17.08	30	.560738	15.47	.576827	16.05
8.504134	16.52	8.518225	17.07	31	8.561666	15.43	8.577790	16.03
.505125	16.52	.519249	17.05	32	.562592	15.43	.578752	16 02
.506116	16.48	.520272	17.03	33	.563518	15 42	.579713	16.00
.507105	16.47	.521294	17.02	34	.564443	15.40	.580673	15.98
.508092	16.47	.522315	16.98	35	.565367	15.37	.581632	15.97
.509079	16.43	.523334	16.98	36	.566289	15 37	.582590	15.95
.510065	16.40	.524353	16.95	37	.567211	15.35	.583547	15.93
.511049	16.40	.525370	16.95	38	.568132	15.33	.584503	15.92
.512033	16.37	.526387	16.92	39	.569052	15.30	.585458	15.90
.513015	16.35	.527402	16.90	40	.569970	15.30	.586412	15.88
8.513996	16.33	8.528416	16.88	41	8.570888	15.28	8.587365	15.88
.514976	16.32	.529429	16.87	42	.571805	15.27	.588318	15.85
.515955	16.28	.530441	16.85	43	.572721	15.25	.589269	15.83
.516932	16.28	.531452	16.83	44	.573636	15.22	.590219	15.83
.517909	16.25	.532462	16.82	45	.574549	15.22	.591169	15.80
.518884	16.25	.533471	16.78	46	.575462	15.20	.592117	15.80
.519859	16.22	.534478	16.78	47	.576374	15.18	.593065	15.78
.520832	16.20	.535485	16.75	48	.577285	15.17	.594012	15.75
.521804	16.18	.536490	16.75	49	.578195	15.15	.594957	15 75
.522775	16.17	.537495	16.72	50	.579104	15.13	.595902	15.73
8.523745	16.15	8.538498	16.72	51	8.580012	15.12	8.596846	15 72
.524714	16.13	.539501	16.68	52	.580919	15.10	.597789	15.70
.525682	16.10	.540502	16.67	53	.581825	15.08	.598731	15.68
.526648	16.10	.541502	16.65	54	.582730	15.07	.599672	15.67
.527614	16.07	.542501	16.63	55	.583634	15.05	.600612	15.65
.528578	16.07	.543499	16.63	56	.584537	15.05	.601551	15.63
.529542	16.03	.544497	16.60	57	.585440	15.02	.602490	15.62
.5305 4	16.02	.545493	16.58	58	.586341	15.00	.603427	15.60
.531465	16.00	.546488	16.57	59	.587241	15.00	.604363	15.60
8.532425	15.98	8.547482	16.53	60	8.588141	14.97	8.605299	15.58

16°					17°			
′	Vers.	D. 1″.	Ex. sec.	D. 1″.	′	Vers.	D. 1″.	Ex. sec.
0	8.588141	14.97	8.605299	15.58	0	8.640434	14.08	8.659838
1	.589039	14.95	.606234	15.55	1	.641279	14.07	.660721
2	.589936	14.95	.607167	15.55	2	.642123	14.05	.661604
3	.590833	14.93	.608100	15.53	3	.642966	14.05	.662486
4	.591729	14.90	.609032	15.52	4	.643809	14.02	.663367
5	.592623	14.90	.609963	15.50	5	.644650	14.02	.664248
6	.593517	14.88	.610893	15.50	6	.645491	14.00	.665127
7	.594410	14.87	.611823	15.47	7	.646331	13.98	.666006
8	.595302	14.83	.612751	15.45	8	.647170	13.97	.666884
9	.596192	14.83	.613678	15.45	9	.648008	13.95	.667761
10	.597082	14.82	.614605	15.43	10	.648845	13.95	.668637
11	8.597971	14.82	8.615531	15.42	11	8.649682	13.93	8.669513
12	.598860	14.78	.616456	15.38	12	.650518	13.92	.670388
13	.599747	14.77	.617379	15.38	13	.651353	13.90	.671262
14	.600633	14.75	.618302	15.38	14	.652187	13.88	.672135
15	.601518	14.75	.619225	15.35	15	.653020	13.87	.673008
16	.602403	14.72	.620146	15.33	16	.653852	13.87	.673879
17	.603286	14.72	.621066	15.33	17	.654684	13.85	.674750
18	.604169	14.70	.621986	15.30	18	.655515	13.83	.675620
19	.605051	14.67	.622904	15.30	19	.656345	13.82	.676490
20	.605931	14.67	.623822	15.28	20	.657174	13.82	.677358
21	8.606811	14.65	8.624739	15.27	21	8.658003	13.78	8.678226
22	.607690	14.63	.625655	15.25	22	.658830	13.78	.679093
23	.608568	14.62	.626570	15.23	23	.659657	13.77	.679960
24	.609445	14.60	.627484	15.23	24	.660483	13.75	.680825
25	.610321	14.60	.628398	15.20	25	.661308	13.73	.681690
26	.611197	14.57	.629310	15.20	26	.662132	13.73	.682554
27	.612071	14.57	.630222	15.18	27	.662956	13.72	.683417
28	.612945	14.53	.631133	15.17	28	.663779	13.70	.684280
29	.613817	14.53	.632043	15.15	29	.664601	13.68	.685141
30	.614689	14.52	.632952	15.13	30	.665422	13.67	.686002
31	8.615560	14.50	8.633860	15.13	31	8.666242	13.67	8.686863
32	.616430	14.48	.634768	15.10	32	.667062	13.65	.687722
33	.617299	14.47	.635674	15.10	33	.667881	13.63	.688581
34	.618167	14.45	.636580	15.08	34	.668699	13.62	.689439
35	.619034	14.45	.637485	15.07	35	.669516	13.60	.690296
36	.619901	14.42	.638389	15.05	36	.670332	13.60	.691153
37	.620766	14.42	.639292	15.05	37	.671148	13.58	.692008
38	.621631	14.40	.640195	15.02	38	.671963	13.57	.692863
39	.622495	14.38	.641096	15.02	39	.672777	13.55	.693718
40	.623358	14.37	.641997	15.00	40	.673590	13.55	.694571
41	8.624220	14.35	8.642897	14.98	41	8.674403	13.53	8.695424
42	.625081	14.33	.643796	14.97	42	.675215	13.52	.696276
43	.625941	14.33	.644694	14.95	43	.676026	13.50	.697127
44	.626801	14.30	.645591	14.95	44	.676836	13.48	.697978
45	.627659	14.30	.646488	14.93	45	.677645	13.48	.698828
46	.628517	14.28	.647384	14.92	46	.678454	13.47	.699677
47	.629374	14.27	.648279	14.90	47	.679262	13.45	.700525
48	.630230	14.25	.649173	14.88	48	.680069	13.43	.701373
49	.631085	14.23	.650066	14.87	49	.680875	13.43	.702220
50	.631939	14.22	.650958	14.87	50	.681681	13.42	.703066
51	8.632792	14.22	8.651850	14.85	51	8.682486	13.40	8.703912
52	.633645	14.18	.652741	14.83	52	.683290	13.38	.704756
53	.634496	14.18	.653631	14.82	53	.684093	13.38	.705600
54	.635347	14.17	.654520	14.80	54	.684896	13.35	.706444
55	.636197	14.15	.655408	14.80	55	.685697	13.35	.707286
56	.637046	14.13	.656296	14.77	56	.686498	13.35	.708128
57	.637894	14.13	.657182	14.77	57	.687299	13.32	.708969
58	.638742	14.10	.658068	14.77	58	.688098	13.32	.709810
59	.639588	14.10	.658954	14.73	59	.688897	13.30	.710650
60	8.640434	14.08	8.659838	14.72	60	8.689695	13.28	8.711489

18°					19°				
′	Vers.	D. 1″.	Ex. sec.	D. 1″.	′	Vers.	D. 1″.	Ex. sec.	D. 1″.
0	8.689695	13.28	8.711489	13.97	0	8.736248	12.58	8.760578	13.30
1	.690492	13.28	.712327	13.95	1	.737003	12.57	.761376	13.30
2	.691289	13.25	.713164	13.95	2	.737757	12.55	.762174	13.28
3	.692084	13.25	.714001	13.95	3	.738510	12.55	.762971	13.27
4	.692879	13.25	.714838	13.92	4	.739263	12.53	.763767	13.27
5	.693674	13.22	.715673	13.92	5	.740015	12.52	.764563	13.25
6	.694467	13.22	.716508	13.90	6	.740766	12.50	.765358	13.23
7	.695260	13.20	.717342	13.88	7	.741516	12.50	.766152	13.23
8	.696052	13.18	.718175	13.88	8	.742266	12.50	.766946	13.22
9	.696843	13.18	.719008	13.87	9	.743016	12.47	.767739	13.20
10	.697634	13.17	.719840	13.85	10	.743764	12.47	.768531	13.20
11	8.698424	13.15	8.720671	13.85	11	8.744512	12.45	8.769323	13.18
12	.699213	13.13	.721502	13.83	12	.745259	12.43	.770114	13.18
13	.700001	13.13	.722332	13.82	13	.746006	12.45	.770905	13.17
14	.700789	13.12	.723161	13.80	14	.746752	12.42	.771695	13.15
15	.701576	13.10	.723989	13.80	15	.747497	12.42	.772484	13.15
16	.702362	13.08	.724817	13.78	16	.748242	12.40	.773273	13.13
17	.703147	13.08	.725644	13.78	17	.748986	12.38	.774061	13.13
18	.703932	13.07	.726471	13.77	18	.749729	12.38	.774849	13.12
19	.704716	13.05	.727297	13.75	19	.750472	12.37	.775636	13.10
20	.705499	13.05	.728122	13.73	20	.751214	12.35	.776422	13.08
21	8.706282	13.02	8.728946	13.73	21	8.751955	12.35	8.777207	13.10
22	.707063	13.02	.729770	13.72	22	.752696	12.33	.777993	13.07
23	.707844	13.02	.730593	13.70	23	.753436	12.32	.778777	13.07
24	.708625	12.98	.731415	13.70	24	.754175	12.32	.779561	13.05
25	.709404	12.98	.732237	13.68	25	.754914	12.30	.780344	13.05
26	.710183	12.98	.733058	13.67	26	.755652	12.28	.781127	13.03
27	.710961	12.95	.733878	13.67	27	.756389	12.28	.781909	13.02
28	.711739	12.95	.734698	13.65	28	.757126	12.27	.782690	13.02
29	.712516	12.93	.735517	13.63	29	.757862	12.27	.783471	13.00
30	.713292	12.92	.736335	13.63	30	.758598	12.25	.784251	13.00
31	8.714067	12.92	8.737153	13.62	31	8.759333	12.23	8.785031	12.98
32	.714842	12.90	.737970	13.60	32	.760067	12.23	.785810	12.97
33	.715616	12.88	.738786	13.60	33	.760801	12.22	.786588	12.97
34	.716389	12.87	.739602	13.58	34	.761534	12.20	.787366	12.97
35	.717161	12.87	.740417	13.57	35	.762266	12.20	.788144	12.93
36	.717933	12.85	.741231	13.57	36	.762998	12.18	.788920	12.93
37	.718704	12.85	.742045	13.55	37	.763729	12.17	.789696	12.93
38	.719475	12.82	.742858	13.53	38	.764459	12.17	.790472	12.92
39	.720244	12.82	.743670	13.53	39	.765189	12.15	.791247	12.90
40	.721013	12.82	.744482	13.52	40	.765918	12.15	.792021	12.90
41	8.721782	12.78	8.745293	13.50	41	8.766647	12.12	8.792795	12.88
42	.722549	12.78	.746103	13.50	42	.767374	12.12	.793568	12.87
43	.723316	12.78	.746913	13.48	43	.768102	12.10	.794340	12.87
44	.724083	12.75	.747722	13.47	44	.768828	12.10	.795112	12.87
45	.724848	12.75	.748530	13.47	45	.769554	12.10	.795884	12.83
46	.725613	12.73	.749338	13.45	46	.770280	12.08	.796654	12.85
47	.726377	12.72	.750145	13.43	47	.771005	12.07	.797425	12.82
48	.727140	12.72	.750951	13.43	48	.771729	12.05	.798194	12.82
49	.727903	12.70	.751757	13.42	49	.772452	12.05	.798963	12.82
50	.728665	12.70	.752562	13.42	50	.773175	12.05	.799732	12.80
51	8.729427	12.67	8.753367	13.40	51	8.773898	12.02	8.800500	12.78
52	.730187	12.67	.754171	13.38	52	.774619	12.02	.801267	12.78
53	.730947	12.67	.754974	13.37	53	.775340	12.02	.802034	12.77
54	.731707	12.63	.755776	13.37	54	.776061	12.00	.802800	12.75
55	.732465	12.63	.756578	13.37	55	.776781	11.98	.803565	12.75
56	.733223	12.63	.757380	13.33	56	.777500	11.97	.804330	12.75
57	.733981	12.60	.758180	13.33	57	.778218	11.97	.805095	12.73
58	.734737	12.60	.758980	13.33	58	.778936	11.97	.805859	12.72
59	.735493	12.58	.759780	13.30	59	.779654	11.93	.806622	12.72
60	8.736248	12.58	8.760578	13.30	60	8.780370	11.95	8.807385	12.7

20°					21°			
′	Vers.	D. 1″.	Ex. sec.	D. 1″.	′	Vers.	D. 1″.	Ex. se
0	8.780370	11.95	8.807385	12.70	0	8.822296	11.35	8.85214
1	.781087	11.92	.808147	12.68	1	.822977	11.35	.85287
2	.781802	11.92	.808908	12.68	2	.823658	11.33	.85360
3	.782517	11.90	.809669	12.68	3	.824338	11.33	.8543
4	.783231	11.90	.810430	12.67	4	.825018	11.32	.8550
5	.783945	11.88	.811190	12.65	5	.825697	11.32	.8557
6	.784658	11.88	.811949	12.65	6	.826375	11.30	.85651
7	.785371	11.87	.812708	12.63	7	.827054	11.28	.8572
8	.786083	11.85	.813466	12.63	8	.827731	11.28	.8579
9	.786794	11.85	.814224	12.62	9	.828408	11.28	.8586
10	.787505	11.83	.814981	12.60	10	.829085	11.27	.8594
11	8.788215	11.82	8.815737	12.60	11	8.829761	11.25	8.86014
12	.788924	11.82	.816493	12.60	12	.830436	11.25	.8608
13	.789633	11.82	.817249	12.58	13	.831111	11.23	.8615
14	.790342	11.78	.818004	12.57	14	.831785	11.23	.86231
15	.791049	11.78	.818758	12.57	15	.832459	11.22	.8630
16	.791756	11.78	.819512	12.55	16	.833132	11.20	.8637
17	.792463	11.77	.820265	12.55	17	.833804	11.20	.8644
18	.793169	11.75	.821018	12.53	18	.834476	11.20	.8652
19	.793874	11.75	.821770	12.52	19	.835148	11.18	.8659
20	.794579	11.73	.822521	12.52	20	.835819	11.17	.8666
21	8.795283	11.73	8.823272	12.52	21	8.836489	11.17	8.8673
22	.795987	11.72	.824023	12.50	22	.837159	11.17	.8680
23	.796690	11.70	.824773	12.48	23	.837829	11.15	.8688
24	.797392	11.70	.825522	12.48	24	.838498	11.13	.8695
25	.798094	11.68	.826271	12.47	25	.839166	11.13	.8702
26	.798795	11.68	.827019	12.47	26	.839834	11.12	.8709
27	.799496	11.67	.827767	12.45	27	.840501	11.12	.8716
28	.800196	11.67	.828514	12.45	28	.841168	11.10	.8723
29	.800896	11.63	.829261	12.43	29	.841834	11.10	.8731
30	.801594	11.65	.830007	12.42	30	.842500	11.08	.8738
31	8.802293	11.63	8.830752	12.42	31	8.843165	11.07	8.8745
32	.802991	11.62	.831497	12.42	32	.843829	11.07	.8752
33	.803688	11.60	.832242	12.40	33	.844493	11.07	.8759
34	.804384	11.60	.832986	12.38	34	.845157	11.05	.8766
35	.805080	11.60	.833729	12.38	35	.845820	11.05	.8773
36	.805776	11.58	.834472	12.38	36	.846483	11.03	.8781
37	.806471	11.57	.835215	12.37	37	.847145	11.02	.8788
38	.807165	11.57	.835957	12.35	38	.847806	11.02	.8795
39	.807859	11.55	.836698	12.35	39	.848467	11.00	.8802
40	.808552	11.53	.837439	12.33	40	.849127	11.00	.8809
41	8.809244	11.53	8.838179	12.33	41	8.849787	11.00	8.8816
42	.809936	11.53	.838919	12.32	42	.850447	10.98	.8823
43	.810628	11.52	.839658	12.30	43	.851106	10.97	.8830
44	.811319	11.50	.840396	12.32	44	.851764	10.97	.8837
45	.812009	11.50	.841135	12.28	45	.852422	10.95	.8844
46	.812699	11.48	.841872	12.28	46	.853079	10.95	.8852
47	.813388	11.48	.842609	12.28	47	.853736	10.93	.8859
48	.814077	11.47	.843346	12.27	48	.854392	10.93	.8866
49	.814765	11.45	.844082	12.25	49	.855048	10.92	.8873
50	.815452	11.45	.844817	12.25	50	.855703	10.92	.8880
51	8.816139	11.43	8.845552	12.25	51	8.856358	10.90	8.8887
52	.816825	11.43	.846287	12.23	52	.857012	10.90	.8894
53	.817511	11.42	.847021	12.22	53	.857666	10.88	.8901
54	.818196	11.42	.847754	12.22	54	.858319	10.88	.8908
55	.818881	11.40	.848487	12.22	55	.858972	10.87	.8915
56	.819565	11.40	.849220	12.20	56	.859624	10.87	.8922
57	.820249	11.38	.849952	12.18	57	.860276	10.85	.8929
58	.820932	11.37	.850683	12.18	58	.860927	10.85	.89
59	.821614	11.37	.851414	12.17	59	.861578	10.83	.894
60	8.822296	11.35	8.852144	12.17	60	8.862228	10.82	8.89

	30°					31°			
′	Vers.	D. 1″.	Ex. sec.	D. 1″.	′	Vers.	D. 1″.	Ex. sec.	D. 1″.
0	9.127022	7.87	9.189492	9.07	0	9.154828	7.58	9.221762	8.85
1	.127494	7.85	.190036	9.07	1	.155283	7.58	.222293	8.87
2	.127965	7.85	.190580	9.07	2	.155738	7.58	.222825	8.83
3	.128436	7.83	.191124	9.07	3	.156193	7.58	.223355	8.85
4	.128906	7.83	.191668	9.05	4	.156648	7.57	.223886	8.85
5	.129376	7.83	.192211	9.05	5	.157102	7.57	.224417	8.83
6	.129846	7.83	.192754	9.05	6	.157556	7.57	.224947	8.83
7	.130316	7.82	.193297	9.05	7	.158010	7.57	.225477	8.83
8	.130785	7.83	.193840	9.03	8	.158464	7.55	.226007	8.83
9	.131255	7.82	.194382	9.05	9	.158917	7.55	.226537	8.82
10	.131724	7.80	.194925	9.03	10	.159370	7.55	.227066	8.82
11	9.132192	7.80	9.195467	9.03	11	9.159823	7.55	9.227595	8.83
12	.132660	7.82	.196009	9.02	12	.160276	7.53	.228125	8.80
13	.133129	7.78	.196550	9.03	13	.160728	7.53	.228653	8.82
14	.133596	7.80	.197092	9.02	14	.161180	7.53	.229182	8.82
15	.134064	7.73	.197633	9.02	15	.161632	7.52	.229711	8.80
16	.134531	7.78	.198174	9.02	16	.162083	7.53	.230239	8.80
17	.134998	7.78	.198715	9.00	17	.162535	7.52	.230767	8.80
18	.135465	7.77	.199255	9.00	18	.162986	7.52	.231295	8.78
19	.135931	7.77	.199795	9.00	19	.163437	7.50	.231822	8.80
20	.136397	7.77	.200335	9.00	20	.163887	7.52	.232350	8.78
21	9.136863	7.77	9.200875	9.00	21	9.164338	7.50	9.232877	8.78
22	.137329	7.75	.201415	8.98	22	.164788	7.48	.233404	8.78
23	.137794	7.77	.201954	9.00	23	.165237	7.50	.233931	8.78
24	.138260	7.73	.202494	8.97	24	.165687	7.48	.234458	8.77
25	.138724	7.75	.203032	8.98	25	.166136	7.48	.234984	8.77
26	.139189	7.73	.203571	8.98	26	.166585	7.48	.235510	8.77
27	.139653	7.73	.204110	8.97	27	.167034	7.48	.236036	8.77
28	.140117	7.73	.204648	8.97	28	.167483	7.47	.236562	8.77
29	.140581	7.73	.205186	8.97	29	.167931	7.47	.237088	8.75
30	.141045	7.72	.205724	8.97	30	.168379	7.47	.237613	8.77
31	9.141508	7.72	9.206262	8.95	31	9.168827	7.47	9.238139	8.75
32	.141971	7.72	.206799	8.97	32	.169275	7.45	.238664	8.75
33	.142434	7.70	.207337	8.95	33	.169722	7.45	.239189	8.73
34	.142896	7.70	.207874	8.93	34	.170169	7.45	.239713	8.75
35	.143358	7.70	.208410	8.95	35	.170616	7.43	.240238	8.73
36	.143820	7.70	.208947	8.93	36	.171062	7.45	.240762	8.73
37	.144282	7.68	.209483	8.95	37	.171509	7.43	.241286	8.73
38	.144743	7.68	.210020	8.93	38	.171955	7.42	.241810	8.72
39	.145204	7.68	.210556	8.92	39	.172400	7.43	.242333	8.73
40	.145665	7.68	.211091	8.93	40	.172846	7.42	.242857	8.72
41	9.146126	7.67	9.211627	8.92	41	9.173291	7.42	9.243380	8.72
42	.146586	7.67	.212162	8.92	42	.173736	7.42	.243903	8.72
43	.147046	7.67	.212697	8.92	43	.174181	7.42	.244426	8.72
44	.147506	7.67	.213232	8.92	44	.174626	7.40	.244949	8.70
45	.147966	7.65	.213767	8.90	45	.175070	7.40	.245471	8.72
46	.148425	7.65	.214301	8.92	46	.175514	7.40	.245994	8.70
47	.148884	7.65	.214836	8.90	47	.175958	7.40	.246516	8.70
48	.149343	7.63	.215370	8.90	48	.176402	7.38	.247038	8.68
49	.149801	7.63	.215904	8.88	49	.176845	7.38	.247559	8.70
50	.150259	7.63	.216437	8.90	50	.177288	7.38	.248081	8.68
51	9.150717	7.63	9.216971	8.88	51	9.177731	7.38	9.248602	8.68
52	.151175	7.63	.217504	8.88	52	.178174	7.37	.249123	8.68
53	.151633	7.62	.218037	8.88	53	.178616	7.37	.249644	8.68
54	.152090	7.62	.218570	8.87	54	.179058	7.37	.250165	8.68
55	.152547	7.60	.219102	8.88	55	.179500	7.37	.250686	8.67
56	.153003	7.62	.219635	8.87	56	.179942	7.35	.251206	8.67
57	.153460	7.60	.220167	8.87	57	.180383	7.37	.251726	8.67
58	.153916	7.60	.220699	8.87	58	.180825	7.33	.252246	8.6
9	.154372	7.60	.221231	8.85	59	.181265	7.35	.252766	8.
	9.154828	7.58	9.221762	8.85	60	9.181706	7.35	9.253286	8

TABLE XXVI.—LOGARITHMIC VERSED SINES

20°					21°				
′	Vers.	D. 1″.	Ex. sec.	D. 1″.	′	Vers.	D. 1″.	Ex. sec.	D. 1″
0	8.780370	11.95	8.807385	12.70	0	8.822296	11.35	8.852144	12.1
1	.781087	11.92	.808147	12.68	1	.822977	11.35	.852874	12.1
2	.781802	11.92	.808908	12.68	2	.823658	11.33	.853604	12.1
3	.782517	11.90	.809669	12.68	3	.824338	11.33	.854332	12.1
4	.783231	11.90	.810430	12.67	4	.825018	11.32	.855061	12.1
5	.783945	11.88	.811190	12.65	5	.825697	11.32	.855789	12.1
6	.784658	11.88	.811949	12.65	6	.826375	11.30	.856516	12.1
7	.785371	11.87	.812708	12.63	7	.827054	11.28	.857243	12.1
8	.786083	11.85	.813466	12.63	8	.827731	11.28	.857969	12.
9	.786794	11.85	.814224	12.62	9	.828408	11.28	.858695	12.
10	.787505	11.83	.814981	12.60	10	.829085	11.27	.859420	12.0
11	8.788215	11.82	8.815737	12.60	11	8.829761	11.25	8.860145	12.0
12	.788924	11.82	.816493	12.60	12	.830436	11.25	.860869	12.
13	.789633	11.82	.817249	12.58	13	.831111	11.23	.861593	12.
14	.790342	11.78	.818004	12.57	14	.831785	11.23	.862316	12.
15	.791049	11.78	.818758	12.57	15	.832459	11.22	.863039	12.0
16	.791756	11.78	.819512	12.55	16	.833132	11.20	.863761	12.0
17	.792463	11.77	.820265	12.55	17	.833804	11.20	.864483	12.0
18	.793169	11.75	.821018	12.53	18	.834476	11.20	.865204	12.0
19	.793874	11.75	.821770	12.52	19	.835148	11.18	.865925	12.0
20	.794579	11.73	.822521	12.52	20	.835819	11.17	.866646	11.9
21	8.795283	11.73	8.823272	12.52	21	8.836489	11.17	8.867365	12.00
22	.795987	11.72	.824023	12.50	22	.837159	11.17	.868085	11.9
23	.796690	11.70	.824773	12.48	23	.837829	11.15	.868804	11.9
24	.797392	11.70	.825522	12.48	24	.838498	11.13	.869522	11.9
25	.798094	11.68	.826271	12.47	25	.839166	11.13	.870240	11.
26	.798795	11.68	.827019	12.47	26	.839834	11.12	.870957	11.9
27	.799496	11.67	.827767	12.45	27	.840501	11.12	.871674	11.9
28	.800196	11.67	.828514	12.45	28	.841168	11.10	.872390	11.9
29	.800896	11.63	.829261	12.43	29	.841834	11.10	.873106	11.9
30	.801594	11.65	.830007	12.42	30	.842500	11.08	.873822	11.
31	8.802293	11.63	8.830752	12.42	31	8.843165	11.07	8.874537	11.90
32	.802991	11.62	.831497	12.42	32	.843829	11.07	.875251	11.9
33	.803688	11.60	.832242	12.40	33	.844493	11.07	.875965	11.8
34	.804384	11.60	.832986	12.38	34	.845157	11.05	.876678	11.8
35	.805080	11.60	.833729	12.38	35	.845820	11.05	.877391	11.8
36	.805776	11.58	.834472	12.38	36	.846483	11.03	.878104	11.8
37	.806471	11.57	.835215	12.37	37	.847145	11.02	.878816	11.8
38	.807165	11.57	.835957	12.35	38	.847806	11.02	.879528	11.8
39	.807859	11.55	.836698	12.35	39	.848467	11.00	.880239	11.8
40	.808552	11.53	.837439	12.33	40	.849127	11.00	.880949	11.8
41	8.809244	11.53	8.838179	12.33	41	8.849787	11.00	8.881659	11.8
42	.809936	11.53	.838919	12.32	42	.850447	10.98	.882369	11.8
43	.810628	11.52	.839658	12.30	43	.851106	10.97	.883078	11.8
44	.811319	11.50	.840396	12.32	44	.851764	10.97	.883787	11.80
45	.812009	11.50	.841135	12.28	45	.852422	10.95	.884495	11.80
46	.812699	11.48	.841872	12.28	46	.853079	10.95	.885203	11.7
47	.813388	11.48	.842609	12.28	47	.853736	10.93	.885910	11.7
48	.814077	11.47	.843346	12.27	48	.854392	10.93	.886617	11.7
49	.814765	11.45	.844082	12.25	49	.855048	10.92	.887323	11.7
50	.815452	11.45	.844817	12.25	50	.855703	10.92	.888029	11.7
51	8.816139	11.43	8.845552	12.25	51	8.856358	10.90	8.888734	11.7
52	.816825	11.43	.846287	12.23	52	.857012	10.90	.889439	11.7
53	.817511	11.42	.847021	12.22	53	.857666	10.88	.890144	11.7
54	.818196	11.42	.847754	12.22	54	.858319	10.88	.890848	11.72
55	.818881	11.40	.848487	12.22	55	.858972	10.87	.891551	11.72
56	.819565	11.40	.849220	12.20	56	.859624	10.87	.892254	11.72
57	.820249	11.38	.849952	12.18	57	.860276	10.85	.892957	11.70
58	.820932	11.37	.850683	12.18	58	.860927	10.85	.893659	11.70
59	.821614	11.37	.851414	12.17	59	.861578	10.83	.894361	11.
60	8.822296	11.35	8.852144	12.17	60	8.862228	10.82	8.895062	11

22°					23°			
Vers.	D. 1".	Ex. sec.	D. 1".	′	Vers.	D. 1".	Ex. sec.	D. 1".
8.862228	10.82	8.895062	11.68	0	8.900341	10.33	8.936315	11.23
.862877	10.83	.895763	11.67	1	.900961	10.35	.936989	11.23
.863527	10.80	.896463	11.67	2	.901582	10.32	.937663	11.22
.864175	10.80	.897163	11.65	3	.902201	10.33	.938336	11.22
.864823	10.80	.897862	11.65	4	.902821	10.32	.939009	11.22
.865471	10.78	.898561	11.63	5	.903440	10.30	.939682	11.20
.866118	10.78	.899259	11.63	6	.904058	10.30	.940354	11.20
.866765	10.77	.899957	11.63	7	.904676	10.28	.941026	11.20
.867411	10.77	.900655	11.62	8	.905293	10.28	.941698	11.18
.868057	10.75	.901352	11.60	9	.905910	10.28	.942369	11.17
.868702	10.73	.902048	11.62	10	.906527	10.27	.943039	11.18
8.869346	10.75	8.902745	11.58	11	8.907143	10.27	8.943710	11.15
.869991	10.72	.903440	11.60	12	.907759	10.25	.944379	11.17
.870634	10.72	.904136	11.57	13	.908374	10.25	.945049	11.15
.871277	10.72	.904830	11.58	14	.908989	10.23	.945718	11.13
.871920	10.70	.905525	11.57	15	.909603	10.23	.946386	11.13
.872562	10.70	.906219	11.55	16	.910217	10.22	.947054	11.13
.873204	10.68	.906912	11.55	17	.910830	10.22	.947722	11.12
.873845	10.68	.907605	11.53	18	.911443	10.22	.948389	11.12
.874486	10.67	.908298	11.53	19	.912056	10.20	.949056	11.12
.875126	10.67	.908990	11.52	20	.912668	10.18	.949723	11.10
8.875766	10.65	8.909681	11.52	21	8.913279	10.20	8.950389	11.10
.876405	10.65	.910372	11.52	22	.913891	10.17	.951055	11.08
.877044	10.63	.911063	11.52	23	.914501	10.17	.951720	11.08
.877682	10.63	.911754	11.48	24	.915111	10.17	.952385	11.07
.878320	10.62	.912443	11.50	25	.915721	10.17	.953049	11.07
.878957	10.62	.913133	11.48	26	.916331	10.15	.953713	11.07
.879594	10.60	.913822	11.47	27	.916940	10.13	.954377	11.05
.880230	10.60	.914510	11.47	28	.917548	10.13	.955040	11.05
.880866	10.58	.915198	11.47	29	.918156	10.13	.955703	11.05
.881501	10.58	.915886	11.45	30	.918764	10.12	.956366	11.03
8.882136	10.58	8.916573	11.45	31	8.919371	10.10	8.957028	11.03
.882771	10.57	.917260	11.43	32	.919977	10.12	.957690	11.02
.883405	10.55	.917946	11.43	33	.920584	10.10	.958351	11.02
.884038	10.55	.918632	11.43	34	.921190	10.08	.959012	11.00
.884671	10.53	.919318	11.42	35	.921795	10.08	.959672	11.00
.885303	10.53	.920003	11.40	36	.922400	10.07	.960332	11.00
.885935	10.53	.920687	11.42	37	.923004	10.07	.960992	10.98
.886567	10.52	.921372	11.38	38	.923608	10.07	.961651	10.98
.887198	10.52	.922055	11.40	39	.924212	10.05	.962310	10.98
.887829	10.50	.922739	11.37	40	.924815	10.05	.962969	10.97
8.888459	10.48	8.923421	11.38	41	8.925418	10.03	8.963627	10.97
.889088	10.48	.924104	11.37	42	.926020	10.03	.964285	10.95
.889717	10.48	.924786	11.35	43	.926622	10.03	.964942	10.95
.890346	10.47	.925467	11.37	44	.927224	10.02	.965599	10.95
.890974	10.47	.926149	11.33	45	.927825	10.00	.966256	10.93
.891602	10.45	.926829	11.35	46	.928425	10.00	.966912	10.93
.892229	10.45	.927510	11.32	47	.929025	10.00	.967568	10.92
.892856	10.43	.928189	11.33	48	.929625	9.98	.968223	10.92
.893482	10.43	.928869	11.32	49	.930224	9.98	.968878	10.92
.894108	10.42	.929548	11.30	50	.930823	9.97	.969533	10.90
8.894733	10.42	8.930226	11.32	51	8.931421	9.97	8.970187	10.90
.895358	10.42	.930905	11.28	52	.932019	9.97	.970841	10.88
.895983	10.40	.931582	11.30	53	.932617	9.95	.971494	10.88
.896607	10.38	.932260	11.27	54	.933214	9.95	.972147	10.88
.897230	10.38	.932936	11.28	55	.933811	9.93	.972800	10.87
.897853	10.38	.933613	11.27	56	.934407	9.93	.973452	10.87
.898476	10.37	.934289	11.27	57	.935003	9.92	.974104	10.87
.899098	10.35	.934965	11.25	58	.935598	9.92	.974756	10.85
.899719	10.37	.935640	11.25	59	.936193	9.92	.975407	10.85
8.900341	10.33	8.936315	11.23	60	8.936788	9.90	8.976058	10.8

24°					25°			
′	Vers.	D. 1″.	Ex. sec.	D. 1″.	′	Vers.	D. 1″.	Ex. sec.
0	8.936788	9.90	8.976058	10.83	0	8.971703	9.50	9.014428
1	.937382	9.90	.976708	10.83	1	.972273	9.48	.015056
2	.937976	9.88	.977358	10.83	2	.972842	9.48	.015685
3	.938569	9.88	.978008	10.82	3	.973411	9.48	.016312
4	.939162	9.87	.978657	10.82	4	.973980	9.47	.016940
5	.939754	9.87	.979306	10.80	5	.974548	9.47	.017567
6	.940346	9.87	.979954	10.80	6	.975116	9.45	.018194
7	.940938	9.85	.980602	10.80	7	.975683	9.45	.018821
8	.941529	9.85	.981250	10.80	8	.976250	9.43	.019447
9	.942120	9.83	.981898	10.78	9	.976816	9.43	.020073
10	.942710	9.83	.982545	10.77	10	.977382	9.43	.020698
11	8.943300	9.82	8.983191	10.77	11	8.977948	9.43	9.021323
12	.943889	9.83	.983837	10.77	12	.978514	9.42	.021948
13	.944479	9.80	.984483	10.77	13	.979079	9.40	.022572
14	.945067	9.80	.985129	10.75	14	.979643	9.40	.023197
15	.945655	9.80	.985774	10.75	15	.980207	9.40	.023820
16	.946243	9.80	.986419	10.73	16	.980771	9.40	.024444
17	.946831	9.78	.987063	10.73	17	.981335	9.38	.025067
18	.947418	9.77	.987707	10.73	18	.981898	9.37	.025690
19	.948004	9.77	.988351	10.72	19	.982460	9.38	.026312
20	.948590	9.77	.988994	10.72	20	.983023	9.37	.026934
21	8.949176	9.75	8.989637	10.70	21	8.983585	9.35	9.027556
22	.949761	9.75	.990279	10.72	22	.984146	9.35	.028177
23	.950346	9.75	.990922	10.68	23	.984707	9.35	.028798
24	.950931	9.73	.991563	10.70	24	.985268	9.33	.029419
25	.951515	9.73	.992205	10.68	25	.985828	9.33	.030039
26	.952099	9.72	.992846	10.68	26	.986388	9.33	.030659
27	.952682	9.72	.993487	10.67	27	.986948	9.32	.031279
28	.953265	9.70	.994127	10.67	28	.987507	9.32	.031899
29	.953847	9.70	.994767	10.65	29	.988066	9.32	.032518
30	.954429	9.70	.995406	10.67	30	.988625	9.30	.033136
31	8.955011	9.68	8.996046	10.65	31	8.989183	9.28	9.033755
32	.955592	9.68	.996685	10.63	32	.989740	9.30	.034373
33	.956173	9.67	.997323	10.63	33	.990298	9.28	.034991
34	.956753	9.68	.997961	10.63	34	.990855	9.27	.035608
35	.957334	9.65	.998599	10.62	35	.991411	9.28	.036225
36	.957913	9.65	.999236	10.62	36	.991968	9.25	.036842
37	.958492	9.65	8.999873	10.62	37	.992523	9.27	.037458
38	.959071	9.65	9.000510	10.60	38	.993079	9.25	.038074
39	.959650	9.63	.001146	10.62	39	.993634	9.25	.038690
40	.960228	9.62	.001783	10.58	40	.994189	9.23	.039305
41	8.960805	9.62	9.002418	10.58	41	8.994743	9.23	9.039920
42	.961382	9.62	.003053	10.58	42	.995297	9.23	.040535
43	.961959	9.60	.003688	10.58	43	.995851	9.22	.041150
44	.962535	9.60	.004323	10.57	44	.996404	9.22	.041764
45	.963111	9.60	.004957	10.57	45	.996957	9.20	.042378
46	.963687	9.58	.005591	10.55	46	.997509	9.22	.042991
47	.964262	9.58	.006224	10.57	47	.998062	9.18	.043604
48	.964837	9.57	.006858	10.53	48	.998613	9.20	.044217
49	.965411	9.57	.007490	10.55	49	.999165	9.18	.044830
50	.965985	9.57	.008123	10.53	50	8.999716	9.17	.045442
51	8.966559	9.55	9.008755	10.53	51	9.000266	9.18	9.046054
52	.967132	9.55	.009387	10.52	52	.000817	9.17	.046665
53	.967705	9.53	.010018	10.52	53	.001367	9.15	.047276
54	.968277	9.53	.010649	10.52	54	.001916	9.17	.047887
55	.968849	9.53	.011280	10.50	55	.002466	9.13	.048498
56	.969421	9.52	.011910	10.50	56	.003014	9.15	.049108
57	.969992	9.52	.012540	10.50	57	.003563	9.13	.049718
58	.970563	9.50	.013170	10.48	58	.004111	9.13	.05032
59	.971133	9.50	.013799	10.48	59	.004659	9.12	.05093
60	8.971703	9.50	9.014428	10.47	60	9.005206	9.12	9.0515

26°					27°			
Vers.	D. 1".	Ex. sec.	D. 1"	'	Vers.	D. 1".	Ex. sec.	D. 1".
9.005206	9.12	9.051546	10.15	0	9.037401	8.77	9.087520	9.83
.005753	9.12	.052155	10.13	1	.037927	8.75	.088110	9.83
.006300	9.10	.052763	10.13	2	.038452	8.77	.088700	9.83
.006846	9.10	.053371	10.13	3	.038978	8.75	.089290	9.83
.007392	9.10	.053979	10.12	4	.039503	8.73	.089880	9.82
.007938	9.08	.054586	10.12	5	.040027	8.75	.090469	9.82
.008483	9.08	.055193	10.12	6	.040552	8.73	.091058	9.82
.009028	9.07	.055800	10.10	7	.041076	8.72	.091647	9.80
.009572	9.07	.056406	10.10	8	.041599	8.73	.092235	9.80
.010116	9.07	.057012	10.10	9	.042123	8.72	.092823	9.80
.010660	9.05	.057618	10.10	10	.042646	8.70	.093411	9.78
9.011203	9.05	9.058224	10.08	11	9.043168	8.72	9.093998	9.80
.011746	9.05	.058829	10.08	12	.043691	8.70	.094586	9.78
.012289	9.03	.059434	10.07	13	.044213	8.70	.095173	9.77
.012831	9.03	.060038	10.08	14	.044735	8.68	.095759	9.78
.013373	9.03	.060643	10.07	15	.045256	8.68	.096346	9.77
.013915	9.02	.061247	10.05	16	.045777	8.68	.096932	9.77
.014456	9.02	.061850	10.07	17	.046298	8.67	.097518	9.75
.014997	9.02	.062454	10.05	18	.046818	8.67	.098103	9.77
.015538	9.00	.063057	10.03	19	.047338	8.67	.098689	9.75
.016078	9.00	.063659	10.05	20	.047858	8.65	.099274	9.73
9.016618	8.98	9.064262	10.03	21	9.048377	8.65	9.099858	9.75
.017157	9.00	.064864	10.03	22	.048896	8.65	.100443	9.73
.917697	8.97	.065466	10.02	23	.049415	8.63	.101027	9.73
.018235	8.98	.066067	10.02	24	.049933	8.63	.101611	9.72
.018774	8.97	.066668	10.02	25	.050451	8.63	.102194	9.73
.019312	8.97	.067269	10.02	26	.050969	8.63	.102778	9.72
.019850	8.95	.067870	10.00	27	.051487	8.62	.103361	9.70
.020387	8.95	.068470	10.00	28	.052004	8.60	.103943	9.72
.020924	8.95	.069070	10.00	29	.052520	8.62	.104526	9.70
.021461	8.93	.069670	9.98	30	.053037	8.60	.105108	9.70
9.021997	8.93	9.070269	9.98	31	9.053553	8.60	9.105690	9.68
.022533	8.93	.070868	9.98	32	.054069	8.58	.106271	9.70
.023069	8.92	.071467	9.97	33	.054584	8.58	.106853	9.68
.023604	8.92	.072065	9.97	34	.055099	8.58	.107434	9.68
.024139	8.90	.072663	9.97	35	.055614	8.58	.108015	9.67
.024673	8.92	.073261	9.97	36	.056129	8.57	.108595	9.67
.025208	8.90	.073859	9.95	37	.056643	8.57	.109175	9.67
.025742	8.88	.074456	9.95	38	.057157	8.55	.109755	9.67
.026275	8.88	.075053	9.93	39	.057670	8.55	.110335	9.65
.026808	8.88	.075649	9.95	40	.058183	8.55	.110914	9.67
9.027341	8.88	9.076246	9.93	41	9.058696	8.55	9.111494	9.63
.027874	8.87	.076842	9.92	42	.059209	8.53	.112072	9.65
.028406	8.87	.077437	9.93	43	.059721	8.53	.112651	9.63
.028938	8.85	.078033	9.92	44	.060233	8.53	.113229	9.63
.029469	8.85	.078628	9.92	45	.060745	8.52	.113807	9.63
.030000	8.85	.079223	9.90	46	.061256	8.52	.114385	9.63
.030531	8.85	.079817	9.92	47	.061767	8.50	.114963	9.62
.031062	8.83	.080412	9.90	48	.062277	8.52	.115540	9.62
.031592	8.83	.081006	9.88	49	.062788	8.50	.116117	9.60
.032122	8.82	.081599	9.90	50	.063298	8.48	.116693	9.62
9.032651	8.82	9.082193	9.88	51	9.063807	8.50	9.117270	9.60
.033180	8.82	.082786	9.87	52	.064317	8.48	.117846	9.60
.033709	8.80	.083378	9.88	53	.064826	8.48	.118422	9.58
.034237	8.80	.083971	9.87	54	.065335	8.47	.118997	9.60
.034765	8.80	.084563	9.87	55	.065843	8.47	.119573	9.58
.035293	8.78	.085155	9.87	56	.066351	8.47	.120148	9.58
.035820	8.78	.085747	9.85	57	.066859	8.45	.120723	9.57
.036347	8.78	.086338	9.85	58	.067366	8.47	.121297	9.57
.036874	8.78	.086929	9.85	59	.067874	8.43	.121871	9.5
9.037401	8.77	9.087520	9.83	60	9.068380	8.45	9.122445	

TABLE XXVI.—LOGARITHMIC VERSED SINES

28°					29°				
′	Vers.	D. 1″.	Ex. sec.	D. 1″.	′	Vers.	D. 1″.	Ex. sec.	D. 1″
0	9.068380	8.45	9.122445	9.57	0	9.098229	8.15	9.156410	9.3
1	.068887	8.43	.123019	9.57	1	.098718	8.13	.156968	9.3
2	.069393	8.43	.123593	9.55	2	.099206	8.12	.157527	9.
3	.069899	8.43	.124166	9.55	3	.099693	8.13	.158084	9.3
4	.070405	8.42	.124739	9.53	4	.100181	8.12	.158642	9.
5	.070910	8.42	.125311	9.55	5	.100668	8.12	.159200	9.
6	.071415	8.40	.125884	9.53	6	.101155	8.12	.159757	9.
7	.071919	8.42	.126456	9.53	7	.101642	8.10	.160314	9.
8	.072424	8.40	.127028	9.52	8	.102128	8.10	.160870	9.
9	.072928	8.40	.127599	9.53	9	.102614	8.10	.161427	9.
10	.073432	8.38	.128171	9.52	10	.103100	8.08	.161983	9.
11	9.073935	8.38	9.128742	9.52	11	9.103585	8.08	9.162539	9.
12	.074438	8.38	.129313	9.50	12	.104070	8.08	.163095	9.
13	.074941	8.37	.129883	9.50	13	.104555	8.08	.163650	9.
14	.075443	8.38	.130453	9.50	14	.105040	8.07	.164205	9.
15	.075946	8.35	.131023	9.50	15	.105524	8.07	.164760	9
16	.076447	8.37	.131593	9.50	16	.106008	8.05	.165315	9
17	.076949	8.35	.132163	9.48	17	.106491	8.07	.165870	9.
18	.077450	8.35	.132732	9.48	18	.106975	8.05	.166424	9.
19	.077951	8.35	.133301	9.48	19	.107458	8.05	.166978	9.
20	.078452	8.33	.133870	9.47	20	.107941	8.03	.167532	9
21	9.078952	8.33	9.134438	9.47	21	9.108423	8.05	9.168085	9.
22	.079452	8.33	.135006	9.47	22	.108906	8.03	.168639	9
23	.079952	8.32	.135574	9.47	23	.109388	8.02	.169192	9
24	.080451	8.32	.136142	9.45	24	.109869	8.03	.169745	9
25	.080950	8.32	.136709	9.47	25	.110351	8.02	.170297	9
26	.081449	8.32	.137277	9.45	26	.110832	8.02	.170850	9.
27	.081948	8.30	.137844	9.43	27	.111313	8.00	.171402	9.
28	.082446	8.30	.138410	9.45	28	.111793	8.00	.171954	9
29	.082944	8.28	.138977	9.43	29	.112273	8.00	.172505	9.
30	.083441	8.30	.139543	9.43	30	.112753	8.00	.173057	9.
31	9.083939	8.28	9.140109	9.42	31	9.113233	8.00	9.173608	9.
32	.084436	8.27	.140674	9.43	32	.113713	7.98	.174159	9.
33	.084932	8.28	.141240	9.42	33	.114192	7.98	.174710	9.
34	.085429	8.27	.141805	9.42	34	.114671	7.97	.175260	9.
35	.085925	8.25	.142370	9.40	35	.115149	7.97	.175810	9.
36	.086420	8.27	.142934	9.42	36	.115627	7.97	.176360	9.
37	.086916	8.25	.143499	9.40	37	.116105	7.97	.176910	9.
38	.087411	8.25	.144063	9.40	38	.116583	7.97	.177460	9.
39	.087906	8.23	.144627	9.38	39	.117061	7.95	.178009	9.
40	.088400	8.25	.145190	9.40	40	.117538	7.95	.178558	9.
41	9.088895	8.23	9.145754	9.38	41	9.118015	7.93	9.179107	9.
42	.089389	8.22	.146317	9.38	42	.118491	7.95	.179656	9.
43	.089882	8.23	.146880	9.37	43	.118968	7.93	.180204	9.
44	.090376	8.22	.147442	9.38	44	.119444	7.92	.180752	9.
45	.090869	8.22	.148005	9.37	45	.119919	7.90	.181300	9.
46	.091362	8.20	.148567	9.37	46	.120395	7.92	.181848	9.
47	.091854	8.20	.149129	9.35	47	.120870	7.92	.182395	9.
48	.092346	8.20	.149690	9.35	48	.121345	7.92	.182943	9.
49	.092838	8.20	.150251	9.37	49	.121820	7.90	.183490	9.
50	.093330	8.18	.150813	9.33	50	.122294	7.90	.184036	9.
51	9.093821	8.18	9.151373	9.35	51	9.122768	7.90	9.184583	9.
52	.094312	8.18	.151934	9.33	52	.123242	7.88	.185129	9.
53	.094803	8.17	.152494	9.35	53	.123715	7.90	.185675	9.
54	.095293	8.17	.153055	9.32	54	.124189	7.88	.186221	9.
55	.095783	8.17	.153614	9.33	55	.124662	7.87	.186767	9.
56	.096273	8.17	.154174	9.32	56	.125134	7.88	.187312	9.
57	.096763	8.15	.154733	9.33	57	.125607	7.87	.187858	9
58	.097252	8.15	.155293	9.30	58	.126079	7.87	.188403	9
59	.097741	8.13	.155851	9.32	59	.126551	7.85	.188947	
60	9.098229	8.15	9.156410	9.30	60	9.127022	7.87	9.189492	

30°					31°			
Vers.	D. 1".	Ex. sec.	D. 1".	′	Vers.	D. 1".	Ex. sec.	D. 1".
9.127022	7.87	9.189492	9.07	0	9.154828	7.58	9.221762	8.85
.127494	7.85	.190036	9.07	1	.155283	7.58	.222293	8.87
.127965	7.85	.190580	9.07	2	.155738	7.58	.222825	8.83
.128436	7.83	.191124	9.07	3	.156193	7.58	.223355	8.85
.128906	7.83	.191668	9.05	4	.156648	7.57	.223886	8.85
.129376	7.83	.192211	9.05	5	.157102	7.57	.224417	8.83
.129846	7.83	.192754	9.05	6	.157556	7.57	.224947	8.83
.130316	7.82	.193297	9.05	7	.158010	7.57	.225477	8.83
.130785	7.83	.193840	9.03	8	.158464	7.55	.226007	8.83
.131255	7.82	.194382	9.05	9	.158917	7.55	.226537	8.82
.131724	7.80	.194925	9.03	10	.159370	7.55	.227066	8.82
9.132192	7.80	9.195467	9.03	11	9.159823	7.55	9.227595	8.83
.132660	7.82	.196009	9.02	12	.160276	7.53	.228125	8.80
.133129	7.78	.196550	9.03	13	.160728	7.53	.228653	8.82
.133596	7.80	.197092	9.02	14	.161180	7.53	.229182	8.82
.134064	7.78	.197633	9.02	15	.161632	7.52	.229711	8.80
.134531	7.78	.198174	9.02	16	.162083	7.53	.230239	8.80
.134998	7.78	.198715	9.00	17	.162535	7.52	.230767	8.80
.135465	7.77	.199255	9.00	18	.162986	7.52	.231295	8.78
.135931	7.77	.199795	9.00	19	.163437	7.50	.231822	8.80
.136397	7.77	.200335	9.00	20	.163887	7.52	.232350	8.78
9.136863	7.77	9.200875	9.00	21	9.164338	7.50	9.232877	8.78
.137329	7.75	.201415	8.98	22	.164788	7.48	.233404	8.78
.137794	7.77	.201954	9.00	23	.165237	7.50	.233931	8.78
.138260	7.73	.202494	8.97	24	.165687	7.48	.234458	8.77
.138724	7.75	.203032	8.98	25	.166136	7.48	.234984	8.77
.139189	7.73	.203571	8.98	26	.166585	7.48	.235510	8.77
.139653	7.73	.204110	8.97	27	.167034	7.48	.236036	8.77
.140117	7.73	.204648	8.97	28	.167483	7.47	.236562	8.77
.140581	7.73	.205186	8.97	29	.167931	7.47	.237088	8.75
.141045	7.72	.205724	8.97	30	.168379	7.47	.237613	8.77
9.141508	7.72	9.206262	8.95	31	9.168827	7.47	9.238139	8.75
.141971	7.72	.206799	8.97	32	.169275	7.45	.238664	8.75
.142434	7.70	.207337	8.95	33	.169722	7.45	.239189	8.73
.142896	7.70	.207874	8.93	34	.170169	7.45	.239713	8.75
.143358	7.70	.208410	8.95	35	.170616	7.43	.240238	8.73
.143820	7.70	.208947	8.93	36	.171062	7.45	.240762	8.73
.144282	7.68	.209483	8.95	37	.171509	7.43	.241286	8.73
.144743	7.68	.210020	8.93	38	.171955	7.42	.241810	8.72
.145204	7.68	.210556	8.92	39	.172400	7.43	.242334	8.73
.145665	7.68	.211091	8.93	40	.172846	7.42	.242857	8.72
9.146126	7.67	9.211627	8.92	41	9.173291	7.42	9.243380	8.72
.146586	7.67	.212162	8.92	42	.173736	7.42	.243903	8.72
.147046	7.67	.212697	8.92	43	.174181	7.42	.244426	8.72
.147506	7.67	.213232	8.92	44	.174626	7.40	.244949	8.70
.147966	7.65	.213767	8.90	45	.175070	7.40	.245471	8.72
.148425	7.65	.214301	8.92	46	.175514	7.40	.245994	8.70
.148884	7.65	.214836	8.90	47	.175958	7.40	.246516	8.70
.149343	7.63	.215370	8.90	48	.176402	7.38	.247038	8.68
.149801	7.63	.215904	8.88	49	.176845	7.38	.247559	8.70
.150259	7.63	.216437	8.90	50	.177288	7.38	.248081	8.68
9.150717	7.63	9.216971	8.88	51	9.177731	7.38	9.248602	8.68
.151175	7.63	.217504	8.88	52	.178174	7.37	.249123	8.68
.151633	7.62	.218037	8.88	53	.178616	7.37	.249644	8.68
.152090	7.62	.218570	8.87	54	.179058	7.37	.250165	8.68
.152547	7.60	.219102	8.88	55	.179500	7.37	.250686	8.67
.153003	7.62	.219635	8.87	56	.179942	7.35	.251206	8.67
.153460	*7.60*	.220167	8.87	57	.180383	7.37	.251726	8.67
153916	*7.60*	.220699	8.87	58	.180825	7.33	.252246	8.67
.154372	*7.60*	.221231	8.85	59	.181265	7.35	.252766	8.6
9.154828	*7.58*	9.221762	8.85	60	9.181706	7.35	9.253286	

TABLE XXVI.—LOGARITHMIC VERSED SINES

44°					45°				
′	Vers.	D. 1″.	Ex. sec.	D. 1″.	′	Vers.	D. 1″.	Ex. sec.	D. 1″.
0	9.448181	5.20	9.591247	7.23	0	9.466709	5.08	9.617224	7.20
1	.448493	5.22	.591681	7.25	1	.467014	5.08	.617656	7.18
2	.448806	5.20	.592116	7.25	2	.467319	5.08	.618087	7.18
3	.449118	5.22	.592551	7.23	3	.467624	5.07	.618518	7.18
4	.449431	5.20	.592985	7.23	4	.467928	5.08	.618949	7.18
5	.449743	5.20	.593419	7.25	5	.468233	5.07	.619380	7.18
6	.450055	5.18	.593854	7.23	6	.468537	5.07	.619811	7.18
7	.450366	5.20	.594288	7.23	7	.468841	5.07	.620242	7.18
8	.450678	5.20	.594722	7.23	8	.469145	5.07	.620673	7.18
9	.450990	5.18	.595156	7.25	9	.469449	5.07	.621104	7.18
10	.451301	5.18	.595591	7.23	10	.469753	5.07	.621535	7.18
11	9.451612	5.20	9.596025	7.23	11	9.470057	5.05	9.621966	7.17
12	.451924	5.18	.596459	7.23	12	.470360	5.07	.622396	7.18
13	.452235	5.18	.596893	7.22	13	.470664	5.05	.622827	7.18
14	.452546	5.17	.597326	7.23	14	.470967	5.05	.623258	7.17
15	.452856	5.17	.597760	7.23	15	.471270	5.05	.623688	7.18
16	.453167	5.18	.598194	7.23	16	.471573	5.05	.624119	7.17
17	.453478	5.17	.598628	7.22	17	.471876	5.05	.624549	7.18
18	.453788	5.17	.599061	7.23	18	.472179	5.05	.624980	7.17
19	.454098	5.17	.599495	7.22	19	.472482	5.03	.625410	7.18
20	.454408	5.17	.599928	7.23	20	.472784	5.05	.625841	7.17
21	9.454718	5.17	9.600362	7.22	21	9.473087	5.03	9.626271	7.17
22	.455028	5.17	.600795	7.23	22	.473389	5.03	.626701	7.17
23	.455338	5.17	.601229	7.22	23	.473691	5.03	.627131	7.17
24	.455648	5.15	.601662	7.22	24	.473993	5.03	.627561	7.17
25	.455957	5.17	.602095	7.22	25	.474295	5.03	.627991	7.17
26	.456267	5.15	.602528	7.23	26	.474597	5.03	.628421	7.17
27	.456576	5.15	.602962	7.22	27	.474899	5.02	.628851	7.17
28	.456885	5.15	.603395	7.22	28	.475200	5.03	.629281	7.17
29	.457194	5.15	.603828	7.22	29	.475502	5.02	.629711	7.17
30	.457503	5.13	.604261	7.22	30	.475803	5.02	.630141	7.17
31	9.457811	5.15	9.604694	7.20	31	9.476104	5.02	9.630571	7.17
32	.458120	5.15	.605126	7.22	32	.476405	5.02	.631001	7.15
33	.458429	5.13	.605559	7.22	33	.476706	5.02	.631430	7.17
34	.458737	5.13	.605992	7.22	34	.477007	5.02	.631860	7.17
35	.459045	5.13	.606425	7.20	35	.477308	5.00	.632290	7.15
36	.459353	5.13	.606857	7.22	36	.477608	5.02	.632719	7.17
37	.459661	5.13	.607290	7.20	37	.477909	5.00	.633149	7.15
38	.459969	5.13	.607722	7.22	38	.478209	5.00	.633578	7.17
39	.460277	5.12	.608155	7.20	39	.478509	5.00	.634008	7.15
40	.460584	5.13	.608587	7.22	40	.478809	5.00	.634437	7.15
41	9.460892	5.12	9.609020	7.20	41	9.479109	5.00	9.634866	7.17
42	.461199	5.12	.609452	7.20	42	.479409	5.00	.635296	7.15
43	.461506	5.12	.609884	7.20	43	.479709	5.00	.635725	7.15
44	.461813	5.12	.610316	7.22	44	.480009	4.98	.636154	7.15
45	.462120	5.12	.610749	7.20	45	.480308	5.00	.636583	7.15
46	.462427	5.12	.611181	7.20	46	.480608	4.98	.637012	7.15
47	.462734	5.10	.611613	7.20	47	.480907	4.98	.637441	7.15
48	.463040	5.12	.612045	7.20	48	.481206	4.98	.637870	7.15
49	.463347	5.10	.612477	7.18	49	.481505	4.98	.638299	7.15
50	.463653	5.10	.612908	7.20	50	.481804	4.98	.638728	7.15
51	9.463959	5.10	9.613340	7.20	51	9.482103	4.97	9.639157	7.15
52	.464265	5.10	.613772	7.20	52	.482401	4.98	.639586	7.15
53	.464571	5.10	.614204	7.18	53	.482700	4.97	.640015	7.13
54	.464877	5.10	.614635	7.20	54	.482998	4.97	.640443	7.15
55	.465183	5.08	.615067	7.20	55	.483296	4.98	.640872	7.15
56	.465488	5.10	.615499	7.18	56	.483595	4.97	.641301	7.13
57	.465794	5.08	.615930	7.20	57	.483893	4.97	.641729	7.15
58	.466099	5.08	.616362	7.18	58	.484191	4.95	.642158	7.15
59	.466404	5.08	.616793	7.18	59	.484488	4.97	.642586	7.13
60	9.466709	5.08	9.617224	7.20	60	9.484786	4.97	9.643015	7.1

34°					35°				
′	Vers.	D. 1″.	Ex. sec.	D. 1″.	′	Vers.	D. 1″.	Ex. sec.	D. 1″.
0	9.232901	6.88	9.314326	8.32	0	9.257314	6.67	9.343949	8.15
1	.233314	6.88	.314825	8.30	1	.257714	6.68	.344438	8.15
2	.233727	6.87	.315323	8.30	2	.258115	6.67	.344927	8.15
3	.234139	6.88	.315821	8.30	3	.258515	6.67	.345416	8.13
4	.234552	6.87	.316319	8.30	4	.258915	6.65	.345904	8.15
5	.234964	6.87	.316817	8.28	5	.259314	6.67	.346393	8.13
6	.235376	6.87	.317314	8.28	6	.259714	6.65	.346881	8.13
7	.235788	6.85	.317811	8.30	7	.260113	6.65	.347369	8.13
8	.236199	6.87	.318309	8.28	8	.260512	6.65	.347857	8.13
9	.236611	6.85	.318806	8.28	9	.260911	6.65	.348345	8.13
10	.237022	6.85	.319303	8.27	10	.261310	6.65	.348833	8.13
11	9.237433	6.85	9.319799	8.28	11	9.261709	6.63	9.349321	8.12
12	.237844	6.83	.320296	8.27	12	.262107	6.63	.349808	8.12
13	.238254	6.85	.320792	8.28	13	.262505	6.63	.350295	8.12
14	.238665	6.83	.321289	8.27	14	.262903	6.63	.350782	8.12
15	.239075	6.83	.321785	8.27	15	.263301	6.62	.351269	8.12
16	.239485	6.82	.322281	8.25	16	.263698	6.63	.351756	8.12
17	.239894	6.83	.322776	8.27	17	.264096	6.62	.352243	8.12
18	.240304	6.82	.323272	8.27	18	.264493	6.62	.352730	8.10
19	.240713	6.82	.323763	8.25	19	.264890	6 62	.353216	8.10
20	.241122	6.82	.324263	8.25	20	.265287	6.60	.353702	8.10
21	9.241531	6.82	9.324758	8.25	21	9.265683	6.62	9.354188	8.10
22	.241940	6.82	.325253	8.25	22	.266080	6.60	.354674	8.10
23	.242348	6.80	.325748	8 25	23	.266476	6.60	.355160	8.10
24	.242756	6.80	.326243	8.23	24	.266872	6.58	.355646	8.08
25	.243164	6.80	.326737	8.25	25	.267267	6.60	.356131	8.10
26	.243572	6.80	.327232	8.23	26	.267663	6.58	.356617	8.08
27	.243980	6.78	.327725	8.23	27	.268058	6.58	.357102	8.08
28	.244387	6.78	.328220	8.23	28	.268453	6.58	.357587	8.08
29	.244794	6.78	.328714	8.22	29	.268848	6.58	.358072	8.08
30	.245201	6.78	.329207	8.23	30	.269243	6.58	.358557	8.08
31	9.245608	6.77	9.329701	8.23	31	9.269638	6.57	9.359042	8.07
32	.246014	6.78	.330195	8.22	32	.270032	6.57	.359526	8.08
33	.246421	6.77	.330688	8.22	33	.270426	6.57	.360011	8.07
34	.246827	6.77	.331181	8.22	34	.270820	6.57	.360495	8.07
35	.247233	6.77	.331674	8.22	35	.271214	6.57	.360979	8.07
36	.247639	6.75	.332167	8.20	36	.271608	6.55	.361463	8.07
37	.248044	6.75	.332659	8.22	37	.272001	6.55	.361947	8.07
38	.248449	6.75	.333152	8.20	38	.272394	6.55	.362431	8.05
39	.248854	6.75	.333644	8.22	39	.272787	6.55	.362914	8.07
40	.249259	6.75	.334137	8.20	40	.273180	6.53	.363398	8.05
41	9.249664	6.73	9.334629	8.20	41	9.273572	6.55	9.363881	8.05
42	.250068	6.75	.335121	8.18	42	.273965	6.53	.364364	8.05
43	.250473	6.73	.335612	8.20	43	.274357	6.53	.364847	8.05
44	.250877	6.73	.336104	8.18	44	.274749	6.53	.365330	8.05
45	.251281	6.72	.336595	8.20	45	.275141	6.52	.365813	8.03
46	.251684	6.73	.337087	8.18	46	.275532	6.53	.366295	8.05
47	.252088	6.72	.337578	8.18	47	.275924	6.52	.366778	8.03
48	.252491	6.72	.338069	8.18	48	.276315	6.52	.367260	8.03
49	.252894	6.72	.338560	8.17	49	.276706	6.52	.367742	8.03
50	.253297	6.70	.339050	8.18	50	.277097	6.52	.368224	8.03
51	9.253699	6.72	9.339541	8.17	51	9.277488	6.50	9.368706	8.03
52	.254102	6.70	.340031	8.18	52	.277878	6.50	.369188	8.03
53	.254504	6.70	.340522	8.17	53	.278268	6.50	.369670	8.02
54	.254906	6.70	.341012	8.17	54	.278658	6.50	.370151	8.02
55	.255308	6.68	.341502	8.15	55	.279048	6.50	.370632	8.03
56	.255709	6.70	.341991	8.17	56	.279438	6.48	.371114	8.02
57	.256111	6.68	.342481	8.17	57	.279827	6.50	.371595	8.0
58	.256512	6.68	.342971	8.15	58	.280217	6.48	.372076	8
59	.256913	6.68	.343460	8.15	59	.280606	6.48	.372556	
60	9.257314	6.67	9.343949	8.15	60	9.280995	6.47	9.373037	

36°					37°				
′	Vers.	D. 1.	Ex. sec.	D. 1″.	′	Vers.	D. 1″.	Ex. sec.	D. 1″.
0	9.280995	6.47	9.373037	8.02	0	9.303983	6.28	9.401634	7.88
1	.281383	6.48	.373518	8.00	1	.304360	6.30	.402107	7.88
2	.281772	6.47	.373998	8.00	2	.304738	6.28	.402580	7.87
3	.282160	6.47	.374478	8.00	3	.305115	6.28	.403052	7.87
4	.282548	6.47	.374958	8.00	4	.305492	6.27	.403524	7.88
5	.282936	6.47	.375438	8.00	5	.305868	6.28	.403997	7.88
6	.283324	6.47	.375918	8.00	6	.306245	6.27	.404469	7.87
7	.283712	6.45	.376398	7.98	7	.306621	6.28	.404941	7.87
8	.284099	6.45	.376877	8.00	8	.306998	6.27	.405412	7.87
9	.284486	6.45	.377357	7.98	9	.307374	6.25	.405884	7.87
10	.284873	6.45	.377836	7.98	10	.307749	6.27	.406356	7.85
11	9.285260	6.45	9.378315	7.98	11	9.308125	6.27	9.406827	7.85
12	.285647	6.43	.378794	7.98	12	.308501	6.25	.407298	7.87
13	.286033	6.43	.379273	7.98	13	.308876	6.25	.407770	7.85
14	.286419	6.43	.379752	7.98	14	.309251	6.25	.408241	7.85
15	.286805	6.43	.380231	7.97	15	.309626	6.25	.408712	7.85
16	.287191	6.43	.380709	7.98	16	.310001	6.23	.409183	7.83
17	.287577	6.42	.381188	7.97	17	.310375	6.25	.409653	7.85
18	.287962	6.40	.381666	7.97	18	.310750	6.23	.410124	7.83
19	.288348	6.42	.382144	7.97	19	.311124	6.23	.410594	7.85
20	.288733	6.42	.382622	7.97	20	.311498	6.23	.411065	7.83
21	9.289118	6.40	9.383100	7.95	21	9.311872	6.22	9.411535	7.83
22	.289502	6.42	.383577	7.97	22	.312245	6.23	.412005	7.83
23	.289887	6.40	.384055	7.95	23	.312619	6.22	.412475	7.83
24	.290271	6.40	.384532	7.97	24	.312992	6.22	.412945	7.83
25	.290655	6.40	.385010	7.95	25	.313365	6.22	.413415	7.82
26	.291039	6.40	.385487	7.95	26	.313738	6.22	.413884	7.83
27	.291423	6.40	.385964	7.95	27	.314111	6.22	.414354	7.82
28	.291807	6.38	.386441	7.95	28	.314484	6.20	.414823	7.83
29	.292190	6.38	.386918	7.93	29	.314856	6.20	.415293	7.82
30	.292573	6.38	.387394	7.95	30	.315228	6.20	.415762	7.82
31	9.292956	6.38	9.387871	7.93	31	9.315600	6.20	9.416231	7.82
32	.293339	6.38	.388347	7.95	32	.315972	6.20	.416700	7.80
33	.293722	6.37	.388824	7.93	33	.316344	6.20	.417168	7.82
34	.294104	6.37	.389300	7.93	34	.316716	6.18	.417637	7.82
35	.294486	6.37	.389776	7.93	35	.317087	6.18	.418106	7.80
36	.294868	6.37	.390252	7.92	36	.317458	6.18	.418574	7.80
37	.295250	6.37	.390727	7.93	37	.317829	6.18	.419042	7.82
38	.295632	6.37	.391203	7.92	38	.318200	6.18	.419511	7.80
39	.296014	6.35	.391678	7.93	39	.318571	6.17	.419979	7.80
40	.296395	6.35	.392154	7.92	40	.318941	6.17	.420447	7.80
41	9.296776	6.35	9.392629	7.92	41	9.319311	6.18	9.420915	7.78
42	.297157	6.35	.393104	7.92	42	.319682	6.15	.421382	7.80
43	.297538	6.33	.393579	7.92	43	.320051	6.17	.421850	7.78
44	.297918	6.35	.394054	7.92	44	.320421	6.17	.422317	7.80
45	.298299	6.33	.394529	7.90	45	.320791	6.15	.422785	7.78
46	.298679	6.33	.395003	7.92	46	.321160	6.17	.423252	7.78
47	.299059	6.33	.395478	7.90	47	.321530	6.15	.423719	7.78
48	.299439	6.33	.395952	7.90	48	.321899	6.13	.424186	7.78
49	.299819	6.32	.396426	7.90	49	.322267	6.15	.424653	7.78
50	.300198	6.32	.396900	7.90	50	.322636	6.15	.425120	7.78
51	9·300577	6.33	9.397374	7.90	51	9.323005	6.13	9.425587	7.77
52	.300957	6.30	.397848	7.90	52	.323373	6.13	.426053	7.78
53	.301335	6.32	.398322	7.88	53	.323741	6.13	.426520	7.77
54	.301714	6.32	.398795	7.90	54	.324109	6.13	.426986	7.77
55	.302093	6.30	.399269	7.88	55	.324477	6.13	.427452	7.77
56	.302471	6.30	.399742	7.88	56	.324845	6.12	.427918	7.77
7	.302849	6.30	.400215	7.88	57	.325212	6.13	.428384	7.77
	.303227	6.30	.400688	7.88	58	.325580	6.12	.428850	7.77
	.303605	6.30	.401161	7.88	59	.325947	6.12	.429316	7.77
	9.303983	6.28	9.401634	7.88	60	9.326314	6.12	9.429782	7.7

38°					39°				
′	Vers.	D. 1″.	Ex. sec.	D. 1″.	′	Vers.	D. 1″.	Ex. sec.	D. 1″.
0	9.326314	6.12	9.429782	7.75	0	9.348021	5.93	9.457518	7.65
1	.326681	6.10	.430247	7.77	1	.348377	5.95	.457977	7.65
2	.327047	6.12	.430713	7.75	2	.348734	5.93	.458436	7.65
3	.327414	6.10	.431178	7.75	3	.349090	5.93	.458895	7.63
4	.327780	6.10	.431643	7.75	4	.349446	5.93	.459353	7.65
5	.328146	6.10	.432108	7.75	5	.349802	5.93	.459812	7.63
6	.328512	6.10	.432573	7.75	6	.350158	5.93	.460270	7.65
7	.328878	6.08	.433038	7.75	7	.350514	5.92	.460729	7.63
8	.329243	6.10	.433503	7.73	8	.350869	5.93	.461187	7.63
9	.329609	6.08	.433967	7.75	9	.351225	5.92	.461645	7.63
10	.329974	6.08	.434432	7.73	10	.351580	5.92	.462103	7.63
11	9.330339	6.08	9.434896	7.75	11	9.351935	5.92	9.462561	7.63
12	.330704	6.08	.435361	7.73	12	.352290	5.90	.463019	7.63
13	.331069	6.07	.435825	7.73	13	.352644	5.92	.463477	7.62
14	.331433	6.08	.436289	7.73	14	.352999	5.90	.463934	7.63
15	.331798	6.07	.436753	7.73	15	.353353	5.90	.464392	7.62
16	.332162	6.07	.437217	7.72	16	.353707	5.92	.464849	7.63
17	.332526	6.07	.437680	7.73	17	.354062	5.88	.465307	7.62
18	.332890	6.07	.438144	7.73	18	.354415	5.90	.465764	7.62
19	.333254	6.05	.438608	7.72	19	.354769	5.90	.466221	7.62
20	.333617	6.07	.439071	7.72	20	.355123	5.88	.466678	7.62
21	9.333981	6.05	9.439534	7.72	21	9.355476	5.88	9.467135	7.62
22	.334344	6.05	.439997	7.72	22	.355829	5.88	.467592	7.62
23	.334707	6.05	.440460	7.72	23	.356182	5.88	.468049	7.62
24	.335070	6.03	.440923	7.72	24	.356535	5.88	.468506	7.60
25	.335432	6.05	.441386	7.72	25	.356888	5.88	.468962	7.60
26	.335795	6.03	.441849	7.72	26	.357241	5.87	.469418	7.62
27	.336157	6.03	.442312	7.70	27	.357593	5.87	.469875	7.60
28	.336519	6.03	.442774	7.72	28	.357945	5.87	.470331	7.60
29	.336881	6.03	.443237	7.70	29	.358297	5.87	.470787	7.60
30	.337243	6.03	.443699	7.70	30	.358649	5.87	.471243	7.60
31	9.337605	6.02	9.444161	7.70	31	9.359001	5.87	9.471699	7.60
32	.337966	6.03	.444623	7.70	32	.359353	5.85	.472155	7.60
33	.338328	6.02	.445085	7.70	33	.359704	5.87	.472611	7.60
34	.338689	6.02	.445547	7.70	34	.360056	5.85	.473067	7.58
35	.339050	6.02	.446009	7.68	35	.360407	5.85	.473522	7.60
36	.339411	6.00	.446470	7.70	36	.360758	5.83	.473978	7.58
37	.339771	6.02	.446932	7.68	37	.361108	5.85	.474433	7.58
38	.340132	6.00	.447393	7.70	38	.361459	5.85	.474888	7.58
39	.340492	6.00	.447855	7.68	39	.361810	5.83	.475343	7.58
40	340852	6.00	.448316	7.68	40	.362160	5.83	.475798	7.58
41	9.341212	6.00	9.448777	7.68	41	9.362510	5.83	9.476253	7.58
42	.341572	6.00	.449238	7.68	42	.362860	5.83	.476708	7.58
43	.341932	5.98	.449699	7.68	43	.363210	5.83	.477163	7.58
44	.342291	6.00	.450160	7.67	44	.363560	5.82	.477618	7.57
45	.342651	5.98	.450620	7 68	45	.363909	5.83	.478072	7.58
46	.343010	5.98	.451081	7.67	46	.364259	5.82	.478527	7.57
47	.343369	5.98	.451541	7.68	47	.364608	5.82	.478981	7.57
48	.343728	5.97	.452002	7.67	48	.364957	5.82	.479435	7.58
49	.344086	5.98	.452462	7.67	49	.365306	5.82	.479890	7.57
50	.344445	5.97	.452922	7.67	50	.365655	5.80	.480344	7.57
51	9.344803	5.97	9.453382	7.67	51	9.366003	5.82	9.480798	7.57
52	.345161	5.97	.453842	7.67	52	.366352	5.80	.481252	7.55
53	.345519	5.97	.454302	7.67	53	.366700	5.80	.481705	7.57
54	.345877	5.97	.454762	7.65	54	.367048	5.80	.482159	7.57
55	.346235	5.95	.455221	7.67	55	.367396	5.80	.482613	7.55
56	.346592	5.97	.455681	7.65	56	.367744	5.78	.483066	7.57
57	.346950	5.95	.456140	7.67	57	.368091	5.80	.483520	[illegible]
58	.347307	5.95	.456600	7.65	58	.368439	5.78	.483973	[illegible]
59	.347664	5.95	.457059	7.65	59	.368786	5.78	.484426	[illegible]
60	9.348021	5.93	9.457518	7.65	60	9.369133	5.78	9.484879	[illegible]

TABLE XXVI.—LOGARITHMIC VERSED SINES

40°					41°				
′	Vers.	D. 1″.	Ex. sec.	D. 1″.	′	Vers.	D. 1″.	Ex. sec.	D. 1″.
0	9.369133	5.78	9.484879	7.55	0	9.389681	5.62	9.511901	7.45
1	.369480	5.78	.485332	7.55	1	.390018	5.63	.512348	7.47
2	.369827	5.78	.485785	7.55	2	.390356	5.63	.512796	7.45
3	.370174	5.77	.486238	7.55	3	.390694	5.62	.513243	7.47
4	.370520	5.78	.486691	7.55	4	.391031	5.62	.513691	7.45
5	.370867	5.77	.487144	7.53	5	.391368	5.62	.514138	7.45
6	.371213	5.77	.487596	7.55	6	.391705	5.62	.514585	7.47
7	.371559	5.77	.488049	7.53	7	.392042	5.62	.515033	7.45
8	.371905	5.77	.488501	7.53	8	.392379	5.62	.515480	7.45
9	.372251	5.75	.488953	7.55	9	.392716	5.60	.515927	7.45
10	.372596	5.77	.489406	7.53	10	.393052	5.60	.516374	7.43
11	9.372942	5.75	9.489858	7.53	11	9.393388	5.60	9.516820	7.45
12	.373287	5.75	.490310	7.53	12	.393724	5.62	.517267	7.45
13	.373632	5.75	.490762	7.53	13	.394061	5.58	.517714	7.43
14	.373977	5.75	.491214	7.52	14	.394396	5.60	.518160	7.45
15	.374322	5.75	.491665	7.53	15	.394732	5.60	.518607	7.43
16	.374667	5.73	.492117	7.53	16	.395068	5.58	.519053	7.45
17	.375011	5.75	.492569	7.52	17	.395403	5.58	.519500	7.43
18	.375356	5.73	.493020	7.52	18	.395738	5.60	.519946	7.43
19	.375700	5.73	.493471	7.53	19	.396074	5.58	.520392	7.43
20	.376044	5.73	.493923	7.52	20	.396409	5.57	.520838	7.43
21	9.376388	5.73	9.494374	7.52	21	9.396743	5.58	9.521284	7.43
22	.376732	5.72	.494825	7.52	21	.397078	5.58	.521730	7.43
23	.377075	5.73	.495276	7.52	23	.397413	5.57	.522176	7.42
24	.377419	5.72	.495727	7.52	24	.397747	5.57	.522621	7.43
25	.377762	5.72	.496178	7.50	25	.398081	5.57	.523067	7.43
26	.378105	5.72	.496628	7.52	26	.398415	5.57	.523513	7.42
27	.378448	5.72	.497079	7.52	27	.398749	5.57	.523958	7.43
28	.378791	5.70	.497530	7.50	28	.399083	5.57	.524401	7.42
29	.379133	5.72	.497980	7.52	29	.399417	5.55	.524849	7.42
30	.379476	5.70	.498430	7.50	30	.399750	5.57	.525294	7.42
31	9.379818	5.72	9.498881	7.48	31	9.400084	5.55	9.525739	7.42
32	.380161	5.70	.499331	7.52	32	.400417	5.55	.526184	7.42
33	.380503	5.70	.499781	7.50	33	.400750	5.55	.526629	7.42
34	.380845	5.68	.500231	7.50	34	.401083	5.55	.527074	7.42
35	.381186	5.70	.500681	7.50	35	.401416	5.53	.527519	7.42
36	.381528	5.68	.501131	7.50	36	.401748	5.55	.527964	7.42
37	.381869	5.70	.501581	7.48	37	.402081	5.53	.528409	7.40
38	.382211	5.68	.502030	7.50	38	.402413	5.53	.528853	7.42
39	.382552	5.68	.502480	7.48	39	.402745	5.53	.529298	7.40
40	.382893	5.68	.502929	7.50	40	.403077	5.53	.529742	7.42
41	9.383234	5.67	9.503379	7.48	41	9.403409	5.53	9.530187	7.40
42	.383574	5.68	.503828	7.48	42	.403741	5.53	.530631	7.40
43	.383915	5.67	.504277	7.48	43	.404073	5.52	.531075	7.40
44	.384255	5.67	.504726	7.48	44	.404404	5.53	.531519	7.40
45	.384595	5.67	.505175	7.48	45	.404736	5.52	.531963	7.40
46	.384935	5.67	.505624	7.48	46	.405067	5.52	.532407	7.40
47	.385275	5.67	.506073	7.48	47	.405398	5.52	.532851	7.40
48	.385615	5.67	.506522	7.48	48	.405729	5.50	.533295	7.40
49	.385955	5.65	.506971	7.47	49	.406059	5.52	.533739	7.38
50	.386294	5.67	.507419	7.48	50	.406390	5.52	.534182	7.40
51	9.386634	5.65	9.507868	7.47	51	9.406721	5.50	9.534626	7.40
52	.386973	5.65	.508316	7.48	52	.407051	5.50	.535070	7.38
53	.387312	5.65	.508765	7.47	53	.407381	5.50	.535513	7.38
54	.387651	5.63	.509213	7.47	54	.407711	5.50	.535956	7.40
55	.387989	5.65	.509661	7.47	55	.408041	5.50	.536400	7.38
[illegible]	.388328	5.63	.510109	7.47	56	.408371	5.48	.536843	7.38
[illegible]	.388666	5.65	.510557	7.47	57	.408700	5.50	.537286	7.38
[illegible]	.389005	5.63	.511005	7.47	58	.409030	5.48	.537729	7.38
[illegible]	.389343	5.63	.511453	7.47	59	.409359	5.48	.538172	[illegible]
[illegible]	9.389681	5.62	9.511901	7.45	60	9.409688	5.43	9.538615	[illegible]

54°					55°				
′	Vers.	D. 1″.	Ex. sec.	D. 1″.	′	Vers.	D. 1″.	Ex. sec.	D. 1″.
0	9.615124	4.12	9.845905	7.03	0	9.629841	4.05	9.871250	7.05
1	.615371	4.13	.846327	7.03	1	.630084	4.03	.871673	7.05
2	.615619	4.13	.846749	7.02	2	.630326	4.05	.872096	7.05
3	.615867	4.13	.847170	7.03	3	.630569	4.03	.872519	7.05
4	.616115	4.12	.847592	7.03	4	.630811	4.05	.872942	7.07
5	.616362	4.13	.848014	7.03	5	.631054	4.03	.873366	7.05
6	.616610	4.12	.848436	7.03	6	.631296	4.03	.873789	7.05
7	.616857	4.12	.848858	7.03	7	.631538	4.03	.874212	7.07
8	.617104	4.12	.849280	7.03	8	.631780	4.03	.874636	7.05
9	.617351	4.13	.849702	7.03	9	.632022	4.03	.875059	7.05
10	.617599	4.10	.850124	7.03	10	.632264	4.02	.875482	7.07
11	9.617845	4.12	9.850546	7.03	11	9.632505	4.03	9.875906	7.05
12	.618092	4.12	.850968	7.03	12	.632747	4.03	.876329	7.05
13	.618339	4.12	.851390	7.03	13	.632989	4.02	.876752	7.07
14	.618586	4.12	.851812	7.03	14	.633230	4.03	.877176	7.05
15	.618833	4.10	.852234	7.03	15	.633472	4.02	.877599	7.07
16	.619079	4.12	.852656	7.03	16	.633713	4.02	.878023	7.05
17	.619326	4.10	.853078	7.03	17	.633954	4.03	.878446	7.07
18	.619572	4.10	.853500	7.05	18	.634196	4.02	.878870	7.07
19	.619818	4.12	.853923	7.03	19	.634437	4.02	.879294	7.05
20	.620065	4.10	.854345	7.03	20	.634678	4.02	.879717	7.07
21	9.620311	4.10	9.854767	7.03	21	9.634919	4.00	9.880141	7.07
22	.620557	4.10	.855189	7.05	22	.635159	4.02	.880565	7.05
23	.620803	4.08	.855612	7.03	23	.635400	4.02	.880988	7.07
24	.621048	4.10	.856034	7.03	24	.635641	4.00	.881412	7.07
25	.621294	4.10	.856456	7.03	25	.635881	4.02	.881836	7.07
26	.621540	4.10	.856878	7.05	26	.636122	4.00	.882260	7.05
27	.621786	4.08	.857301	7.03	27	.636362	4.02	.882683	7.07
28	.622031	4.08	.857723	7 03	28	.636603	4.00	.883107	7.07
29	.622276	4.10	.858145	7.05	29	.636843	4.00	.883531	7.07
30	.622522	4.08	.858568	7.03	30	.637083	4.00	.883955	7.07
31	9.622767	4.08	9.858990	7.05	31	9.637323	4.00	9.884379	7.07
32	.623012	4.08	.859413	7.03	32	.637563	4.00	.884803	7.07
33	.623257	4.08	.859835	7.05	33	.637803	4.00	.885227	7.07
34	.623502	4.08	.860258	7.03	34	.638043	4.00	.885651	7.07
35	.623747	4.08	.860680	7.05	35	.638283	3.98	.886075	7.07
36	.623992	4.08	.861103	7.03	36	.638522	4.00	.886499	7.07
37	.624237	4.07	.861525	7.05	37	.638762	3.98	.886923	7.07
38	.624481	4.08	.861948	7.03	38	.639001	4.00	.887347	7.08
39	.624726	4.07	.862370	7.05	39	.639241	3.98	.887772	7.07
40	.624970	4.08	.862793	7.03	40	.639480	3.98	.888196	7.07
41	9.625215	4 07	9.863215	7.05	41	9.639719	3.98	9.888620	7.07
42	.625459	4.07	.863638	7.05	42	.639958	3.98	.889044	7.08
43	.625703	4.07	.864061	7.03	43	.640197	3.98	.889469	7.07
44	.625947	4.07	.864483	7.05	44	.640436	3.98	.889893	7.07
45	.626191	4.07	.864906	7.05	45	.640675	3.98	.890317	7.08
46	.626435	4.07	.865329	7.05	46	.640914	3.98	.890742	7.07
47	.626679	4.07	.865752	7.03	47	.641153	3.97	.891166	7.08
48	.626923	4.05	.866174	7.05	48	.641391	3.98	.891591	7.07
49	.627166	4.07	.866597	7.05	49	.641630	3.97	.892015	7.08
50	.627410	4.07	.867020	7.05	50	.641868	3.98	.892440	7.07
51	9.627654	4.05	9.867443	7.05	51	9.642107	3.97	9.892864	7.08
52	.627897	4.05	.867866	7.05	52	.642345	3.97	.893289	7.08
53	.628140	4.07	.868289	7.05	53	.642583	3.98	.893714	7.07
54	.628384	4.05	.868712	7.05	54	.642822	3.97	.894138	7.08
55	.628627	4.05	.869135	7.05	55	.643060	3.97	.894563	7.08
56	.628870	4.05	.869558	7.05	56	.643298	3.95	.894988	7.07
57	.620113	4.05	.869981	7.05	57	.643535	3.97	.895412	7.08
58	.620356	4.03	.870404	7.05	58	.643773	3.97	.895837	7.0
59	.620598	4.05	.870827	7.05	59	.644011	3.97	.896262	7.
60	9.620841	4.05	9.871250	7.05	60	9.644249	3.95	9.896687	7

44°					45°				
′	Vers.	D. 1″.	Ex. sec.	D. 1″.	′	Vers.	D. 1″.	Ex. sec.	D.
0	9.448181	5.20	9.591247	7.23	0	9.466709	5.08	9.617224	7
1	.448493	5.22	.591681	7.25	1	.467014	5.08	.617656	7
2	.448806	5.20	.592116	7.25	2	.467319	5.08	.618087	7
3	.449118	5.22	.592551	7.23	3	.467624	5.07	.618518	7
4	.449431	5.20	.592985	7.23	4	.467928	5.08	.618949	7
5	.449743	5.20	.593419	7.25	5	.468233	5.07	.619380	7
6	.450055	5.18	.593854	7.23	6	.468537	5.07	.619811	7
7	.450366	5.20	.594288	7.23	7	.468841	5.07	.620242	7
8	.450678	5.20	.594722	7.23	8	.469145	5.07	.620673	7
9	.450990	5.18	.595156	7.25	9	.469449	5.07	.621104	7
10	.451301	5.18	.595591	7.23	10	.469753	5.07	.621535	7
11	9.451612	5.20	9.596025	7.23	11	9.470057	5.05	9.621966	7
12	.451924	5.18	.596459	7.23	12	.470360	5.07	.622396	7
13	.452235	5.18	.596893	7.22	13	.470664	5.05	.622827	7
14	.452546	5.17	.597326	7.23	14	.470967	5.05	.623258	7
15	.452856	5.17	.597760	7.23	15	.471270	5.05	.623688	7
16	.453167	5.18	.598194	7.23	16	.471573	5.05	.624119	7
17	.453478	5.17	.598628	7.22	17	.471876	5.05	.624549	7
18	.453788	5.17	.599061	7.23	18	.472179	5.05	.624980	7
19	.454098	5.17	.599495	7.22	19	.472482	5.03	.625410	7
20	.454408	5.17	.599928	7.23	20	.472784	5.05	.625841	7
21	9.454718	5.17	9.600362	7.22	21	9.473087	5.03	9.626271	7
22	.455028	5.17	.600795	7.23	22	.473389	5.03	.626701	7
23	.455338	5.17	.601229	7.22	23	.473691	5.03	.627131	7
24	.455648	5.15	.601662	7.22	24	.473993	5.03	.627561	7
25	.455957	5.17	.602095	7.22	25	.474295	5.03	.627991	7
26	.456267	5.15	.602528	7.23	26	.474597	5.03	.628421	7
27	.456576	5.15	.602962	7.22	27	.474899	5.02	.628851	7
28	.456885	5.15	.603395	7.22	28	.475200	5.03	.629281	7
29	.457194	5.15	.603828	7.22	29	.475502	5.02	.629711	7
30	.457503	5.13	.604261	7.22	30	.475803	5.02	.630141	7
31	9.457811	5.15	9.604694	7.20	31	9.476104	5.02	9.630571	7
32	.458120	5.15	.605126	7.22	32	.476405	5.02	.631001	7
33	.458429	5.13	.605559	7.22	33	.476706	5.02	.631430	7
34	.458737	5.13	.605992	7.22	34	.477007	5.02	.631860	7
35	.459045	5.13	.606425	7.20	35	.477308	5.00	.632290	7
36	.459353	5.13	.606857	7.22	36	.477608	5.02	.632719	7
37	.459661	5.13	.607290	7.20	37	.477909	5.00	.633149	7
38	.459969	5.13	.607722	7.22	38	.478209	5.00	.633578	7
39	.460277	5.12	.608155	7.20	39	.478509	5.00	.634008	7
40	.460584	5.13	.608587	7.22	40	.478809	5.00	.634437	7
41	9.460892	5.12	9.609020	7.20	41	9.479109	5.00	9.634866	7
42	.461199	5.12	.609452	7.20	42	.479409	5.00	.635296	7
43	.461506	5.12	.609884	7.20	43	.479709	5.00	.635725	7
44	.461813	5.12	.610316	7.22	44	.480009	4.98	.636154	7
45	.462120	5.12	.610749	7.20	45	.480308	5.00	.636583	7
46	.462427	5.12	.611181	7.20	46	.480608	4.98	.637012	7
47	.462734	5.10	.611613	7.20	47	.480907	4.98	.637441	7
48	.463040	5.12	.612045	7.20	48	.481206	4.98	.637870	7
49	.463347	5.10	.612477	7.18	49	.481505	4.98	.638299	7
50	.463653	5.10	.612908	7.20	50	.481804	4.98	.638728	7
51	9.463959	5.10	9.613340	7.20	51	9.482103	4.97	9.639157	7
52	.464265	5.10	.613772	7.20	52	.482401	4.98	.639586	7
53	.464571	5.10	.614204	7.18	53	.482700	4.97	.640015	7
54	.464877	5.10	.614635	7.20	54	.482998	4.97	.640443	7
55	.465183	5.08	.615067	7.20	55	.483296	4.98	.640872	7
56	.465488	5.10	.615499	7.18	56	.483595	4.97	.641301	7
57	.465794	5.08	.615930	7.20	57	.483893	4.97	.641729	7
58	.466099	5.08	.616362	7.18	58	.484191	4.95	.642158	7
59	.466404	5.08	.616793	7.18	59	.484488	4.97	.642586	
60	9.466709	5.08	9.617224	7.20	60	9.484786	4.97	9.643015	

46°					47°				
′	Vers.	D. 1.	Ex. sec.	D. 1″.	′	Vers.	D. 1″.	Ex. sec.	D. 1″.
0	9.484786	4.97	9.643015	7.13	0	9.502429	4.85	9.668646	7.10
1	.485084	4.95	.643443	7.15	1	.502720	4.83	.669072	7.10
2	.485381	4.95	.643872	7.13	2	.503010	4.83	.669498	7.10
3	.485673	4.97	.644300	7.13	3	.503300	4.85	.669924	7.10
4	.485976	4.95	.644728	7.13	4	.503591	4.83	.670350	7.10
5	.486273	4.95	.645156	7.15	5	.503881	4.83	.670776	7.08
6	.486570	4.93	.645585	7.13	6	.504171	4.82	.671201	7.10
7	.486866	4.95	.646013	7.13	7	.504460	4.83	.671627	7.10
8	.487163	4.95	.646441	7.13	8	.504750	4.83	.672053	7.10
9	.487460	4.93	.646869	7.13	9	.505040	4.82	.672479	7.08
10	.487756	4.95	.647297	7.13	10	.505329	4.82	.672904	7.10
11	9.488053	4.93	9.647725	7.13	11	9.505618	4.83	9.673330	7.10
12	.488349	4.93	.648153	7.13	12	.505908	4.82	.673756	7.08
13	.488645	4.93	.648581	7.13	13	.506197	4.82	.674181	7.10
14	.488941	4.93	.649009	7.12	14	.506486	4.82	.674607	7.08
15	.489237	4.93	.649436	7.13	15	.506775	4.80	.675032	7.10
16	.489533	4.92	.649864	7.13	16	.507063	4.82	.675458	7.08
17	.489828	4.93	.650292	7.13	17	.507352	4.80	.675883	7.10
18	.490124	4.92	.650720	7.12	18	.507640	4.82	.676309	7.08
19	.490419	4.92	.651147	7.13	19	.507929	4.80	.676734	7.08
20	.490714	4.93	.651575	7.12	20	.508217	4.80	.677159	7.08
21	9.491010	4.92	9.652002	7.13	21	9.508505	4.80	9.677584	7.10
22	.491305	4.92	.652430	7.12	22	.508793	4.80	.678010	7.08
23	.491600	4.90	.652857	7.13	23	.509081	4.80	.678435	7.08
24	.491894	4.92	.653285	7.12	24	.509369	4.80	.678860	7.08
25	.492189	4.92	.653712	7.13	25	.509657	4.80	.679285	7.08
26	.492484	4.90	.654140	7.12	26	.509945	4.78	.679710	7.10
27	.492778	4.90	.654567	7.12	27	.510232	4.80	.680136	7.08
28	.493072	4.92	.654994	7.12	28	.510520	4.78	.680561	7.08
29	.493367	4.90	.655421	7.13	29	.510807	4.78	.680986	7.08
30	.493661	4.90	.655849	7.12	30	.511094	4.78	.681411	7.08
31	9.493955	4.90	9.656276	7.12	31	9.511381	4.78	9.681836	7.07
32	.494249	4.88	.656703	7.12	32	.511668	4.78	.682260	7.08
33	.494542	4.90	.657130	7.12	33	.511955	4.77	.682685	7.08
34	.494836	4.90	.657557	7.12	34	.512241	4.78	.683110	7.08
35	.495130	4.88	.657984	7.12	35	.512528	4.78	.683535	7.08
36	.495423	4.88	.658411	7.12	36	.512815	4.77	.683960	7.08
37	.495716	4.88	.658838	7.12	37	.513101	4.77	.684385	7.07
38	.496009	4.88	.659265	7.10	38	.513387	4.77	.684809	7.08
39	.496302	4.88	.659691	7.12	39	.513673	4.77	.685234	7.08
40	.496595	4.88	.660118	7.12	40	.513959	4.77	.685659	7.07
41	9.496888	4.88	9.660545	7.12	41	9.514245	4.77	9.686083	7.08
42	.497181	4.87	.660972	7.10	42	.514531	4.77	.686508	7.08
43	.497473	4.88	.661398	7.12	43	.514817	4.75	.686933	7.07
44	.497766	4.87	.661825	7.12	44	.515102	4.77	.687357	7.08
45	.498058	4.87	.662252	7.10	45	.515388	4.75	.687782	7.07
46	.498350	4.88	.662678	7.12	46	.515673	4.77	.688206	7.08
47	.498643	4.87	.663105	7.10	47	.515959	4.75	.688631	7.07
48	.498935	4.85	.663531	7.12	48	.516244	4.75	.689055	7.07
49	.499226	4.87	.663958	7.10	49	.516529	4.75	.689479	7.08
50	.499518	4.87	.664384	7.10	50	.516814	4.73	.689904	7.07
51	9.499810	4.85	9.664810	7.12	51	9.517098	4.75	9.690328	7.07
52	.500101	4.87	.665237	7.10	52	.517383	4.75	.690752	7.08
53	.500393	4.85	.665663	7.10	53	.517668	4.73	.691177	7.07
54	.500684	4.85	.666089	7.10	54	.517952	4.73	.691601	7.07
55	.500975	4.85	.666515	7.12	55	.518236	4.75	.692025	7.07
56	.501266	4.85	.666942	7.10	56	.518521	4.73	.692449	7.07
57	.501557	4.85	.667368	7.10	57	.518805	4.73	.692873	7.08
58	.501848	4.85	.667794	7.10	58	.519089	4.73	.693298	7.07
59	.502139	4.83	.668220	7.10	59	.519373	4.73	.693722	7.0
60	9.502429	4.85	9.668646	7.10	60	9.519657	4.72	9.694146	7.

TABLE XXVI.—LOGARITHMIC VERSED SINES

48°					49°				
′	Vers.	D. 1″.	Ex. sec.	D. 1″.	′	Vers.	D. 1″.	Ex. sec.	D. 1
0	9.519657	4.72	9.694146	7.07	0	9.536484	4.62	9.719541	7.
1	.519940	4.73	.694570	7.07	1	.536761	4.62	.719964	7.
2	.520224	4.72	.694994	7.07	2	.537038	4.62	.720386	7.
3	.520507	4.73	.695418	7.07	3	.537315	4.62	.720809	7.
4	.520791	4.72	.695842	7.07	4	.537592	4.62	.721231	7.
5	.521074	4.72	.696266	7.05	5	.537869	4.60	.721653	7.
6	.521357	4.72	.696689	7.07	6	.538145	4.62	.722076	7.
7	.521640	4.72	.697113	7.07	7	.538422	4.60	.722498	7.
8	.521923	4.72	.697537	7.07	8	.538698	4.60	.722921	7.
9	.522206	4.70	.697961	7.07	9	.538974	4.62	.723343	7.
10	.522488	4.72	.698385	7.07	10	.539251	4.60	.723765	7.
11	9.522771	4.72	9.698809	7.05	11	9.539527	4.60	9.724188	7.
12	.523054	4.70	.699232	7.07	12	.539803	4.60	.724610	7.
13	.523336	4.70	.699656	7.07	13	.540079	4.58	.725032	7.
14	.523618	4.70	.700080	7.05	14	.540354	4.60	.725454	7.
15	.523900	4.70	.700503	7.07	15	.540630	4.60	.725877	7.
16	.524182	4.70	.700927	7.05	16	.540906	4.58	.726299	7.
17	.524464	4.70	.701350	7.07	17	.541181	4.58	.726721	7.
18	.524746	4.70	.701774	7.07	18	.541456	4.60	.727143	7.
19	.525028	4.68	.702198	7.05	19	.541732	4.58	.727565	7.
20	.525309	4.70	.702621	7.07	20	.542007	4.58	.727988	7.
21	9.525591	4.68	9.703045	7.05	21	9.542282	4.58	9.728410	7.
22	.525872	4.68	.703468	7.05	22	.542557	4.58	.728832	7.
23	.526153	4.70	.703891	7.07	23	.542832	4.57	.729254	7.
24	.526435	4.68	.704315	7.05	24	.543106	4.58	.729676	7.
25	.526716	4.68	.704738	7.07	25	.543381	4.57	.730098	7.
26	.526997	4.67	.705162	7.05	26	.543655	4.58	.730520	7.
27	.527277	4.68	.705585	7.05	27	.543930	4.57	.730942	7.
28	.527558	4.68	.706008	7.05	28	.544204	4.57	.731364	7.
29	.527839	4.67	.706431	7.07	29	.544478	4.57	.731786	7.
30	.528119	4.68	.706855	7.05	30	.544752	4.57	.732208	7.
31	9.528400	4.67	9.707278	7.05	31	9.545026	4.57	9.732630	7.
32	.528680	4.67	.707701	7.05	32	.545300	4.57	.733052	7.
33	.528960	4.67	.708124	7.05	33	.545574	4.57	.733474	7.
34	.529240	4.67	.708547	7.07	34	.545848	4.55	.733896	7.
35	.529520	4.67	.708971	7.05	35	.546121	4.57	.734317	7.
36	.529800	4.67	.709394	7.05	36	.546395	4.55	.734739	7.
37	.530080	4.65	.709817	7.05	37	.546668	4.55	.735161	7.
38	.530359	4.67	.710240	7.05	38	.546941	4.55	.735583	7.
39	.530639	4.65	.710663	7.05	39	.547214	4.55	.736005	7.
40	.530918	4.67	.711086	7.05	40	.547487	4.55	.736427	7.
41	9.531198	4.65	9.711509	7.05	41	9.547760	4.55	9.736848	7.
42	.531477	4.65	.711932	7.05	42	.548033	4.55	.737270	7.
43	.531756	4.65	.712355	7.05	43	.548306	4.55	.737692	7.
44	.532035	4.65	.712778	7.03	44	.548579	4.53	.738114	7.
45	.532314	4.63	.713200	7.05	45	.548851	4.55	.738535	7.
46	.532592	4.65	.713623	7.05	46	.549124	4.53	.738957	7.
47	.532871	4.65	.714046	7.05	47	.549396	4.53	.739379	7.
48	.533150	4.63	.714469	7.05	48	.549668	4.53	.739800	7.
49	.533428	4.63	.714892	7.05	49	.549940	4.53	.740222	7.
50	.533706	4.65	.715315	7.03	50	.550212	4.53	.740644	7.
51	9.533985	4.63	9.715737	7.05	51	9.550484	4.53	9.741065	7.
52	.534263	4.63	.716160	7.05	52	.550756	4.53	.741487	7.
53	.534541	4.63	.716583	7.03	53	.551028	4.52	.741908	7.
54	.534819	4.63	.717005	7.05	54	.551299	4.53	.742330	7.
55	.535097	4.62	.717428	7.05	55	.551571	4.52	.742751	7.
56	.535374	4.63	.717851	7.03	56	.551842	4.52	.743173	7.
57	.535652	4.62	.718273	7.05	57	.552113	4.52	.743595	7.
58	.535929	4.63	.718696	7.03	58	.552384	4.53	.744016	7
59	.536207	4.62	.719118	7.05	59	.552656	4.52	.744438	
60	9.536484	4.62	9.719541	7.05	60	9.552927	4.50	9.744859	

62°					63°			
Vers.	D. 1.	Ex. sec.	D. 1".	'	Vers.	D. 1".	Ex. sec.	D. 1".
9.724709	3.50	10.053009	7.47	0	9.737200	3.43	10.080153	7.58
.724919	3.50	.053547	7.47	1	.737406	3.43	.080608	7.57
.725129	3.50	.053995	7.47	2	.737612	3.43	.081062	7.57
.725339	3:50	.054443	7.48	3	.737818	3.43	.081516	7.58
.725549	3.50	.054892	7.47	4	.738024	3.43	.081971	7.57
.725759	3.50	.055340	7.47	5	.738230	3.43	.082425	7.58
.725969	3.50	.055788	7.48	6	.738436	3.43	.082880	7.58
.726179	3.48	.056237	7.47	7	.738642	3.42	.083335	7.58
.726388	3.50	.056685	7.48	8	.738847	3.43	.083790	7.58
.726598	3.50	.057134	7.48	9	.739053	3.42	.084245	7.58
.726808	3.48	.057583	7.48	10	.739258	3.43	.084700	7.58
9.727017	3.50	10.058032	7.48	11	9.739464	3.42	10.085155	7.60
.727227	3.48	.058481	7.48	12	.739669	3.43	.085611	7.58
.727436	3.48	.058930	7.48	13	.739875	3.42	.086066	7.60
.727645	3.50	.059379	7.48	14	.740080	3.42	.086522	7.58
.727855	3.48	.059828	7.50	15	.740285	3.42	.086977	7.60
.728064	3.48	.060278	7.48	16	.740490	3.42	.087433	7.60
.728273	3.48	.060727	7.50	17	.740695	3.42	.087889	7.60
.728482	3.48	.061177	7.48	18	.740900	3.42	.088345	7.60
.728691	3.48	.061626	7.50	19	.741105	3.42	.088801	7.62
.728900	3.48	.062076	7.50	20	.741310	3.42	.089258	7.60
9.729109	3.47	10.062526	7.50	21	9.741515	3.40	10.089714	7.62
.729317	3.48	.062976	7.50	22	.741719	3.42	.090171	7.60
.729526	3.48	.063426	7.50	23	.741924	3.42	.090627	7.62
.729735	3.47	.063876	7.52	24	.742129	3.40	.091084	7.62
.729943	3.48	.064327	7.50	25	.742333	3.42	.091541	7.62
.730152	3.47	.064777	7.50	26	.742538	3.40	.091998	7.62
.730360	3.48	.065227	7.52	27	.742742	3.40	.092455	7.62
.730569	3.47	.065678	7.52	28	.742946	3.40	.092912	7.63
.730777	3.47	.066129	7.52	29	.743150	3.42	.093370	7.62
.730985	3.47	.066580	7.50	30	.743355	3.40	.093827	7.63
9.731193	3.47	10.067030	7.53	31	9.743559	3.40	10.094285	7.63
.731401	3.47	.067482	7.52	32	.743762	3.40	.094743	7.62
.731609	3.47	.067933	7.52	33	.743967	3.40	.095200	7.63
.731817	3.47	.068384	7.52	34	.744171	3.40	.095658	7.63
.732025	3.47	.068835	7.53	35	.744375	3.38	.096116	7.65
.732233	3.47	.069287	7.52	36	.744578	3.40	.096575	7.63
.732441	3.45	.069738	7.53	37	.744782	3.40	.097033	7.63
.732648	3.47	.070190	7.53	38	.744986	3.38	.097491	7.65
.732856	3.47	.070642	7.52	39	.745189	3.40	.097950	7.63
.733064	3.45	.071093	7.53	40	.745393	3.38	.098408	7.65
9.733271	3.45	10.071545	7.55	41	9.745596	3.40	10.098867	7.65
.733478	3.47	.071998	7.53	42	.745800	3.38	.099326	7.65
.733686	3.45	.072450	7.53	43	.746003	3.38	.099785	7.65
.733893	3.45	.072902	7.53	44	.746206	3.38	.100244	7.67
.734100	3.45	.073354	7.55	45	.746409	3.40	.100704	7.65
.734307	3.47	.073807	7.55	46	.746613	3.38	.101163	7.67
.734515	3.43	.074260	7.53	47	.746816	3.38	.101623	7.65
.734721	3.45	.074712	7.55	48	.747019	3.38	.102082	7.67
.734928	3.45	.075165	7.55	49	.747222	3.37	.102542	7.67
.735135	3.45	.075618	7.55	50	.747424	3.38	.103002	7.67
9.735342	3.45	10.076071	7.55	51	9.747627	3.38	10.103462	7.67
.735549	3.43	.076524	7.55	52	.747830	3.38	.103922	7.67
.735755	3.45	.076977	7.57	53	.748033	3.37	.104382	7.68
.735962	3.45	.077431	7.55	54	.748235	3.38	.104843	7.67
.736169	3.43	.077884	7.57	55	.748438	3.37	.105303	7.68
.736375	3.43	.078338	7.57	56	.748640	3.38	.105764	7.67
.736581	3.45	.078792	7.55	57	.748843	3.37	.106224	7.68
.736788	3.43	.079245	7.57	58	.749045	3.37	.106685	7.68
.736994	3.43	.079699	7.57	59	.749247	3.37	.107146	7.
9.737200	3.43	10.080153	7.58	60	9.749449	3.38	10.107607	7

52°					53°			
′	Vers.	D. 1″.	Ex. sec.	D. 1″.	′	Vers.	D. 1″.	Ex. sec.
0	9.584714	4.32	9.795372	7.02	0	9.600085	4.22	9.820622
1	.584973	4.32	.795793	7.00	1	.600338	4.22	.821043
2	.585232	4.32	.796213	7.02	2	.600591	4.23	.821464
3	.585491	4.30	.796634	7.02	3	.600845	4.22	.821885
4	.585749	4.32	.797055	7.02	4	.601098	4.22	.822306
5	.586008	4.30	.797476	7.00	5	.601351	4.20	.822727
6	.586266	4.32	.797896	7.02	6	.601603	4.22	.823148
7	.586525	4.30	.798317	7.02	7	.601856	4.22	.823569
8	.586783	4.30	.798738	7.00	8	.602109	4.22	.823990
9	.587041	4.30	.799158	7.02	9	.602362	4.20	.824411
10	.587299	4.30	.799579	7.02	10	.602614	4.20	.824833
11	9.587557	4.30	9.800000	7.02	11	9.602866	4.22	9.825254
12	.587815	4.30	.800421	7.00	12	.603119	4.20	.825675
13	.588073	4.30	.800841	7.02	13	.603371	4.20	.826096
14	.588331	4.28	.801262	7.02	14	.603623	4.20	.826517
15	.588588	4.28	.801683	7.02	15	.603875	4.20	.826938
16	.588846	4.28	.802104	7.00	16	.604127	4.20	.827360
17	.589103	4.30	.802524	7.02	17	.604379	4.20	.827781
18	.589361	4.28	.802945	7.02	18	.604631	4.20	.828202
19	.589618	4.28	.803366	7.02	19	.604883	4.18	.828623
20	.589875	4.28	.803787	7.00	20	.605134	4.20	.829044
21	9.590132	4.28	9.804207	7.02	21	9.605386	4.18	9.829466
22	.590389	4.28	.804628	7.02	22	.605637	4.18	.829887
23	.590646	4.28	.805049	7.02	23	.605888	4.20	.830308
24	.590903	4.28	.805470	7.02	24	.606140	4.18	.830729
25	.591160	4.27	.805891	7.00	25	.606391	4.18	.831151
26	.591416	4.28	.806311	7.02	26	.606642	4.18	.831572
27	.591673	4.27	.806732	7.02	27	.606893	4.18	.831993
28	.591929	4.27	.807153	7.02	28	.607144	4.17	.832415
29	.592185	4.28	.807574	7.02	29	.607394	4.18	.832836
30	.592442	4.27	.807995	7.00	30	.607645	4.18	.833257
31	9.592698	4.27	9.808415	7.02	31	9.607896	4.17	9.833679
32	.592954	4.27	.808836	7.02	32	.608146	4.18	.834100
33	.593210	4.27	.809257	7.02	33	.608397	4.17	.834522
34	.593466	4.25	.809678	7.02	34	.608647	4.17	.834943
35	.593721	4.27	.810099	7.02	35	.608897	4.17	.835364
36	.593977	4.27	.810520	7.00	36	.609147	4.17	.835786
37	.594233	4.25	.810940	7.02	37	.609397	4.17	.836207
38	.594488	4.25	.811361	7.02	38	.609647	4.17	.836629
39	.594743	4.27	.811782	7.02	39	.609897	4.17	.837050
40	.594999	4.25	.812203	7.02	40	.610147	4.17	.837472
41	9.595254	4.25	9.812624	7.02	41	9.610397	4.15	9.837893
42	.595509	4.25	.813045	7.02	42	.610646	4.17	.838315
43	.595764	4.25	.813466	7.00	43	.610896	4.15	.838736
44	.596019	4.25	.813886	7.02	44	.611145	4.15	.839158
45	.596274	4.23	.814307	7.02	45	.611394	4.17	.839579
46	.596528	4.25	.814728	7.02	46	.611644	4.15	.840001
47	.596783	4.25	.815149	7.02	47	.611893	4.15	.840423
48	.597038	4.23	.815570	7.02	48	.612142	4.15	.840844
49	.597292	4.23	.815991	7.02	49	.612391	4.15	.841266
50	.597546	4.25	.816412	7.02	50	.612640	4.13	.841687
51	9.597801	4.23	9.816833	7.02	51	9.612888	4.15	9.842109
52	.598055	4.23	.817254	7.02	52	.613137	4.15	.842531
53	.598309	4.23	.817675	7.02	53	.613386	4.13	.842953
54	.598563	4.23	.818096	7.02	54	.613634	4.15	.843374
55	.598817	4.23	.818517	7.02	55	.613883	4.13	.843796
56	.599071	4.22	.818938	7.02	56	.614131	4.13	.844218
57	.599324	4.23	.819359	7.02	57	.614379	4.13	.844639
58	.599578	4.22	.819780	7.02	58	.614627	4.15	.84506
59	.599831	4.23	.820201	7.02	59	.614876	4.13	.8454
60	9.600085	4.22	9.820622	7.02	60	9.615124	4.12	9.845

66°					67°			
Vers.	D. 1".	Ex. sec.	D. 1".	′	Vers.	D. 1".	Ex. sec.	D.1".
9.773248	3.23	10.163934	7.98	0	9.784809	3.18	10.192931	8.15
.773442	3.23	.164413	7.97	1	.785000	3.18	.193420	8.13
.773636	3.25	.164891	7.98	2	.785191	3.17	.193908	8.15
.773831	3.23	.165370	7.98	3	.785381	3.18	.194397	8.15
.774025	3.23	.165849	7.98	4	.785572	3.18	.194886	8.17
.774219	3.25	.166328	7.98	5	.785763	3.17	.195376	8.15
.774414	3.23	.166807	7.98	6	.785953	3.18	.195865	8.17
.774608	3.23	.167286	8.00	7	.786144	3.17	.196355	8.17
.774802	3.23	.167766	7.98	8	.786334	3.17	.196845	8.17
.774996	3.23	.168245	8.00	9	.786524	3.18	.197335	8.17
.775190	3.23	.168725	8.00	10	.786715	3.17	.197825	8.17
9.775384	3.22	10.169205	8.00	11	9.786905	3.17	10.198315	8.18
.775577	3.23	.169685	8.00	12	.787095	3.17	.198806	8.18
.775771	3.23	.170165	8.02	13	.787285	3.17	.199297	8.18
.775965	3.23	.170646	8.02	14	.787475	3.17	.199788	8.18
.776159	3.22	.171127	8.00	15	.787665	3.17	.200279	8.18
.776352	3.23	.171607	8.02	16	.787855	3.17	.200770	8.20
.776546	3.22	.172088	8.02	17	.788045	3.17	.201262	8.18
.776739	3.23	.172569	8.03	18	.788235	3.17	.201753	8.20
.776933	3.22	.173051	8.02	19	.788425	3.15	.202245	8.20
.777126	3.22	.173532	8.03	20	.788614	3.17	.202737	8.20
9.777319	3.22	10.174014	8.03	21	9.788804	3.15	10.203229	8.22
.777512	3.22	.174496	8.03	22	.788993	3.17	.203722	8.22
.777705	3.23	.174978	8.03	23	.789183	3.15	.204215	8.20
.777899	3.22	.175460	8.03	24	.789372	3.17	.204707	8.22
.778092	3.22	.175942	8.05	25	.789562	3.15	.205200	8.23
.778285	3.20	.176425	8.03	26	.789751	3.15	.205694	8.22
.778477	3.22	.176907	8.05	27	.789940	3.17	.206187	8.23
.778670	3.22	.177390	8.05	28	.790130	3.15	.206681	8.22
.778863	3.22	.177873	8.05	29	.790319	3.15	.207174	8.23
.779056	3.20	.178356	8.05	30	.790508	3.15	.207668	8.23
9.779248	3.22.	10.178839	8.07	31	9.790697	3.15	10.208162	8.25
.779441	3.22	.179323	8.07	32	.790886	3.15	.208657	8.23
.779634	3.20	.179807	8.05	33	.791075	3.15	.209151	8.25
.779826	3.20	.180290	8.07	34	.791264	3.15	.209646	8.25
.780018	3.22	.180774	8.08	35	.791453	3.13	.210141	8.25
.780211	3.20	.181259	8.07	36	.791641	3.15	.210636	8.25
.780403	3.20	.181743	8.07	37	.791830	3.15	.211131	8.27
.780595	3.20	.182227	8.08	38	.792019	3.13	.211627	8.27
.780787	3.22	.182712	8.08	39	.792207	3.15	.212123	8.25
.780980	3.20	.183197	8.08	40	.792396	3.13	.212618	8.28
9.781172	3.20	10.183682	8 08	41	9.792584	3.13	10.213115	8.27
.781364	3.20	.184167	8.10	42	.792772	3.15	.213611	8.27
.781556	3.18	.184653	8.08	43	.792961	3.13	.214107	8.28
.781747	3.20	.185138	8.10	44	.793149	3.13	.214604	8.28
.781939	3.20	.185624	8.10	45	.793337	3.13	.215101	8.28
.782131	3.20	.186110	8.10	46	.793525	3.15	.215598	8.28
.782323	3.18	.186596	8.10	47	.793714	3.13	.216095	8.30
.782514	3.20	.187082	8.10	48	.793902	3.13	.216593	8.28
.782706	3.18	.187568	8.12	49	.794090	3.12	.217090	8.30
.782897	3.20	.188055	8.12	50	.794277	3.13	.217588	8.30
9.783089	3.18	10.188542	8.12	51	9.794465	3.13	10.218086	8.32
.783280	3.18	.189029	8.12	52	.794653	3.13	.218585	8.30
.783471	3.20	.189516	8.12	53	.794841	3.12	.219083	8.32
.783663	3.18	.190003	8.13	54	.795028	3.13	.219582	8.32
.783854	3.18	.190491	8.12	55	.795216	3.13	.220081	8.32
.784045	3.18	.190978	8.13	56	.795404	3.12	.220580	8.32
.784236	*3.18*	*.191466*	8.13	57	.795591	3.13	.221079	8.32
.784427	*3.18*	*.191954*	8.15	58	.795779	3.12	.221578	8.3
.784618	*3.18*	*.192443*	8.13	59	.795966	3.12	.222078	8.
9.784809	*3.18*	*10.192931*	8.15	60	9.796153	3.13	10.222578	8

56°					57°				
′	Vers.	D. 1″.	Ex. sec.	D 1″.	′	Vers.	D. 1″.	Ex. sec.	D. 1
0	9.644249	3.95	9.896687	7.08	0	9.658356	3.87	9.922247	7.
1	.644486	3.97	.897112	7.08	1	.658588	3.88	.922674	7.
2	.644724	3.95	.897537	7.08	2	.658821	3.87	.923102	7.
3	.644961	3.95	.897962	7.08	3	.659053	3.88	.923529	7.
4	.645198	3.95	.898387	7.08	4	.659286	3.87	.923956	7.
5	.645435	3.97	.898812	7.08	5	.659518	3.87	.924384	7.
6	.645673	3.95	.899237	7.08	6	.659750	3.88	.924811	7.
7	.645910	3.95	.899662	7.08	7	.659983	3.87	.925239	7.
8	.646147	3.95	.900087	7.08	8	.660215	3.87	.925666	7.
9	.646384	3.93	.900512	7.10	9	.660447	3.87	.926094	7.
10	.646620	3.95	.900938	7.08	10	.660679	3.85	.926521	7.
11	9.646857	3.95	9.901363	7.08	11	9.660910	3.87	9.926949	7.
12	.647094	3.93	.901788	7.08	12	.661142	3.87	.927377	7.
13	.647330	3.95	.902213	7.10	13	.661374	3.85	.927804	7.
14	.647567	3.93	.902639	7.08	14	.661605	3.87	.928232	7.
15	.647803	3.93	.903064	7.10	15	.661837	3.85	.928660	7.
16	.648039	3.95	.903490	7.08	16	.662068	3.87	.929088	7.
17	.648276	3.93	.903915	7.10	17	.662300	3.85	.929516	7.
18	.648512	3.93	.904341	7.08	18	.662531	3.85	.929944	7.
19	.648748	3.93	.904766	7.10	19	.662762	3.85	.930372	7.
20	.648984	3.93	.905192	7.08	20	.662993	3.85	.930800	7.
21	9.649220	3.93	9.905617	7.10	21	9.663224	3.85	9.931228	7
22	.649456	3.92	.906043	7.10	22	.663455	3.85	.931656	7
23	.649691	3.93	.906469	7.08	23	.663686	3.85	.932085	7
24	.649927	3.93	.906894	7.10	24	.663917	3.85	.932513	7
25	.650163	3.92	.907320	7.10	25	.664148	3.83	.932941	7
26	.650398	3.92	.907746	7.10	26	.664378	3.85	.933369	7
27	.650633	3.93	.908172	7.10	27	.664609	3.83	.933798	7
28	.650869	3.92	.908598	7.10	28	.664839	3.85	.934226	7
29	.651104	3.92	.909024	7.10	29	.665070	3.83	.934655	7
30	.651339	3.92	.909450	7.10	30	.665300	3.83	.935083	7
31	9.651574	3.92	9.909876	7.10	31	9.665530	3.83	9.935512	7
32	.651809	3.92	.910302	7.10	32	.665760	3.83	.935941	7
33	.652044	3.92	.910728	7.10	33	.665990	3.83	.936369	7
34	.652279	3.92	.911154	7.10	34	.666220	3.83	.936798	7
35	.652514	3.90	.911580	7.10	35	.666450	3.83	.937227	7
36	.652748	3.92	.912006	7.10	36	.666680	3.83	.937656	7
37	.652983	3.90	.912432	7.12	37	.666910	3.82	.938085	7
38	.653217	3.92	.912859	7.10	38	.667139	3.83	.938513	7
39	.653452	3.90	.913285	7.10	39	.667369	3.83	.938942	7
40	.653686	3.90	.913711	7.12	40	.667599	3.82	.939371	7
41	9.653920	3.92	9.914138	7.10	41	9.667828	3.82	9.939801	7
42	.654155	3.90	.914564	7.12	42	.668057	3.83	.940230	7
43	.654389	3.90	.914991	7.10	43	.668287	3.82	.940659	7
44	.654623	3.90	.915417	7.12	44	.668516	3.82	.941088	7
45	.654857	3.88	.915844	7.10	45	.668745	3.82	.941517	7
46	.655090	3.90	.916270	7.12	46	.668974	3.82	.941947	7
47	.655324	3.90	.916697	7.12	47	.669203	3.82	.942376	7
48	.655558	3.90	.917124	7.10	48	.669432	3.82	.942806	7
49	.655792	3.88	.917550	7.12	49	.669661	3.80	.943235	7
50	.656025	3.88	.917977	7.12	50	.669889	3.82	.943665	7
51	9.656258	3.90	9.918404	7.12	51	9.670118	3.82	9.944094	7
52	.656492	3.88	.918831	7.12	52	.670347	3.80	.944524	7
53	.656725	3.88	.919258	7.12	53	.670575	3.82	.944953	
54	.656958	3.88	.919685	7.12	54	.670804	3.80	.945383	
55	.657191	3.88	.920112	7.12	55	.671032	3.80	.945813	
56	.657424	3.88	.920539	7.12	56	.671260	3.80	.946243	
57	.657657	3.88	.920966	7.12	57	.671488	3.80	.946673	
58	.657890	3.88	.921393	7.12	58	.671716	3.82	.947103	
59	.658123	3.88	.921820	7.12	59	.671945	3.78	.947533	
60	9.658356	3.87	9.922247	7.12	60	9.672172	3.80	9.947963	

70°					71°				
′	Vers.	D. 1″.	Ex. sec.	D. 1″.	′	Vers.	D. 1″.	Ex. sec.	D.1″.
0	9.818213	3.00	10.284161	8.80	0	9.828938	2.95	10.316296	9.07
1	.818393	3.00	.284689	8.78	1	.829115	2.95	.316840	9.08
2	.818573	3.02	.285216	8.82	2	.829292	2.95	.317385	9.07
3	.818754	3.00	.285745	8.80	3	.829469	2.95	.317929	9.10
4	.818934	3.00	.286273	8.82	4	.829646	2.95	.318475	9.08
5	.819114	3.00	.286802	8.82	5	.829823	2.95	.319020	9.08
6	.819294	3.00	.287331	8.82	6	.830000	2.95	.319565	9.10
7	.819474	3.00	.287860	8.82	7	.830177	2.93	.320111	9.12
8	.819654	3.00	.288389	8.83	8	.830353	2.95	.320658	9.10
9	.819834	3.00	.288919	8.83	9	.830530	2.93	.321204	9.12
10	.820014	3.00	.289449	8.83	10	.830706	2.95	.321751	9.12
11	9.820194	3.00	10.289979	8.85	11	9.830883	2.93	10.322298	9.12
12	.820374	2.98	.290510	8.85	12	.831059	2.95	.322845	9.13
13	.820553	3.00	.291041	8.85	13	.831236	2.93	.323393	9.13
14	.820733	3.00	.291572	8.85	14	.831412	2.95	.323941	9.13
15	.820913	2.98	.292103	8.87	15	.831589	2.93	.324489	9.15
16	.821092	3.00	.292635	8.85	16	.831765	2.93	.325038	9.15
17	.821272	2.98	.293166	8.87	17	.831941	2.93	.325587	9.15
18	.821451	3.00	.293698	8.88	18	.832117	2.93	.326136	9.17
19	.821631	2.98	.294231	8.88	19	.832293	2.93	.326686	9.15
20	.821810	2.98	.294764	8.87	20	.832469	2.93	.327235	9.18
21	9.821989	2.98	10.295296	8.90	21	9.832645	2.93	10.327786	9.17
22	.822168	3.00	.295830	8.88	22	.832821	2.93	.328336	9.18
23	.822348	2.98	.296363	8.90	23	.832997	2.93	.328887	9.18
24	.822527	2.98	.296897	8.90	24	.833173	2.93	.329438	9.18
25	.822706	2.98	.297431	8.90	25	.833349	2.93	.329989	9.20
26	.822885	2.98	.297965	8.92	26	.833525	2.92	.330541	9.20
27	.823064	2.98	.298500	8.90	27	.833700	2.93	.331093	9.20
28	.823243	2.97	.299034	8.93	28	.833876	2.92	.331645	9.22
29	.823421	2.98	.299570	8.92	29	834051	2.93	.332198	9.20
30	.823600	2.98	.300105	8.93	30	.834227	2.92	.332750	9.23
31	9.823779	2.98	10.300641	8.92	31	9.834402	2.93	10.333304	9.22
32	.823958	2.97	.301176	8.95	32	.834578	2.92	.333857	9.23
33	.824136	2.98	.301713	8.93	33	.834753	2.92	.334411	9.23
34	.824315	2.97	.302249	8.95	34	.834928	2.93	.334965	9.25
35	.824493	2.98	.302786	8.95	35	.835104	2.92	.335520	9.23
36	.824672	2.97	.303323	8.95	36	.835279	2.92	.336074	9.25
37	.824850	2.97	.303860	8.97	37	.835454	2.92	.336629	9.27
38	.825028	2.98	.304398	8.97	38	.835629	2.92	.337185	9.27
39	.825207	2.97	.304936	8.97	39	.835804	2.92	.337741	9.27
40	.825385	2.97	.305474	8.97	40	.835979	2.92	.338297	9.27
41	9.825563	2.97	10.306012	8.98	41	9.836154	2.92	10.338853	9.28
42	.825741	2.97	.306551	8.98	42	.836329	2.92	.339410	9.28
43	.825919	2.97	.307090	8.98	43	.836504	2.90	.339967	9.28
44	.826097	2.97	.307629	9.00	44	.836678	2.92	.340524	9.30
45	.826275	2.97	.308169	8.98	45	.836853	2.92	.341082	9.30
46	.826453	2.97	.308708	9.02	46	.837028	2.90	.341640	9.30
47	.826631	2.97	.309249	9.00	47	.837202	2.88	.342198	9.30
48	.826809	2.97	.309789	9.02	48	.837377	2.90	.342756	9.32
49	.826987	2.95	.310330	9.02	49	.837551	2.92	.343315	9.33
50	.827164	2.97	.310871	9.02	50	.837726	2.90	.343875	9.32
51	9.827342	2.95	10.311412	9.02	51	9.837900	2.92	10.344434	9.33
52	.827519	2.97	.311953	9.03	52	.838075	2.90	.344994	9.33
53	.827697	2.95	.312495	9.03	53	.838249	2.90	.345554	9.35
54	.827874	2.97	.313037	9.05	54	.838423	2.90	.346115	9.35
55	.828052	2.95	.313580	9.03	55	.838597	2.90	.346676	9.35
56	.828229	2.95	.314122	9.05	56	.838771	2.90	.347237	9.35
57	.828406	2.97	.314665	9.07	57	.838945	2.90	.347798	9.37
8	.828584	2.95	.315209	9.05	58	.839119	2.90	.348360	9.3
9	.828761	2.95	.315752	9.07	59	.839293	2.90	.348922	9.
	9.828938	2.95	10.316296	9.07	60	9.839467	2.90	10.349485	9

60°					61°			
′	Vers.	D. 1″.	Ex. sec.	D. 1″.	′	Vers.	D. 1″.	Ex. sec.
0	9.698970	3.65	10.000000	7.30	0	9.711968	3.57	10.026397
1	.699189	3.63	.000438	7.28	1	.712182	3.58	.026839
2	.699407	3.65	.000875	7.30	2	.712397	3.57	.027281
3	.699626	3.65	.001313	7.30	3	.712611	3.57	.027724
4	.699845	3.63	.001751	7.30	4	.712825	3.57	.028167
5	.700063	3.65	.002189	7.30	5	.713039	3.57	.028609
6	.700282	3.63	.002627	7.30	6	.713253	3.57	.029052
7	.700500	3.63	.003065	7.30	7	.713467	3.57	.029495
8	.700718	3.63	.003503	7.32	8	.713681	3.57	.029938
9	.700936	3.63	.003942	7.30	9	.713895	3.57	.030381
10	.701154	3.63	.004380	7.30	10	.714109	3.57	.030825
11	9.701372	3.63	10.004818	7.32	11	9.714323	3.55	10.031268
12	.701590	3.63	.005257	7.30	12	.714536	3.57	.031711
13	.701808	3.63	.005695	7.32	13	.714750	3.55	.032155
14	.702026	3.63	.006134	7.32	14	.714963	3.57	.032598
15	.702244	3.63	.006573	7.32	15	.715177	3.55	.033042
16	.702462	3.62	.007012	7.30	16	.715390	3.55	.033486
17	.702679	3.63	.007450	7.32	17	.715603	3.57	.033929
18	.702897	3.62	.007889	7.32	18	.715817	3.55	.034373
19	.703114	3.63	.008328	7.32	19	.716030	3.55	.034817
20	.703332	3.62	.008767	7.33	20	.716243	3.55	.035261
21	9.703549	3.62	10.009207	7.32	21	9.716456	3.55	10.035705
22	.703766	3.62	.009646	7.32	22	.716669	3.55	.036150
23	.703983	3.62	.010085	7.33	23	.716882	3.55	.036594
24	.704200	3.62	.010525	7.32	24	.717095	3.53	.037038
25	.704417	3.62	.010964	7.33	25	.717307	3.55	.037483
26	.704634	3.62	.011404	7.32	26	.717520	3.53	.037928
27	.704851	3.62	.011843	7.33	27	.717732	3.55	.038372
28	.705068	3.62	.012283	7.33	28	.717945	3.53	.038817
29	.705285	3.60	.012723	7.33	29	.718157	3.55	.039262
30	.705501	3.62	.013163	7.33	30	.718370	3.53	.039707
31	9.705718	3.62	10.013603	7.33	31	9.718582	3.53	10.040152
32	.705935	3.60	.014043	7.33	32	.718794	3.55	.040597
33	.706151	3.60	.014483	7.33	33	.719007	3.53	.041042
34	.706367	3.62	.014923	7.33	34	.719219	3.53	.041488
35	.706584	3.60	.015363	7.35	35	.719431	3.53	.041933
36	.706800	3.60	.015804	7.33	36	.719643	3.53	.042379
37	.707016	3.60	.016244	7.33	37	.719855	3.52	.042824
38	.707232	3.60	.016684	7.33	38	.720066	3.53	.043270
39	.707448	3.60	.017125	7.35	39	.720278	3.53	.043716
40	.707664	3.60	.017566	7.35	40	.720490	3.52	.044162
41	9.707880	3.60	10.018007	7.33	41	9.720701	3.53	10.044608
42	.708096	3.58	.018447	7.35	42	.720913	3.52	.045054
43	.708311	3.60	.018888	7.35	43	.721124	3.53	.045500
44	.708527	3.60	.019329	7.35	44	.721336	3.52	.045946
45	.708743	3.58	.019770	7.37	45	.721547	3.52	.046393
46	.708958	3.60	.020212	7.35	46	.721758	3.53	.046839
47	.709174	3.58	.020653	7.35	47	.721970	3.52	.047286
48	.709389	3.58	.021094	7.35	48	.722181	3.52	.047732
49	.709604	3.58	.021535	7.37	49	.722392	3.52	.048179
50	.709819	3.58	.021977	7.37	50	.722603	3.52	.048626
51	9.710035	3.58	10.022419	7.35	51	9.722814	3.50	10.049073
52	.710250	3.58	.022860	7.37	52	.723024	3.52	.049520
53	.710465	3.58	.023302	7.37	53	.723235	3.52	.049967
54	.710680	3.58	.023744	7.37	54	.723446	3.52	.050414
55	.710895	3.57	.024186	7.37	55	.723657	3.50	.050861
56	.711109	3.58	.024628	7.37	56	.723867	3.52	.051309
57	.711324	3.58	.025070	7.37	57	.724078	3.50	.051758
58	.711539	3.57	.025512	7.37	58	.724288	3.50	.052204
59	.711753	3.58	.025954	7.38	59	.724498	3.52	.05265
60	9.711968	3.57	10.026397	7.37	60	9.724709	3.50	10.0530

62°					63°			
Vers.	D. 1.	Ex. sec.	D. 1″.	′	Vers.	D. 1″.	Ex. sec.	D. 1″.
9.724709	3.50	10.053099	7.47	0	9.737200	3.43	10.080153	7.58
.724919	3.50	.053547	7.47	1	.737406	3.43	.080608	7.57
.725129	3.50	.053905	7.47	2	.737612	3.43	.081062	7.57
.725339	3:50	.054443	7.48	3	.737818	3.43	.081516	7.58
.725549	3.50	.054892	7.47	4	.738024	3.43	.081971	7.57
.725759	3.50	.055340	7.47	5	.738230	3.43	.082425	7.58
.725969	3.50	.055788	7.48	6	.738436	3.43	.082880	7.58
.726179	3.48	.056237	7.47	7	.738642	3.42	.083335	7.58
.726388	3.50	.056685	7.48	8	.738847	3.43	.083790	7.58
.726598	3.50	.057134	7.48	9	.739053	3.42	.084245	7.58
.726808	3.48	.057583	7.48	10	.739258	3.43	.084700	7.58
9.727017	3.50	10.058032	7.48	11	9.739464	3.42	10.085155	7.60
.727227	3.48	.058481	7.48	12	.739669	3.43	.085611	7.58
.727436	3.48	.058930	7.48	13	.739875	3.42	.086066	7.60
.727645	3.50	.059379	7.48	14	.740080	3.42	.086522	7.58
.727855	3.48	.059828	7.50	15	.740285	3.42	.086977	7.60
.728064	3.48	.060278	7.48	16	.740490	3.42	.087433	7.60
.728273	3.48	.060727	7.50	17	.740695	3.42	.087889	7.60
.728482	3.48	.061177	7.48	18	.740900	3.42	.088345	7.60
.728691	3.48	.061626	7.50	19	.741105	3.42	.088801	7.62
.728900	3.48	.062076	7.50	20	.741310	3.42	.089258	7.60
9.729109	3.47	10.062526	7.50	21	9.741515	3.40	10.089714	7.62
.729317	3.48	.062976	7.50	22	.741719	3.42	.090171	7.60
.729526	3.48	.063426	7.50	23	.741924	3.42	.090627	7.62
.729735	3.47	.063876	7.52	24	.742129	3.40	.091084	7.62
.729943	3.48	.064327	7.50	25	.742333	3.42	.091541	7.62
.730152	3.47	.064777	7.50	26	.742538	3.40	.091998	7.62
.730360	3.48	.065227	7.52	27	.742742	3.40	.092455	7.62
.730569	3.47	.065678	7.52	28	.742946	3.40	.092912	7.63
.730777	3.47	.066129	7.52	29	.743150	3.42	.093370	7.62
.730985	3.47	.066580	7.50	30	.743355	3.40	.093827	7.63
9.731193	3.47	10.067030	7.53	31	9.743559	3.40	10.094285	7.63
.731401	3.47	.067482	7.52	32	.743762	3.40	.094743	7.62
.731609	3.47	.067933	7.52	33	.743967	3.40	.095200	7.63
.731817	3.47	.068384	7.52	34	.744171	3.40	.095658	7.63
.732025	3.47	.068835	7.53	35	.744375	3.38	.096116	7.65
.732233	3.47	.069287	7.52	36	.744578	3.40	.096575	7.63
.732441	3.45	.069738	7.53	37	.744782	3.40	.097033	7.63
.732648	3.47	.070190	7.53	38	.744986	3.38	.097491	7.65
.732856	3.47	.070642	7.52	39	.745189	3.40	.097950	7.63
.733064	3.45	.071093	7.53	40	.745393	3.38	.098408	7.65
9.733271	3.45	10.071545	7.55	41	9.745596	3.40	10.098867	7.65
.733478	3.47	.071998	7.53	42	.745800	3.38	.099326	7.65
.733686	3.45	.072450	7.53	43	.746003	3.38	.099785	7.65
.733893	3.45	.072902	7.53	44	.746206	3.38	.100244	7.67
.734100	3.45	.073354	7.55	45	.746409	3.40	.100704	7.65
.734307	3.47	.073807	7.55	46	.746613	3.38	.101163	7.67
.734515	3.43	.074260	7.53	47	.746816	3.38	.101623	7.65
.734721	3.45	.074712	7.55	48	.747019	3.38	.102082	7.67
.734928	3.45	.075165	7.55	49	.747222	3.37	.102542	7.67
.735135	3.45	.075618	7.55	50	.747424	3.38	.103002	7.67
9.735342	3.45	10.076071	7.55	51	9.747627	3.38	10.103462	7.67
.735549	3.43	.076524	7.55	52	.747830	3.38	.103922	7.67
.735755	3.45	.076977	7.57	53	.748033	3.37	.104382	7.68
.735962	3.45	.077431	7.55	54	.748235	3.38	.104843	7.67
.736169	3.43	.077884	7.57	55	.748438	3.37	.105303	7.68
.736375	3.43	.078338	7.57	56	.748640	3.38	.105764	7.67
.736581	3.45	.078792	7.55	57	.748843	3.37	.106224	7.68
.736788	3.43	.079245	7.57	58	.749045	3.37	.106685	7.68
.736994	3.43	.079699	7.57	59	.749247	3.37	.107146	7.6
9.737200	3.43	10.080153	7.58	60	9.749449	3.38	10.107607	7

TABLE XXVI.—LOGARITHMIC VERSED SINES

76°					77°				
′	Vers.	D. 1".	Ex. sec.	D. 1".	′	Vers.	D. 1".	Ex. sec.	D. 1".
0	9.879714	2.70	10.496039	11.13	0	9.889329	2.65	10.537241	11.77
1	.879876	2.68	.496707	11.17	1	.889488	2.65	.537947	11.78
2	.880037	2.70	.497377	11.17	2	.889647	2.63	.538654	11.80
3	.880199	2.68	.498047	11.17	3	.889805	2.65	.539362	11.82
4	.880360	2.70	.498717	11.18	4	.889964	2.65	.540071	11.82
5	.880522	2.68	.499388	11.20	5	.890123	2.63	.540780	11.83
6	.880683	2.70	.500060	11.20	6	.890281	2.65	.541490	11.83
7	.880845	2.68	.500732	11.22	7	.890440	2.63	.542200	11.85
8	.881006	2.68	.501405	11.22	8	.890598	2.65	.542911	11.87
9	.881167	2.70	.502078	11.23	9	.890757	2.63	.543623	11.88
10	.881329	2.68	.502752	11.23	10	.890915	2.63	.544336	11.88
11	9.881490	2.68	10.503426	11.27	11	9.891073	2.65	10.545049	11.90
12	.881651	2.68	.504102	11.25	12	.891232	2.63	.545763	11.90
13	.881812	2.68	.504777	11.28	13	.891390	2.63	.546477	11.93
14	.881973	2.68	.505454	11.28	14	.891548	2.63	.547193	11.93
15	.882134	2.68	.506131	11.28	15	.891706	2.63	.547909	11.95
16	.882295	2.68	.506808	11.30	16	.891864	2.63	.548626	11.95
17	.882456	2.68	.507486	11.32	17	.892022	2.63	.549343	11.97
18	.882617	2.67	.508165	11.32	18	.892180	2.63	.550061	11.98
19	.882777	2.68	.508844	11.33	19	.892338	2.63	.550780	12.00
20	.882938	2.68	.509524	11.35	20	.892496	2.63	.551500	12.00
21	9.883099	2.68	10.510205	11.35	21	9.892654	2.63	10.552220	12.02
22	.883259	2.67	.510886	11.37	22	.892812	2.62	.552941	12.03
23	.883420	2.68	.511568	11.37	23	.892969	2.63	.553663	12.03
24	.883581	2.67	.512250	11.38	24	.893127	2.63	.554385	12.07
25	.883741	2.68	.512933	11.40	25	.893285	2.62	.555109	12.07
26	.883902	2.67	.513617	11.40	26	.893442	2.63	.555833	12.07
27	.884062	2.68	.514301	11.42	27	.893600	2.63	.556557	12.10
28	.884223	2.67	.514986	11.43	28	.893758	2.62	.557283	12.10
29	.884383	2.67	.515672	11.43	29	.893915	2.62	.558009	12.12
30	.884543	2.67	.516358	11.45	30	.894072	2.63	.558736	12.12
31	9.884703	2.68	10.517045	11.45	31	9.894230	2.62	10.559463	12.15
32	.884864	2.67	.517732	11.47	32	.894387	2.62	.560192	12.15
33	.885024	2.67	.518420	11.48	33	.894544	2.63	.560921	12.17
34	.885184	2.67	.519109	11.48	34	.894702	2.62	.561651	12.17
35	.885344	2.67	.519798	11.50	35	.894859	2.62	.562381	12.20
36	.885504	2.67	.520488	11.52	36	.895016	2.62	.563113	12.20
37	.885664	2.67	.521179	11.50	37	.895173	2.62	.563845	12.20
38	.885824	2.65	.521870	11.53	38	.895330	2.62	.564577	12.23
39	.885983	2.67	.522562	11.53	39	.895487	2.62	.565311	12.23
40	.886143	2.67	.523254	11.55	40	.895644	2.62	.566045	12.27
41	9.886303	2.67	10.523947	11.57	41	9.895801	2.62	10.566781	12.25
42	.886463	2.65	.524641	11.57	42	.895958	2.62	.567516	12.28
43	.886622	2.67	.525335	11.58	43	.896115	2.62	.568253	12.28
44	.886782	2.65	.526030	11.60	44	.896272	2.60	.568990	12.32
45	.886941	2.67	.526726	11.62	45	.896428	2.62	.569729	12.32
46	.887101	2.65	.527423	11.62	46	.896585	2.62	.570468	12.32
47	.887260	2.67	.528120	11.62	47	.896742	2.60	.571207	12.35
48	.887420	2.65	.528817	11.65	48	.896898	2.62	.571948	12.35
49	.887579	2.67	.529516	11.65	49	.897055	2.60	.572689	12.37
50	.887739	2.65	.530215	11.65	50	.897211	2.62	.573431	12.38
51	9.887898	2.65	10.530914	11.67	51	9.897368	2.60	10.574174	12.38
52	.888057	2.65	.531614	11.68	52	.897524	2.60	.574917	12.42
53	.888216	2.65	.532315	11.70	53	.897680	2.62	.575662	12.42
54	.888375	2.65	.533017	11.70	54	.897837	2.60	.576407	12.43
55	.888534	2.65	.533719	11.72	55	.897993	2.60	.577153	12.45
56	.888693	2.65	.534422	11.73	56	.898149	2.60	.577900	12.45
57	.888852	2.65	.535126	11.73	57	.898305	2.60	.578647	12.45
58	.889011	2.65	.535830	11.75	58	.898461	2.62	.579396	12.48
59	.889170	2.65	.536535	11.77	59	.898618	2.60	.580145	12.50
60	9.889329	2.65	10.537241	11.77	60	9.898774	2.60	10.580895	12.5

66°					67°			
Vers.	D. 1".	Ex. sec.	D. 1".	′	Vers.	D. 1".	Ex. sec.	D.1".
9.773248	3.23	10.163934	7.98	0	9.784809	3.18	10.192931	8.15
.773442	3.23	.164413	7.97	1	.785000	3.18	.193420	8.13
.773636	3.25	.164891	7.98	2	.785191	3.17	.193908	8.15
.773831	3.23	.165370	7.98	3	.785381	3.18	.194397	8.15
.774025	3.23	.165849	7.98	4	.785572	3.18	.194886	8.17
.774219	3.25	.166328	7.98	5	.785763	3.17	.195376	8.15
.774414	3.23	.166807	7.98	6	.785953	3.18	.195865	8.17
.774608	3.23	.167286	8.00	7	.786144	3.17	.196355	8.17
.774802	3.23	.167766	7.98	8	.786334	3.17	.196845	8.17
.774996	3.23	.168245	8.00	9	.786524	3.18	.197335	8.17
.775190	3.23	.168725	8.00	10	.786715	3.17	.197825	8.17
9.775384	3.22	10.169205	8.00	11	9.786905	3.17	10.198315	8.18
.775577	3.23	.169685	8.00	12	.787095	3.17	.198806	8.18
.775771	3.23	.170165	8.02	13	.787285	3.17	.199297	8.18
.775965	3.23	.170646	8.02	14	.787475	3.17	.199788	8.18
.776159	3.22	.171127	8.00	15	.787665	3.17	.200279	8.18
.776352	3.23	.171607	8.02	16	.787855	3.17	.200770	8.20
.776546	3.22	.172088	8.02	17	.788045	3.17	.201262	8.18
.776739	3.23	.172569	8.03	18	.788235	3.17	.201753	8.20
.776933	3.22	.173051	8.02	19	.788425	3.15	.202245	8.20
.777126	3.22	.173532	8.03	20	.788614	3.17	.202737	8.20
9.777319	3.22	10.174014	8.03	21	9.788804	3.15	10.203229	8.22
.777512	3.22	.174496	8.03	22	.788993	3.17	.203722	8.22
.777705	3.23	.174978	8.03	23	.789183	3.15	.204215	8.20
.777899	3.22	.175460	8.03	24	.789372	3.17	.204707	8.22
.778092	3.22	.175942	8.05	25	.789562	3.15	.205200	8.23
.778285	3.20	.176425	8.03	26	.789751	3.15	.205694	8.22
.778477	3.22	.176907	8.05	27	.789940	3.17	.206187	8.23
.778670	3.22	.177390	8.05	28	.790130	3.15	.206681	8.22
.778863	3.22	.177873	8.05	29	.790319	3.15	.207174	8.23
.779056	3.20	.178356	8.05	30	.790508	3.15	.207668	8.23
9.779248	3.22.	10.178839	8.07	31	9.790697	3.15	10.208162	8.25
.779441	3.22	.179323	8.07	32	.790886	3.15	.208657	8.23
.779634	3.20	.179807	8.05	33	.791075	3.15	.209151	8.25
.779826	3.20	.180290	8.07	34	.791264	3.15	.209646	8.25
.780018	3.22	.180774	8.08	35	.791453	3.13	.210141	8.25
.780211	3.20	.181259	8.07	36	.791641	3.15	.210636	8.25
.780403	3.20	.181743	8.07	37	.791830	3.15	.211131	8.27
.780595	3.20	.182227	8.08	38	.792019	3.13	.211627	8.27
.780787	3.22	.182712	8.08	39	.792207	3.15	.212123	8.25
.780980	3.20	.183197	8.08	40	.792396	3.13	.212618	8.28
9.781172	3.20	10.183682	8 08	41	9.792584	3.13	10.213115	8.27
.781364	3.20	.184167	8.10	42	.792772	3.15	.213611	8.27
.781556	3.18	.184653	8.08	43	.792961	3.13	.214107	8.28
.781747	3.20	.185138	8.10	44	.793149	3.13	.214604	8.28
.781939	3.20	.185624	8.10	45	.793337	3.13	.215101	8.28
.782131	3.20	.186110	8.10	46	.793525	3.15	.215598	8.28
.782323	3.18	.186596	8.10	47	.793714	3.13	.216095	8.30
.782514	3.20	.187082	8.10	48	.793902	3.13	.216593	8.28
.782706	3.18	.187568	8.12	49	.794090	3.12	.217090	8.30
.782897	3.20	.188055	8.12	50	.794277	3.13	.217588	8.30
9.783089	3.18	10.188542	8.12	51	9.794465	3.13	10.218086	8.32
.783280	3.18	.189029	8.12	52	.794653	3.13	.218585	8.30
.783471	3.20	.189516	8.12	53	.794841	3.12	.219083	8.32
.783663	3.18	.190003	8.13	54	.795028	3.13	.219582	8.32
.783854	3.18	.190491	8.12	55	.795216	3.13	.220081	8.32
.784045	3.18	.190978	8.13	56	.795404	3.12	.220580	8.32
.784236	3.18	.191466	8.13	57	.795591	3.13	.221079	8.32
.784427	3.18	.191954	8.15	58	.795779	3.12	.221578	[illegible]
.784618	3.18	.192443	8.13	59	.795966	3.12	.222078	[illegible]
9.784809	8.18	10.192931	8.15	60	9.796153	3.13	10.222578	

TABLE XXVI.—LOGARITHMIC VERSED SINES

	68°					69°			
′	Vers.	D. 1″.	Ex. sec.	D. 1″.	′	Vers.	D. 1″.	Ex. sec.	D.
0	9.796153	3.13	10.222578	8.33	0	9.807286	3.07	10.252957	
1	.796341	3.12	.223078	8.33	1	.807470	3.07	.253470	
2	.796528	3.12	.223578	8.35	2	.807654	3.05	.253983	
3	.796715	3.12	.224079	8.33	3	.807837	3.07	.254497	
4	.796902	3.12	.224579	8.35	4	.808021	3.05	.255010	
5	.797089	3.12	.225080	8.35	5	.808204	3.07	.255524	
6	.797276	3.12	.225581	8.37	6	.808388	3.05	.256039	
7	.797463	3.12	.226083	8.35	7	.808571	3.07	.256553	
8	.797650	3.12	.226584	8.37	8	.808755	3.05	.257068	
9	.797837	3.10	.227086	8.37	9	.808938	3.05	.257582	
10	.798023	3.12	.227588	8.37	10	.809121	3.07	.258098	
11	9.798210	3.12	10.228090	8.37	11	9.809305	3.05	10.258613	
12	.798397	3.10	.228592	8.38	12	.809488	3.05	.259129	
13	.798583	3.12	.229095	8.38	13	.809671	3.05	.259644	
14	.798770	3.10	.229598	8.38	14	.809854	3.05	.260160	
15	.798956	3.10	.230101	8.38	15	.810037	3.05	.260677	
16	.799142	3.12	.230604	8.38	16	.810220	3.05	.261193	
17	.799329	3.10	.231107	8.40	17	.810403	3.03	.261710	
18	.799515	3.10	.231611	8.40	18	.810585	3.05	.262227	
19	.799701	3.10	.232115	8.40	19	.810768	3.05	.262744	
20	.799887	3.12	.232619	8.40	20	.810951	3.05	.263262	
21	9.800074	3.10	10.233123	8 40	21	9.811134	3.03	10.263779	
22	.800260	3.10	.233627	8.42	22	.811316	3.05	.264297	
23	.800446	3.08	.234132	8.42	23	.811499	3.03	.264815	
24	.800631	3.10	.234637	8.42	24	.811681	3.05	.265334	
25	.800817	3.10	.235142	8.42	25	.811864	3.03	.265853	
26	.801003	3.10	.235647	8.43	26	.812046	3.03	.266371	
27	.801189	3.10	.236153	8.42	27	.812228	3.03	.266891	
28	.801375	3.08	.236658	8.43	28	.812410	3.05	.267410	
29	.801560	3.10	.237164	8.43	29	.812593	3.03	.267930	
30	.801746	3.08	.237670	8.45	30	.812775	3.03	.268449	
31	9.801931	3.10	10.238177	8.43	31	9.812957	3.03	10.268970	
32	.802117	3.08	.238683	8.45	32	.813139	3.03	.269490	
33	.802302	3.08	.239190	8.45	33	.813321	3.03	.270011	
34	.802487	3.10	.239697	8.45	34	.813503	3.03	.270531	
35	.802673	3.08	.240204	8.47	35	.813685	3.02	.271052	
36	.802858	3.08	.240712	8.45	36	.813866	3.03	.271574	
37	.803043	3.08	.241219	8.47	37	.814048	3.03	.272095	
38	.803228	3.08	.241727	8.47	38	.814230	3.02	.272617	
39	.803413	3.08	.242235	8.48	39	.814411	3.03	.273139	
40	.803598	3.08	.242744	8.47	40	.814593	3.03	.273662	
41	9.803783	3.08	10.243252	8.48	41	9.814775	3.02	10.274184	
42	.803968	3.08	.243761	8.48	42	.814956	3.02	.274707	
43	.804153	3.08	.244270	8.48	43	.815137	3.03	.275230	
44	.804338	3.07	.244779	8.50	44	.815319	3.02	.275753	
45	.804522	3.08	.245289	8.48	45	.815500	3.02	.276277	
46	.804707	3.08	.245798	8.50	46	.815681	3.02	.276801	
47	.804892	3 07	.246308	8.50	47	.815862	3.03	.277325	
48	.805076	3.08	.246818	8.52	48	.816044	3.02	.277849	
49	.805261	3.07	.247329	8.50	49	.816225	3.02	.278374	
50	.805445	3.07	.247839	8.52	50	.816406	3.02	.278899	
51	9.805629	3.08	10.248350	8.52	51	9.816587	3.00	10.279424	
52	.805814	3.07	.248861	8.52	52	.816767	3.02	.279949	
53	.805998	3.07	.249372	8.52	53	.816948	3.02	.280475	
54	.806182	3.07	.249883	8.53	54	.817129	3.02	.281000	
55	.806366	3.07	.250395	8.53	55	.817310	3.00	.281527	
56	.806550	3.07	.250907	8.53	56	.817490	3.02	.282053	
57	.806734	3.07	.251419	8.55	57	.817671	3.02	.282580	
58	.806918	3.07	.251932	8.53	58	.817852	3.00	.283106	
59	.807102	3.07	.252444	8.55	59	.818032	3.02	.283634	
60	9.807286	3.07	10.252957	8.55	60	9.818213	3.00	10.28416	

70°					71°				
′	Vers.	D. 1″.	Ex. sec.	D. 1″	′	Vers.	D. 1″.	Ex. sec.	D.1″.
0	9.818213	3.00	10.284161	8.80	0	9.828938	2.95	10.316296	9.07
1	.818393	3.00	.284689	8.78	1	.829115	2.95	.316840	9.08
2	.818573	3.02	.285216	8.82	2	.829292	2.95	.317385	9.07
3	.818754	3.00	.285745	8.80	3	.829469	2.95	.317929	9.10
4	.818934	3.00	.286273	8.82	4	.829646	2.95	.318475	9.08
5	.819114	3.00	.286802	8.82	5	.829823	2.95	.319020	9.08
6	.819294	3.00	.287331	8.82	6	.830000	2.95	.319565	9.10
7	.819474	3.00	.287860	8.82	7	.830177	2.93	.320111	9.12
8	.819654	3.00	.288389	8.83	8	.830353	2.95	.320658	9.10
9	.819834	3.00	.288919	8.83	9	.830530	2.93	.321204	9.12
10	.820014	3.00	.289449	8.83	10	.830706	2.95	.321751	9.12
11	9.820194	3.00	10.289979	8.85	11	9.830883	2.93	10.322298	9.12
12	.820374	2.98	.290510	8.85	12	.831059	2.95	.322845	9.13
13	.820553	3.00	.291041	8.85	13	.831236	2.93	.323393	9.13
14	.820733	3.00	.291572	8.85	14	.831412	2.95	.323941	9.13
15	.820913	2.98	.292103	8.87	15	.831589	2.93	.324489	9.15
16	.821092	3.00	.292635	8.85	16	.831765	2.93	.325038	9.15
17	.821272	2.98	.293166	8.87	17	.831941	2.93	.325587	9.15
18	.821451	3.00	.293698	8.88	18	.832117	2.93	.326136	9.17
19	.821631	2.98	.294231	8.88	19	.832293	2.93	.326686	9.15
20	.821810	2.98	.294764	8.87	20	.832469	2.93	.327235	9.18
21	9.821989	2.98	10.295296	8.90	21	9.832645	2.93	10.327786	9.17
22	.822168	3.00	.295830	8.88	22	.832821	2.93	.328336	9.18
23	.822348	2.98	.296363	8.90	23	.832997	2.93	.328887	9.18
24	.822527	2.98	.296897	8.90	24	.833173	2.93	.329438	9.18
25	.822706	2.98	.297431	8.90	25	.833349	2.93	.329989	9.20
26	.822885	2.98	.297965	8.92	26	.833525	2.92	.330541	9.20
27	.823064	2.98	.298500	8.90	27	.833700	2.93	.331093	9.20
28	.823243	2.97	.299034	8.93	28	.833876	2.92	.331645	9.22
29	.823421	2.98	.299570	8.92	29	834051	2.93	.332198	9.20
30	.823600	2.98	.300105	8.93	30	.834227	2.92	.332750	9.23
31	9.823779	2.98	10.300641	8.92	31	9.834402	2.93	10.333304	9.22
32	.823958	2.97	.301176	8.95	32	.834578	2.92	.333857	9.23
33	.824136	2.98	.301713	8.93	33	.834753	2.92	.334411	9.23
34	.824315	2.97	.302249	8.95	34	.834928	2.93	.334965	9.25
35	.824493	2.98	.302786	8.95	35	.835104	2.92	.335520	9.23
36	.824672	2.97	.303323	8.95	36	.835279	2.92	.336074	9.25
37	.824850	2.97	.303860	8.97	37	.835454	2.92	.336629	9.27
38	.825028	2.98	.304398	8 97	38	.835629	2.92	.337185	9.27
39	.825207	2.97	.304936	8.97	39	.835804	2.92	.337741	9.27
40	.825385	2.97	.305474	8.97	40	.835979	2.92	.338297	9.27
41	9.825563	2.97	10.306012	8.98	41	9.836154	2.92	10.338853	9.28
42	.825741	2.97	.306551	8.98	42	.836329	2.92	.339410	9.28
43	.825919	2.97	.307090	8.98	43	.836504	2.90	.339967	9.28
44	.826097	2.97	.307629	9.00	44	.836678	2.92	.340524	9.30
45	.826275	2.97	.308169	8.98	45	.836853	2.92	.341082	9.30
46	.826453	2.97	.308708	9.02	46	.837028	2.90	.341640	9.30
47	.826631	2.97	.309249	9.00	47	.837202	2.88	.342198	9.30
48	.826809	2.97	.309789	9.02	48	.837377	2.90	.342756	9.32
49	.826987	2.95	.310330	9.02	49	.837551	2.92	.343315	9.33
50	.827164	2.97	.310871	9.02	50	.837726	2.90	.343875	9.32
51	9.827342	2.95	10.311412	9.02	51	9.837900	2.92	10.344434	9.33
52	.827519	2.97	.311953	9.03	52	.838075	2.90	.344994	9.33
53	.827697	2.95	.312495	9.03	53	.838249	2.90	.345554	9.35
54	.827874	2.97	.313037	9.05	54	.838423	2.90	.346115	9.35
55	.828052	2.95	.313580	9.03	55	.838597	2.90	.346676	9.35
56	.828229	2.95	.314122	9.05	56	.838771	2.90	.347237	9.35
57	.828406	2.97	.314665	9.07	57	.838945	2.90	.347798	9.37
58	.828584	2.95	.315209	9.05	58	.839119	2.90	.348360	9.3
59	.828761	2.95	.315752	9.07	59	.839293	2.90	.348922	9.
60	9.828938	2.95	10.316296	9.07	60	9.839467	2.90	10.349485	9

72°					73°				
′	Vers.	D. 1.	Ex. sec.	D. 1".	′	Vers.	D. 1".	Ex. sec.	D. 1".
0	9.839467	2.90	10.349485	9.38	0	9.849805	2.85	10.383870	9.73
1	.839641	2.90	.350048	9.38	1	.849976	2.85	.384454	9.73
2	.839815	2.90	.350611	9.40	2	.850147	2.83	.385038	9.75
3	.839989	2.88	.351175	9.38	3	.850317	2.85	.385623	9.77
4	.840162	2.90	.351738	9.42	4	.850488	2.83	.386209	9.75
5	.840336	2.90	.352303	9.40	5	.850658	2.85	.386794	9.77
6	.840510	2.88	.352867	9.42	6	.850829	2.83	.387380	9.78
7	.840683	2.90	.353432	9.42	7	.850999	2.83	.387967	9.78
8	.840857	2.88	.353997	9.43	8	.851169	2.85	.388554	9.78
9	.841030	2.90	.354563	9.43	9	.851340	2.83	.389141	9.78
10	.841204	2.88	.355129	9.43	10	.851510	2.83	.389728	9.80
11	9.841377	2.88	10.355695	9.43	11	9.851680	2.83	10.390316	9.82
12	.841550	2.88	.356261	9.45	12	.851850	2.83	.390905	9.80
13	.841723	2.88	.356828	9.45	13	.852020	2.83	.391493	9.82
14	.841896	2.90	.357395	9.47	14	.852190	2.83	.392082	9.83
15	.842070	2.88	.357963	9.47	15	.852360	2.83	.392672	9.83
16	.842243	2.88	.358531	9.47	16	.852530	2.83	.393262	9.85
17	.842416	2.88	.359099	9.48	17	.852700	2.83	.393852	9.85
18	.842589	2.88	.359668	9.48	18	.852870	2.83	.394443	9.85
19	.842762	2.87	.360237	9.48	19	.853040	2.82	.395034	9.85
20	.842934	2.88	.360806	9.50	20	.853209	2.83	.395625	9.87
21	9.843107	2.88	10.361376	9.50	21	9.853379	2.83	10.396217	9.87
22	.843280	2.88	.361946	9.50	22	.853549	2.82	.396809	9.88
23	.843453	2.87	.362516	9.52	23	.853718	2.83	.397402	9.88
24	.843625	2.88	.363087	9.52	24	.853888	2.82	.397995	9.90
25	.843798	2.87	.363658	9.52	25	.854057	2.83	.398589	9.88
26	.843970	2.88	.364229	9.53	26	.854227	2.82	.399182	9.92
27	.844143	2.87	.364801	9.53	27	.854396	2.82	.399777	9.90
28	.844315	2.88	.365373	9.53	28	.854565	2.83	.400371	9.92
29	.844488	2.87	.365945	9.55	29	.854735	2.82	.400966	9.93
30	.844660	2.87	.366518	9.55	30	.854904	2.82	.401562	9.93
31	9.844832	2.87	10.367091	9.57	31	9.855073	2.82	10.402158	9.93
32	.845004	2.88	.367665	9.57	32	.855242	2.82	.402754	9.95
33	.845177	2.87	.368239	9.57	33	.855411	2.82	.403351	9.95
34	.845349	2.87	.368813	9.57	34	.855580	2.82	.403948	9.95
35	.845521	2.87	.369387	9.58	35	.855749	2.82	.404545	9.97
36	.845693	2.87	.369962	9.60	36	.855918	2.82	.405143	9.98
37	.845865	2.87	.370538	9.58	37	.856087	2.80	.405742	9.97
38	.846037	2.85	.371113	9.60	38	.856255	2.82	.406340	9.98
39	.846208	2.87	.371689	9.62	39	.856424	2.82	.406939	10.00
40	.846380	2.87	.372266	9.60	40	.856593	2.82	.407539	10.00
41	9.846552	2.87	10.372842	9.62	41	9.856762	2.80	10.408139	10.00
42	.846724	2.85	.373419	9.63	42	.856930	2.82	.408739	10.02
43	.846895	2.87	.373997	9.63	43	.857099	2.80	.409340	10.02
44	.847067	2.85	.374575	9.63	44	.857267	2.82	.409941	10.03
45	.847238	2.87	.375153	9.63	45	.857436	2.80	.410543	10.03
46	.847410	2.85	.375731	9.65	46	.857604	2.80	.411145	10.03
47	.847581	2.87	.376310	9.67	47	.857772	2.82	.411747	10.05
48	.847753	2.85	.376890	9.65	48	.857941	2.80	.412350	10.07
49	.847924	2.85	.377469	9.67	49	.858109	2.80	.412954	10.05
50	.848095	2.87	.378049	9.68	50	.858277	2.80	.413557	10.07
51	9.848267	2.85	10.378630	9.67	51	9.858445	2.80	10.414161	10.08
52	.848438	2.85	.379210	9.70	52	.858613	2.80	.414766	10.08
53	.848609	2.85	.379792	9.68	53	.858781	2.80	.415371	10.08
54	.848780	2.85	.380373	9.70	54	.858949	2.80	.415976	10.10
55	.848951	2.85	.380955	9.70	55	.859117	2.80	.416582	10.12
56	.849122	2.85	.381537	9.72	56	.859285	2.80	.417189	10.10
57	.849293	2.85	.382120	9.72	57	.859453	2.80	.417795	10.12
58	.849164	2.83	.382703	9.72	58	.859621	2.78	.418402	10.1
59	.849634	2.85	.383286	9.73	59	.859788	2.80	.419010	10.1
60	9.849805	2.85	10.383870	9.73	68	9.859956	2.80	10.419618	10.

74°					75°			
Vers.	D. 1".	Ex. sec.	D. 1".	′	Vers.	D. 1".	Ex. sec.	D. 1".
9.859956	2.80	10.419618	10.13	0	9.869924	2.75	10.456928	10.60
.860124	2.78	.420226	10.15	1	.870089	2.73	.457564	10.62
.860291	2.80	.420835	10.17	2	.870253	2.75	.458201	10.63
.860459	2.78	.421445	10.15	3	.870418	2.73	.458839	10.62
.860626	2.80	.422054	10.17	4	.870582	2.75	.459476	10.65
.860794	2.78	.422664	10.18	5	.870747	2.73	.460115	10.65
.860961	2.78	.423275	10.18	6	.870911	2.75	.460754	10.65
.861128	2.80	.423886	10.20	7	.871076	2.73	.461393	10.67
.861296	2.78	.424498	10.20	8	.871240	2.73	.462033	10.67
.861463	2.78	.425110	10.20	9	.871404	2.73	.462673	10.68
.861630	2.78	.425722	10.22	10	.871568	2.73	.463314	10 70
9.861797	2.78	10.426335	10.22	11	9.871732	2.73	10.463956	10.70
.861964	2.78	.426948	10.23	12	.871896	2.73	.464598	10.70
.862131	2.78	.427562	10.23	13	.872060	2.73	.465240	10.72
.862298	2.78	.428176	10.23	14	.872224	2.73	.465883	10.73
.862465	2.78	.428790	10.27	15	.872388	2.73	.466527	10.73
.862632	2.78	.429406	10.25	16	.872552	2.73	.467171	10.73
.862799	2.77	.430021	10.27	17	.872716	2.73	.467815	10.75
.862965	2.78	.430637	10.27	18	.872880	2.72	.468460	10.77
.863132	2.78	.431253	10.28	19	.873043	2.73	.469106	10.77
.863299	2.77	.431870	10.30	20	.873207	2.73	.469752	10.77
9.863465	2.78	10.432488	10.28	21	9.873371	2.72	10.470398	10.78
.863632	2.78	.433105	10.32	22	.873534	2.73	.471045	10.80
.863799	2.77	.433724	10.30	23	.873698	2.72	.471693	10.80
.863965	2.77	.434342	10.32	24	.873861	2.73	.472341	10.82
.864131	2.78	.434961	10.33	25	.874025	2.72	.472990	10.82
.864298	2.77	.435581	10.33	26	.874188	2.72	.473639	10.83
.864464	2.77	.436201	10.33	27	.874351	2.73	.474289	10.83
.864630	2.78	.436821	10.35	28	.874515	2.72	.474939	10.85
.864797	2.77	.437442	10.37	29	.874678	2.72	.475590	10.87
.864963	2.77	.438064	10.37	30	.874841	2.72	.476242	10.85
9.865129	2.77	10.438686	10.37	31	9.875004	2.72	10.476893	10.88
.865295	2.77	.439308	10.38	32	.875167	2.72	.477546	10.88
.865461	2.77	.439931	10.38	33	.875330	2.72	.478199	10.88
.865627	2.77	.440554	10.40	34	.875493	2.72	.478852	10.90
.865793	2.77	.441178	10.40	35	.875656	2.72	.479506	10.92
.865959	2.75	.441802	10.42	36	.875819	2.72	.480161	10.92
.866124	2.77	.442427	10.42	37	.875982	2.72	.480816	10.93
.866290	2.77	.443052	10.43	38	.876145	2.72	.481472	10.93
.866456	2.77	.443678	10.43	39	.876308	2.70	.482128	10.95
.866622	2.75	.444304	10.45	40	.876470	2.72	.482785	10.95
9.866787	2.77	10.444931	10.45	41	9.876633	2.72	10.483442	10.97
.866953	2.75	.445558	10.45	42	.876796	2.70	.484100	10.98
.867118	2.77	.446185	10.47	43	.876958	2.72	.484759	10.98
.867284	2.75	.446813	10.48	44	.877121	2.70	.485418	10.98
.867449	2.75	.447442	10.48	45	.877283	2.70	.486077	10.98
.867614	2.77	.448071	10.48	46	.877445	2.72	.486738	11.00
.867780	2.75	.448700	10.50	47	.877608	2.70	.487398	11 02
.867945	2.75	.449330	10.52	48	.877770	2.70	.488059	11.03
.868110	2.75	.449961	10.52	49	.877932	2.72	.488721	11.05
.868275	2.77	.450592	10.52	50	.878095	2.70	.489384	11.05
9.868441	2.75	10.451223	10.53	51	9.878257	2.70	10.490047	11.05
.868606	2.75	.451855	10.53	52	.878419	2.70	.490710	11.07
.868771	2.75	.452487	10.55	53	.878581	2.70	.491374	11.08
.868936	2.73	.453120	10.57	54	.878743	2.70	.492039	11.08
.869100	2.75	.453754	10.57	55	.878905	2.70	.492704	11.10
.869265	2.75	.454388	10.57	56	.879067	2.70	.493370	11.10
.869430	2.75	.455022	10.58	57	.879229	2.68	.494036	11.12
.869595	2.75	.455657	10.58	58	.879390	2.70	.494703	11.13
.869760	2.73	.456292	10.60	59	.879552	2.70	.495371	11.1
9.869924	2.75	10.456928	10.60	60	9.879714	2.70	10.496039	11.

TABLE XXVI.—LOGARITHMIC VERSED SINES

76°					77°				
′	Vers.	D. 1″.	Ex. sec.	D. 1″.	′	Vers.	D. 1″.	Ex. sec.	D. 1″.
0	9.879714	2.70	10.496039	11.13	0	9.889329	2.65	10.537241	11.77
1	.879876	2.68	.496707	11.17	1	.889488	2.65	.537947	11.78
2	.880037	2.70	.497377	11.17	2	.889647	2.63	.538654	11.80
3	.880199	2.68	.498047	11.17	3	.889805	2.65	.539362	11.82
4	.880360	2.70	.498717	11.18	4	.889964	2.65	.540071	11.82
5	.880522	2.68	.499388	11.20	5	.890123	2.63	.540780	11.83
6	.880683	2.70	.500060	11.20	6	.890281	2.65	.541490	11.83
7	.880845	2.68	.500732	11.22	7	.890440	2.63	.542200	11.85
8	.881006	2.68	.501405	11.22	8	.890598	2.65	.542911	11.87
9	.881167	2.70	.502078	11.23	9	.890757	2.63	.543623	11.88
10	.881329	2.68	.502752	11.23	10	.890915	2.63	.544336	11.88
11	9.881490	2.68	10.504426	11.27	11	9.891073	2.65	10.545049	11.90
12	.881651	2.68	.504102	11.25	12	.891232	2.63	.545763	11.90
13	.881812	2.68	.504777	11.28	13	.891390	2.63	.546477	11.93
14	.881973	2.64	.505454	11.28	14	.891548	2.63	.547193	11.93
15	.882134	2.63	.506131	11.28	15	.891706	2.63	.547909	11.95
16	.882295	2.68	.505808	11.30	16	.891864	2.63	.548626	11.95
17	.882456	2.68	.507486	11.32	17	.892022	2.63	.549343	11.97
18	.882617	2.67	.508165	11.32	18	.892180	2.63	.550061	11.98
19	.882777	2.68	.503844	11.33	19	.892338	2.63	.550780	12.00
20	.882938	2.68	.509524	11.35	20	.892496	2.63	.551500	12.00
21	9.883099	2.68	10.510205	11.35	21	9.892654	2.63	10.552220	12.02
22	.883260	2.67	.510886	11.37	22	.892812	2.62	.552941	12.03
23	.883420	2.68	.511568	11.37	23	.892969	2.63	.553663	12.03
24	.883581	2.67	.512250	11.38	24	.893127	2.63	.554385	12.07
25	.883741	2.68	.512933	11.40	25	.893285	2.62	.555109	12.07
26	.883902	2.67	.513617	11.40	26	.893442	2.63	.555833	12.07
27	.884062	2.68	.514301	11.42	27	.893600	2.63	.556557	12.10
28	.884223	2.67	.514986	11.43	28	.893758	2.62	.557283	12.10
29	.884383	2.67	.515672	11.43	29	.893915	2.62	.558009	12.12
30	.884543	2.67	.516358	11.45	30	.894072	2.63	.558736	12.12
31	9.884703	2.68	10.517045	11.45	31	9.894230	2.62	10.559463	12.15
32	.884864	2.67	.517732	11.47	32	.894387	2.62	.560192	12.15
33	.885024	2.67	.518420	11.48	33	.894544	2.63	.560921	12.17
34	.885184	2.67	.519109	11.48	34	.894702	2.62	.561651	12.17
35	.885344	2.67	.519798	11.50	35	.894859	2.62	.562381	12.20
36	.885504	2.67	.520488	11.52	36	.895016	2.62	.563113	12.20
37	.885664	2.67	.521179	11.50	37	.895173	2.62	.563845	12.20
38	.885824	2.65	.521870	11.53	38	.895330	2.62	.564577	12.23
39	.885983	2.67	.522562	11.53	39	.895487	2.62	.565311	12.23
40	.886143	2.67	.523254	11.55	40	.895644	2.62	.566045	12.27
41	9.886303	2.67	10.523947	11.57	41	9.895801	2.62	10.566781	12.25
42	.886463	2.65	.524641	11.57	42	.895958	2.62	.567516	12.28
43	.886622	2.67	.525335	11.58	43	.896115	2.62	.568253	12.28
44	.886782	2.65	.526030	11.60	44	.896272	2.60	.568990	12.32
45	.886941	2.67	.526726	11.62	45	.896428	2.62	.569729	12.32
46	.887101	2.65	.527423	11.62	46	.896585	2.62	.570468	12.32
47	.887260	2.67	.528120	11.62	47	.896742	2.60	.571207	12.35
48	.887420	2.65	.528817	11.65	48	.896898	2.62	.571948	12.35
49	.887579	2.67	.529516	11.65	49	.897055	2.60	.572689	12.37
50	.887739	2.65	.530215	11.65	50	.897211	2.62	.573431	12.38
51	9.887898	2.65	10.530914	11.67	51	9.897368	2.60	10.574174	12.38
52	.888057	2.65	.531614	11.68	52	.897524	2.60	.574917	12.42
53	.888216	2.65	.532315	11.70	53	.897680	2.62	.575662	12.42
54	.888375	2.65	.533017	11.70	54	.897837	2.60	.576407	12.43
55	.888534	2.65	.533719	11.72	55	.897993	2.60	.577153	12.45
56	.888693	2.65	.534422	11.73	56	.898149	2.60	.577900	12.45
57	.888852	2.65	.535126	11.73	57	.898305	2.60	.578647	12.48
58	.889011	2.65	.535830	11.75	58	.898461	2.62	.579396	12.48
59	.889170	2.65	.536535	11.77	59	.898618	2.60	.580145	12.50
60	9.889329	2.65	10.537241	11.77	60	9.898774	2.60	10.580895	12.5

78°					79°			
Vers.	D. 1".	Ex. sec.	D. 1".	′	Vers.	D. 1".	Ex. sec.	D. 1".
9.898774	2.60	10.580895	12.50	0	9.908051	2.55	10.627452	13.40
.898930	2.60	.581645	12.53	1	.908204	2.55	.628256	13.40
.899086	2.58	.582397	12.53	2	.908357	2.57	.629060	13.43
.899241	2.60	.583149	12.57	3	.908511	2.55	.629866	13.45
.899397	2.60	.583903	12.57	4	.908664	2.55	.630673	13.45
.899553	2.60	.584657	12.57	5	.908817	2.55	.631480	13.48
.899709	2.60	.585411	12.60	6	.908970	2.55	.632289	13.48
.899865	2.58	.586167	12.60	7	.909123	2.55	.633098	13.52
.900020	2.60	.586923	12.63	8	.909276	2.53	.633909	13.52
.900176	2.58	.587681	12.63	9	.909428	2.55	.634720	13.55
.900331	2.60	.588439	12.65	10	.909581	2.55	.635533	13.55
9.900487	2.58	10.589198	12.65	11	9.909734	2.55	10.636346	13.58
.900642	2.60	.589957	12.68	12	.909887	2.53	.637161	13.58
.900798	2.58	.590718	12.68	13	.910039	2.55	.637976	13.60
.900953	2.58	.591479	12.72	14	.910192	2.55	.638792	13.63
.9 1108	2.60	.592242	12.72	15	.910345	2.53	.639610	13.63
.901264	2.58	.593005	12.73	16	.910497	2.55	.640428	13.67
.901419	2.58	.593769	12.73	17	.910650	2.53	.641248	13.67
.901574	2.58	.594533	12.77	18	.910802	2.55	.642068	13.70
.901729	2.58	.595299	12.78	19	.910955	2.53	.642890	13.72
.901884	2.60	.596066	12.78	20	.911107	2.53	.643713	13.72
9.902040	2.58	10.596833	12.80	21	9.911259	2.55	10.644536	13.75
.902195	2.58	.597601	12.82	22	.911412	2.53	.645361	13.75
.902350	2 57	.598370	12.83	23	.911564	2.53	.646186	13.78
.902504	2.58	.599140	12.85	24	.911716	2.53	.647013	13.80
.902659	2.58	.599911	12.85	25	.911868	2.53	.647841	13.82
.902814	2.58	.600682	12.88	26	.912020	2.53	.648670	13.82
.902969	2.58	.601455	12.88	27	.912172	2.53	.649499	13.85
.903124	2.57	.602228	12.92	28	.912324	2.53	.650330	13.87
.903278	2.58	.603003	12.92	29	.912476	2.53	.651162	13.88
.903433	2.58	.603778	12.93	30	.912628	2.53	.651995	13.90
9.903588	2.57	10.604554	12.95	31	9.912780	2.53	10.652829	13.92
.903742	2.58	.605331	12.95	32	.912932	2.53	.653664	13.95
.903897	2.57	.606108	12.98	33	.913084	2.52	.654501	13.95
.904051	2.58	.606887	13.00	34	.913235	2.53	.655338	13.97
.904206	2.57	.607667	13.00	35	.913387	2.53	.656176	14.00
.904360	2.57	.608447	13.02	36	.913539	2.52	.657016	14.00
.904514	2.57	.609228	13.03	37	.913690	2.53	.657856	14.03
.904668	2.58	.610010	13.07	38	.913842	2.52	.658698	14.03
.904823	2.57	.610794	13 07	39	.913993	2.53	.659540	14.07
.904977	2.57	.611578	13.08	40	.914145	2.52	.660384	14.08
9.905131	2.57	10.612363	13.08	41	9.914296	2.53	10.661229	14.10
.905285	2.57	.613148	13.12	42	.914448	2.52	.662075	14.12
.905439	2.57	.613935	13.13	43	.914599	2.52	.662922	14.13
.905593	2.57	.614723	13.13	44	.914750	2.53	.663770	14.15
.905747	2.57	.615511	13.17	45	.914902	2.52	.664619	14.18
.905901	2.57	.616301	13.17	46	.915053	2.52	.665470	14.18
.906055	2.57	.617091	13.20	47	.915204	2.52	.666321	14.22
.906209	2.57	.617883	13.20	48	.915355	2.52	.667174	14.23
.906363	2.55	.618675	13.22	49	.915506	2.52	.668028	14.25
.906516	2.57	.619468	13.23	50	.915657	2.52	.668883	14.27
9.906670	2.57	10.620262	13.25	51	9.915808	2.52	10.669739	14.28
.906824	2.55	.621057	13.27	52	.915959	2.52	.670596	14.30
.906977	2.57	.621853	13.28	53	.916110	2.52	.671454	14.33
.907131	2.55	.622650	13.30	54	.916261	2.52	.672314	14.33
.907284	2.57	.623448	13.32	55	.916412	2.50	.673174	14.37
.907438	2.55	.624247	13.33	56	.916562	2.52	.674036	14.38
.907591	2.55	.625047	13.35	57	.916713	2.52	.674899	14.40
.907744	2.57	.625848	13.37	58	.916864	2.50	.675763	14.42
.907898	2.55	.626650	13.37	59	.917014	2.52	.676628	14.45
9.908051	2.55	10.627452	13.40	60	9.917165	2.52	10.677495	14.4

TABLE XXVI.—LOGARITHMIC VERSED SINES

80°					81°			
′	Vers.	D. 1″.	Ex. sec.	D. 1″.	′	Vers.	D. 1″.	Ex. sec.
0	9.917165	2.52	10.677495	14.45	0	9.926119	2.47	10.731786
1	.917316	2.50	.678362	14.48	1	.926267	2.47	.732733
2	.917466	2.50	.679231	14.50	2	.926415	2.45	.733680
3	.917616	2.52	.680101	14.52	3	.926562	2.47	.734630
4	.917767	2.50	.680972	14.55	4	.926710	2.47	.735580
5	.917917	2.52	.681845	14.55	5	.926858	2.47	.736532
6	.918068	2.50	.682718	14.58	6	.927006	2.45	.737486
7	.918218	2.50	.683593	14.60	7	.927153	2.47	.738441
8	.918368	2.50	.684469	14.62	8	.927301	2.45	.739398
9	.918518	2.50	.685346	14.63	9	.927448	2.47	.740356
10	.918668	2.50	.686224	14.67	10	.927596	2.45	.741316
11	9.918818	2.50	10.687104	14.68	11	9.927743	2.47	10.742277
12	.918968	2.50	.687985	14.70	12	.927891	2.45	.743239
13	.919118	2.50	.688867	14.72	13	.928038	2.45	.744204
14	.919268	2.50	.689750	14.73	14	.928185	2.47	.745169
15	.919418	2.50	.690634	14.77	15	.928333	2.45	.746137
16	.919568	2.50	.691520	14.78	16	.928480	2.45	.747105
17	.919718	2.50	.692407	14.80	17	.928627	2.45	.748076
18	.919868	2.50	.693295	14.83	18	.928774	2.45	.749048
19	.920018	2.48	.694185	14.83	19	.928921	2.45	.750021
20	.920167	2.50	.695075	14.87	20	.929068	2.45	.750996
21	9.920317	2.48	10.695967	14.90	21	9.929215	2.45	10.751973
22	.920466	2.50	.696861	14.90	22	.929362	2.45	.752951
23	.920616	2.50	.697755	14.93	23	.929509	2.45	.753931
24	.920766	2.48	.698651	14.95	24	.929656	2.45	.754912
25	.920915	2.48	.699548	14.97	25	.929803	2.45	.755895
26	.921064	2.50	.700446	15.00	26	.929950	2.45	.756880
27	.921214	2.48	.701346	15.02	27	.930097	2.43	.757866
28	.921363	2.48	.702247	15.03	28	.930243	2.45	.758854
29	.921512	2.50	.703149	15.05	29	.930390	2.45	.759844
30	.921662	2.48	.704052	15.08	30	.930537	2.43	.760835
31	9.921811	2.48	10.704957	15.10	31	9.930683	2.45	10.761827
32	.921960	2.48	.705863	15.13	32	.930830	2.43	.762822
33	.922109	2.48	.706771	15.15	33	.930976	2.45	.763818
34	.922258	2.48	.707680	15.17	34	.931123	2.43	.764815
35	.922407	2.48	.708590	15.18	35	.931269	2.45	.765815
36	.922556	2.48	.709501	15.22	36	.931416	2.43	.766816
37	.922705	2.48	.710414	15.23	37	.931562	2.43	.767819
38	.922854	2.48	.711328	15.25	38	.931708	2.45	.768823
39	.923003	2.48	.712243	15.28	39	.931855	2.43	.769829
40	.923152	2.48	.713160	15.30	40	.932001	2.43	.770837
41	9.923301	2.47	10.714078	15.33	41	9.932147	2.43	10.771846
42	.923449	2.48	.714998	15.35	42	.932293	2.43	.772858
43	.923598	2.48	.715919	15.37	43	.932439	2.43	.773870
44	.923747	2.47	.716841	15 38	44	.932585	2.43	.774885
45	.923895	2.48	.717764	15.42	45	.932731	2.43	.775902
46	.924044	2.47	.718689	15.45	46	.932877	2.43	.776920
47	.924192	2.48	.719616	15.45	47	.933023	2.43	.777940
48	.924341	2.47	.720543	15.48	48	.933169	2.43	.778961
49	.924489	2.47	.721472	15.52	49	.933315	2.42	.779985
50	.924637	2.48	.722403	15.53	50	.933460	2.43	.781010
51	9.924786	2.47	10.723335	15.55	51	9.933606	2.43	10.782037
52	.924934	2.47	.724268	15.58	52	.933752	2.42	.783065
53	.925082	2.48	.725203	15.60	53	.933897	2.43	.784096
54	.925231	2.47	.726139	15.63	54	.934043	2.43	.785128
55	.925379	2.47	.727077	15.65	55	.934189	2.42	.786162
56	.925527	2.47	.728016	15.67	56	.934334	2 43	.787198
57	.925675	2.47	.728956	15.70	57	.934480	2.42	.788236
58	.925823	2.47	.729898	15.73	58	.934625	2.42	.78927
59	.925971	2.47	.730842	15.73	59	.934770	2.43	.79031
60	9.926119	2.47	10.731786	15.78	60	9.934916	2.42	10.7913

TABLE XXVII.—NATURAL SINES AND COSINES.

′	10°		11°		12°		13°		14°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24954	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	.19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
	Cosin	*Sine*	*Cosin*	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	
′	*79°*		*78°*		77°		76°		75°		′

TABLE XXVII.—NATURAL SINES AND COSINES.

′	15°		16°		17°		18°		19°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.25882	.96593	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	.25910	.96585	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59
2	.25938	.96578	.27620	.96110	.29293	.95613	.30957	.95088	.32612	.94533	58
3	.25966	.96570	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57
4	.25994	.96562	.27676	.96094	.29348	.95596	.31012	.95070	.32667	.94514	56
5	.26022	.96555	.27704	.96086	.29376	.95588	.31040	.95061	.32694	.94504	55
6	.26050	.96547	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54
7	.26079	.96540	.27759	.96070	.29432	.95571	.31095	.95043	.32749	.94485	53
8	.26107	.96532	.27787	.96062	.29460	.95562	.31123	.95033	.32777	.94476	52
9	.26135	.96524	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	.26191	.96509	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49
12	.26219	.96502	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48
13	.26247	.96494	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47
14	.26275	.96486	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46
15	.26303	.96479	.27983	.96005	.29654	.95502	.31316	.94970	.32969	.94409	45
16	.26331	.96471	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44
17	.26359	.96463	.28039	.95989	.29710	.95485	.31372	.94952	.33024	.94390	43
18	.26387	.96456	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.94380	42
19	.26415	.96448	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.94370	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	.26471	.96433	.28150	.95956	.29821	.95450	.31482	.94915	.33134	.94351	39
22	.26500	.96425	.28178	.95948	.29849	.95441	.31510	.94906	.33161	.94342	38
23	.26528	.96417	.28206	.95940	.29876	.95433	.31537	.94897	.33189	.94332	37
24	.26556	.96410	.28234	.95931	.29904	.95424	.31565	.94888	.33216	.94322	36
25	.26584	.96402	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35
26	.26612	.96394	.28290	.95915	.29960	.95407	.31620	.94869	.33271	.94303	34
27	.26640	.96386	.28318	.95907	.29987	.95398	.31648	.94860	.33298	.94293	33
28	.26668	.96379	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32
29	.26696	.96371	.28374	.95890	.30043	.95380	.31703	.94842	.33353	.94274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94832	.33381	.94264	30
31	.26752	.96355	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29
32	.26780	.96347	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28
33	.26808	.96340	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27
34	.26836	.96332	.28513	.95849	.30182	.95337	.31841	.94795	.33490	.94225	26
35	.26864	.96324	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25
36	.26892	.96316	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24
37	.26920	.96308	.28597	.95824	.30265	.95310	.31923	.94768	.33573	.94196	23
38	.26948	.96301	.28625	.95816	.30292	.95301	.31951	.94758	.33600	.94186	22
39	.26976	.96293	.28652	.95807	.30320	.95293	.31979	.94749	.33627	.94176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	.27032	.96277	.28708	.95791	.30376	.95275	.32034	.94730	.33682	.94157	19
42	.27060	.96269	.28736	.95782	.30403	.95266	.32061	.94721	.33710	.94147	18
43	.27088	.96261	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17
44	.27116	.96253	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16
45	.27144	.96246	.28820	.95757	.30486	.95240	.32144	.94693	.33792	.94118	15
46	.27172	.96238	.28847	.95749	.30514	.95231	.32171	.94684	.33819	.94108	14
47	.27200	.96230	.28875	.95740	.30542	.95222	.32199	.94674	.33846	.94098	13
48	.27228	.96222	.28903	.95732	.30570	.95213	.32227	.94665	.33874	.94088	12
49	.27256	.96214	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	.27312	.96198	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9
52	.27340	.96190	.29015	.95698	.30680	.95177	.32337	.94627	.33983	.94049	8
53	.27368	.96182	.29042	.95690	.30708	.95168	.32364	.94618	.34011	.94039	7
54	.27396	.96174	.29070	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6
55	.27424	.96166	.29098	.95673	.30763	.95150	.32419	.94599	.34065	.94019	5
56	.27452	.96158	.29126	.95664	.30791	.95142	.32447	.94590	.34093	.94009	4
57	.27480	.96150	.29154	.95656	.30819	.95133	.32474	.93580	.34120	.93999	3
58	.27508	.96142	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2
59	.27536	.96134	.29209	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1
60	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	74°		73°		72°		71°		70°		

TABLE XXVII.—NATURAL SINES AND COSINES.

′	20°		21°		22°		23°		24°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93959	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93503	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35484	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93431	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93379	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	69°		68°		67°		66°		65°		

TABLE XXVI.—LOGARITHMIC VERSED SIGNS AND EXTERNAL SECANTS.

88°					89°				
′	Vers.	D. 1″.	Ex. sec.	$q+l$	′	Vers.	D. 1″.	Ex. sec.	$q+l$
				15. 29*					
0	9.984573	2.17	11.441753	9086	0	9.992354	2.13	11.750498	
1	.984703	2.18	.445517	9215	1	.992482	2.15	.757925	
2	.984834	2.18	.449311	9345	2	.992611	2.13	.765477	
3	.984965	2.18	.453137	9474	3	.992739	2.15	.773158	
4	.985096	2.17	.456994	9603	4	.992868	2.13	.780973	
5	.985226	2.18	.460883	9732	5	.992996	2.13	.788926	
6	.985357	2.17	.464805	9862	6	.993124	2.15	.797022	
7	.985487	2.18	.468761	9991	7	.993253	2.13	.805268	
8	.985618	2.17	.472751	◆120	8	.993381	2.13	.813668	
9	.985748	2.18	.476775	0249	9	.993509	2.13	.822229	
10	.985879	2.17	.480834	0378	10	.993637	2.13	.830956	
11	9.986009	2.18	11.484929	0507	11	9.993765	2.15	11.839858	
12	.986140	2.17	.489061	0636	12	.993894	2.13	.848940	
13	.986270	2.17	.493230	0765	13	.994022	2.13	.858211	
14	.986400	2.18	.497437	0894	14	.994150	2.13	.867679	
15	.986531	2.17	.501683	1023	15	.994278	2.13	.877351	
16	.986661	2.17	.505968	1152	16	.994406	2.13	.887239	
17	.986791	2.17	.510293	1281	17	.994534	2.13	.897350	
18	.986921	2.17	.514659	1410	18	.994662	2.12	.907697	
19	.987051	2.17	.519066	1539	19	.994789	2.13	.918290	
20	.987181	2.17	.523516	1668	20	.994917	2.13	.929141	
21	9.987311	2.17	11.528010	1797	21	9.995045	2.13	11.940264	
22	.987441	2.17	.532548	1925	22	.995173	2.13	.951672	
23	.987571	2.17	.537131	2054	23	.995301	2.12	.963381	
24	.987701	2.17	.541760	2183	24	.995428	2.13	.975408	
25	.987831	2.17	.546437	2312	25	.995556	2.12	11.987769	
26	.987961	2.17	.551161	2440	26	.995683	2.13	12.000485	
27	.988091	2.17	.555935	2569	27	.995811	2.13	.013578	
28	.988221	2.15	.560759	2698	28	.995939	2.12	.027069	
29	.988350	2.17	.565634	2826	29	.996066	2.12	.040984	
30	.988480	2.17	.570561	2955	30	.996193	2.13	.055352	
31	9.988610	2.15	11.575542	3083	31	9.996321	2.12	12.070202	
32	.988739	2.17	.580578	3212	32	.996448	2.13	.085569	
33	.988869	2.15	.585670	3340	33	.996576	2.12	.101490	
34	.988998	2.17	.590819	3469	34	.996703	2.12	.118008	
35	.989128	2.15	.596027	3597	35	.996830	2.12	.135168	
36	.989257	2.17	.601295	3726	36	.996957	2.13	.153024	
37	.989387	2.15	.606625	3854	37	.997085	2.12	.171634	
38	.989516	2.17	.612018	3983	38	.997212	2.12	.191066	
39	.989646	2.15	.617475	4111	39	.997339	2.12	.211396	
40	.989775	2.15	.622998	4239	40	.997466	2.12	.232712	
41	9.989904	2.17	11.628589	4368	41	9.997593	2.12	12.255116	
42	.990034	2.15	.634250	4496	42	.997720	2.12	.278724	
43	.990163	2.15	.639982	4624	43	.997847	2.12	.303674	
44	.990292	2.15	.645788	4752	44	.997974	2.12	.330129	
45	.990421	2.15	.651668	4881	45	.998101	2.12	.358285	
46	.990550	2.15	.657626	5009	46	.998228	2.12	.388375	
47	.990679	2.15	.663663	5137	47	.998355	2.10	.420686	
48	.990808	2.15	.669781	5265	48	.998481	2.12	.455575	
49	.990937	2.15	.675984	5393	49	.998608	2.12	.493490	
50	.991066	2.15	.682272	5521	50	.998735	2.12	.535009	
51	9.991195	2.15	11.688649	5649	51	9.998862	2.10	12.580803	
52	.991324	2.15	.695117	5777	52	.998988	2.12	.632172	
53	.991453	2.15	.701679	5905	53	.999115	2.10	.690291	
54	.991582	2.13	.708338	6033	54	.999241	2.12	.757364	
55	.991710	2.15	.715097	6161	55	.999368	2.10	.836672	
56	.991839	2.15	.721958	6289	56	.999494	2.12	12.932708	
57	.991968	2.13	.728925	6417	57	.999621	2.10	13.058774	
58	.992096	2.15	.736002	6545	58	.999747	2.12	.234991	
59	.992225	2.15	.743192	6673	59	.999874	2.10	.536148	
60	9.992354	2.13	11.750498	6801	60	10.000000	2.10	Inf. pos.	
				15.30*					

TABLE XXVII.—NATURAL SINES AND COSINES.

′	30°		31°		32°		33°		34°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82822	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85431	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	.86295	.52026	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82544	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82478	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84245	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	.56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55630	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55654	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	.82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57238	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82936	.57310	.81949	
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	
	59°		58°		57°		56°		55°		

TABLE XXVII.—NATURAL SINES AND COSINES.

′	5°		6°		7°		8°		9°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	.15672	.98764	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	.15701	.98760	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	.15730	.98755	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	.15758	.98751	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	.15787	.98746	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	.15816	.98741	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	.15845	.98737	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	.15873	.98732	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	.15902	.98728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	.15931	.98723	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	.15959	.98718	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	.15988	.98714	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	.16017	.98709	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	.16046	.98704	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	.16074	.98700	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	.16103	.98695	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	.16132	.98690	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	.16160	.98686	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	.16189	.98681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	.16246	.98671	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	.16275	.98667	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	.16304	.98662	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	.16333	.98657	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	.16361	.98652	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	.16390	.98648	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	.16419	.98643	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	.16447	.98638	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	.16476	.98633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	.16533	.98624	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	.16562	.98619	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	.16591	.98614	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	.16620	.98609	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	.16648	.98604	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	.16677	.98600	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	.16706	.98595	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	.16734	.98590	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	.16763	.98585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	.16792	.98580	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	.16820	.98575	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	.16849	.98570	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	.16878	.98565	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	.16906	.98561	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	.16935	.98556	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	.16964	.98551	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	.16992	.98546	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	.17021	.98541	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	.17050	.98536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	.17107	.98526	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	.17136	.98521	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	.17164	.98516	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	.17193	.98511	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	.17222	.98506	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	.17250	.98501	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	.17279	.98496	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	.17308	.98491	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	.17336	.98486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17365	.98481	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	84°		83°		82°		81°		80°		

TABLE XXVII.—NATURAL SINES AND COSINES.

′	10°		11°		12°		13°		14°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24954	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	.19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	79°		78°		77°		76°		75°		

TABLE XXVII.—NATURAL SINES AND COSINES.

′	15°		16°		17°		18°		19°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.25882	.96593	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	.25910	.96585	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59
2	.25938	.96578	.27620	.96110	.29293	.95613	.30957	.95088	.32612	.94533	58
3	.25966	.96570	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57
4	.25994	.96562	.27676	.96094	.29348	.95596	.31012	.95070	.32667	.94514	56
5	.26022	.96555	.27704	.96086	.29376	.95588	.31040	.95061	.32694	.94504	55
6	.26050	.96547	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54
7	.26079	.96540	.27759	.96070	.29432	.95571	.31095	.95043	.32749	.94485	53
8	.26107	.96532	.27787	.96062	.29460	.95562	.31123	.95033	.32777	.94476	52
9	.26135	.96524	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	.26191	.96509	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49
12	.26219	.96502	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48
13	.26247	.96494	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47
14	.26275	.96486	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46
15	.26303	.96479	.27983	.96005	.29654	.95502	.31316	.94970	.32969	.94409	45
16	.26331	.96471	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44
17	.26359	.96463	.28039	.95989	.29710	.95485	.31372	.94952	.33024	.94390	43
18	.26387	.96456	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.94380	42
19	.26415	.96448	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.94370	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	.26471	.96433	.28150	.95956	.29821	.95450	.31482	.94915	.33134	.94351	39
22	.26500	.96425	.28178	.95948	.29849	.95441	.31510	.94906	.33161	.94342	38
23	.26528	.96417	.28206	.95940	.29876	.95433	.31537	.94897	.33189	.94332	37
24	.26556	.96410	.28234	.95931	.29904	.95424	.31565	.94888	.33216	.94322	36
25	.26584	.96402	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35
26	.26612	.96394	.28290	.95915	.29960	.95407	.31620	.94869	.33271	.94303	34
27	.26640	.96386	.28318	.95907	.29987	.95398	.31648	.94860	.33298	.94293	33
28	.26668	.96379	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32
29	.26696	.96371	.28374	.95890	.30043	.95380	.31703	.94842	.33353	.94274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94832	.33381	.94264	30
31	.26752	.96355	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29
32	.26780	.96347	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28
33	.26808	.96340	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27
34	.26836	.96332	.28513	.95849	.30182	.95337	.31841	.94795	.33490	.94225	26
35	.26864	.96324	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25
36	.26892	.96316	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24
37	.26920	.96308	.28597	.95824	.30265	.95310	.31923	.94768	.33573	.94196	23
38	.26948	.96301	.28625	.95816	.30292	.95301	.31951	.94758	.33600	.94186	22
39	.26976	.96293	.28652	.95807	.30320	.95293	.31979	.94749	.33627	.94176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	.27032	.96277	.28708	.95791	.30376	.95275	.32034	.94730	.33682	.94157	19
42	.27060	.96269	.28736	.95782	.30403	.95266	.32061	.94721	.33710	.94147	18
43	.27088	.96261	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17
44	.27116	.96253	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16
45	.27144	.96246	.28820	.95757	.30486	.95240	.32144	.94693	.33792	.94118	15
46	.27172	.96238	.28847	.95749	.30514	.95231	.32171	.94684	.33819	.94108	14
47	.27200	.96230	.28875	.95740	.30542	.95222	.32199	.94674	.33846	.94098	13
48	.27228	.96222	.28903	.95732	.30570	.95213	.32227	.94665	.33874	.94088	12
49	.27256	.96214	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	.27312	.96198	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9
52	.27340	.96190	.29015	.95698	.30680	.95177	.32337	.94627	.33983	.94049	8
53	.27368	.96182	.29042	.95690	.30708	.95168	.32364	.94618	.34011	.94039	7
54	.27396	.96174	.29070	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6
55	.27424	.96166	.29098	.95673	.30763	.95150	.32419	.94599	.34065	.94019	5
56	.27452	.96158	.29126	.95664	.30791	.95142	.32447	.94590	.34093	.94009	4
57	.27480	.96150	.29154	.95656	.30819	.95133	.32474	.93580	.34120	.93999	3
58	.27508	.96142	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2
59	.27536	.96134	.29209	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1
60	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	0
	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	
′	74°		73°		72°		71°		70°		′

TABLE XXVII.—NATURAL SINES AND COSINES.

′	20°		21°		22°		23°		24°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93959	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93503	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35484	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93431	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93379	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	
	69°		68°		67°		66°		65°		′

′	25°		26°		27°		28°		29°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	.48506	.87448	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	.48532	.87434	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	.48557	.87420	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	.48583	.87406	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	.48608	.87391	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	.48634	.87377	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	.48659	.87363	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	.48684	.87349	52
9	.42199	.90520	.44072	.89764	.45632	.88981	.47178	.88172	.48710	.87335	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	.48735	.87321	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	.48761	.87306	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	.48786	.87292	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	.48811	.87278	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	.48837	.87264	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	.48862	.87250	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	.48888	.87235	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	.48913	.87221	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	.48938	.87207	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	.48964	.87193	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	.49014	.87164	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	.49040	.87150	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	.49065	.87136	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	.49090	.87121	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	.49116	.87107	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	.49141	.87093	34
27	.42972	.90296	.44542	.89532	.46007	.88741	.47639	.87923	.49166	.87079	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	.49192	.87064	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	.49217	.87050	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	.49242	.87036	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	.49268	.87021	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	.49293	.87007	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	.49318	.86993	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	.49344	.86978	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	.49369	.86964	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	.49394	.86949	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	.49419	.86935	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	.49445	.86921	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	.49470	.86906	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	.49495	.86892	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	.49521	.86878	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	.49546	.86863	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	.49571	.86849	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	.49596	.86834	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	.49622	.86820	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	.49647	.86805	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	.49672	.86791	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	.49697	.86777	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	.49723	.86762	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	.49748	.86748	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	.49773	.86733	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	.49798	.86719	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	.49824	.86704	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	.49849	.86690	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	.49874	.86675	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	.49899	.86661	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	.49924	.86646	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	.49950	.86632	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	.49975	.86617	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	0
	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	64°		63°		62°		61°		60°		

TABLE XXVII.—NATURAL SINES AND COSINES.

′	30°		31°		32°		33°		34°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82822	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85431	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	.86295	.52026	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82544	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82478	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84245	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	.56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55630	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55654	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	.82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57238	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82936	.57310	.81949	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	
	59°		58°		57°		56°		55°		

TABLE XXVII.—NATURAL SINES AND COSINES.

′	35°		36°		37°		38°		39°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	.57381	.81899	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59
2	.57405	.81882	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58
3	.57429	.81865	.58849	.80850	.60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	61658	.78729	.63022	.77641	56
5	.57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	.57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	.63135	.77550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	.57619	.81731	.59037	.80713	.60437	.79671	.61818	.78604	.63180	.77513	49
12	.57643	.81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61909	.78532	.63271	.77439	45
16	.57738	.81647	.59154	.80627	.60553	.79583	.61932	.78514	.63293	.77421	44
17	.57762	.81631	.59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	.78460	.63361	.77366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39
22	.57881	.81546	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.77310	38
23	.57904	.81530	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37
24	.57928	.81513	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36
25	.57952	.81496	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	.57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	.63518	.77236	34
27	.57999	.81462	.59412	.80438	.60807	.79388	.62183	.78315	.63540	.77218	33
28	.58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	.63563	.77199	32
29	.58047	.81428	.59459	.80403	60853	.79353	.62229	.78279	.63585	.77181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	.59506	.80368	.60899	.79318	.62274	.78243	.63630	.77144	29
32	.58118	.81378	.59529	.80351	.60922	.79300	.62297	.78225	.63653	.77125	28
33	.58141	.81361	.59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	.59576	.80316	.60968	.79264	.62342	.78188	.63698	.77088	26
35	.58189	.81327	.59599	.80299	.60991	.79247	.62365	.78170	.63720	.77070	25
36	.58212	.81310	.59622	.80282	.61015	.79229	.62388	.78152	.63742	.77051	24
37	.58236	.81293	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	.59669	.80247	.61061	.79193	.62433	.78116	.63787	.77014	22
39	.58283	.81259	.59693	.80230	.61084	.79176	.62456	.78098	63810	.76996	21
40	.58307	.81242	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	.59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.76940	18
43	.58378	.81191	.59786	.80160	.61176	.79105	.62547	.78025	.63899	.76921	17
44	.58401	.81174	.59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15
46	.58449	.81140	.59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	.59902	.80073	.61291	.79016	.62660	.77934	.64011	.76828	12
49	.58519	.81089	.59926	.80056	.61314	.78998	.62683	.77916	.64033	.76810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	.58567	.81055	.59972	.80021	.61360	.78962	.62728	.77879	.64078	.76772	9
52	.58590	.81038	.59995	80003	.61383	.78944	.62751	.77861	.64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	.64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77824	.64145	.76717	6
55	.58661	.80987	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	.60089	.79934	.61474	.78873	.62842	.77788	.64190	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3
58	.58731	.80936	.60135	.79899	.61520	.78837	.62887	.77751	.64234	.76642	2
59	*.58755*	*.80919*	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76623	1
60	*.58779*	*.80902*	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	
′	54°		53°		52°		51°		50°		′

TABLE XXVII.—NATURAL SINES AND COSINES.

′	40°		41°		42°		43°		44°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.64368	.76530	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	.76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76380	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.76173	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.64812	.76154	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.64834	.76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	.76116	.66175	.74973	.67473	.73803	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	35
36	.65077	.75927	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75908	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.65276	.75756	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.65320	.75719	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.65364	.75680	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.65562	.75509	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	
	49°		48°		47°		46°		45°		

′	0°		1°		2°		3°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.00000	Infinite.	.01746	57.2900	.03492	28.6363	.05241	19.0811	60
1	.00029	3437.75	.01775	56.3506	.03521	28.3994	.05270	18.9755	59
2	.00058	1718.87	.01804	55.4415	.03550	28.1664	.05299	18.8711	58
3	.00087	1145.92	.01833	54.5613	.03579	27.9372	.05328	18.7678	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	.05591	17.8863	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	.05795	17.2558	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	.05883	16.9990	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	33
28	.00815	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	.06525	15.3254	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	15
46	.01338	74.7292	.03084	32.4213	.04833	20.6932	.06584	15.1893	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	2
59	*.01716*	*58.2612*	.03463	28.8771	.05212	19.1879	.06963	14.3607	1
60	*.01746*	*57.2900*	.03492	28.6363	.05241	19.0811	.06993	14.3007	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	89°		88°		87°		86°		

TABLE XXVIII.—NATURAL TANGENTS AND COTANGENTS.

′	20°		21°		22°		23°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	40
21	.37090	2.69612	.39089	2 55827	.41115	2.43220	.43170	2.31641	39
22	.37123	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43239	2.31271	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	33
28	.37322	2 67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38863	.43932	2.27626	17
44	.37853	2.64177	.39862	2.50864	41899	2.38668	.43966	2.27447	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	5
56	.38253	2.61418	.40267	2 48340	.42310	2.36349	.44384	2.25309	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	1
60	*.38386*	*2.60509*	.40403	2.47509	.42447	2.35585	.44523	2.24604	0
′	*Cotang*	*Tang*	Cotang	Tang	Cotang	Tang	Cotang	Tang	
	69°		68°		67°		66°		

TABLE XXVIII.—NATURAL TANGENTS AND COTANGENTS.

′	24°		25°		26°		27°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.44523	2.24604	.46631	2.14451	.48773	2.05030	.50953	1.96261	60
1	.44558	2.24428	.46666	2.14288	.48809	2.04879	.50989	1.96120	59
2	.44593	2.24252	.46702	2.14125	.48845	2.04728	.51026	1.95979	58
3	.44627	2.24077	.46737	2.13963	.48881	2.04577	.51063	1.95838	57
4	.44662	2.23902	.46772	2.13801	.48917	2.04426	.51099	1.95698	56
5	.44697	2.23727	.46808	2.13639	.48953	2.04276	.51136	1.95557	55
6	.44732	2.23553	.46843	2.13477	.48989	2.04125	.51173	1.95417	54
7	.44767	2.23378	.46879	2.13316	.49026	2.03975	.51209	1.95277	53
8	.44802	2.23204	.46914	2.13154	.49062	2.03825	.51246	1.95137	52
9	.44837	2.23030	.46950	2.12993	.49098	2.03675	.51283	1.94997	51
10	.44872	2.22857	.46985	2.12832	.49134	2.03526	.51319	1.94858	50
11	.44907	2.22683	.47021	2.12671	.49170	2.03376	.51356	1.94718	49
12	.44942	2.22510	.47056	2.12511	.49206	2.03227	.51393	1.94579	48
13	.44977	2.22337	.47092	2.12350	.49242	2.03078	.51430	1.94440	47
14	.45012	2.22164	.47128	2.12190	.49278	2.02929	.51467	1.94301	46
15	.45047	2.21992	.47163	2.12030	.49315	2.02780	.51503	1.94162	45
16	.45082	2.21819	.47199	2.11871	.49351	2.02631	.51540	1.94023	44
17	.45117	2.21647	.47234	2.11711	.49387	2.02483	.51577	1.93885	43
18	.45152	2.21475	.47270	2.11552	.49423	2.02335	.51614	1.93746	42
19	.45187	2.21304	.47305	2.11392	.49459	2.02187	.51651	1.93608	41
20	.45222	2.21132	.47341	2.11233	.49495	2.02039	.51688	1.93470	40
21	.45257	2.20961	.47377	2.11075	.49532	2.01891	.51724	1.93332	39
22	.45292	2.20790	.47412	2.10916	.49568	2.01743	.51761	1.93195	38
23	.45327	2.20619	.47448	2.10758	.49604	2.01596	.51798	1.93057	37
24	.45362	2.20449	.47483	2.10600	.49640	2.01449	.51835	1.92920	36
25	.45397	2.20278	.47519	2.10442	.49677	2.01302	.51872	1.92782	35
26	.45432	2.20108	.47555	2.10284	.49713	2.01155	.51909	1.92645	34
27	.45467	2.19938	.47590	2.10126	.49749	2.01008	.51946	1.92508	33
28	.45502	2.19769	.47626	2.09969	.49786	2.00862	.51983	1.92371	32
29	.45538	2.19599	.47662	2.09811	.49822	2.00715	.52020	1.92235	31
30	.45573	2.19430	.47698	2.09654	.49858	2.00569	.52057	1.92098	30
31	.45608	2.19261	.47733	2.09498	.49894	2.00423	.52094	1.91962	29
32	.45643	2.19092	.47769	2.09341	.49931	2.00277	.52131	1.91826	28
33	.45678	2.18923	.47805	2.09184	.49967	2.00131	.52168	1.91690	27
34	.45713	2.18755	.47840	2.09028	.50004	1.99986	.52205	1.91554	26
35	.45748	2.18587	.47876	2.08872	.50040	1.99841	.52242	1.91418	25
36	.45784	2.18419	.47912	2.08716	.50076	1.99695	.52279	1.91282	24
37	.45819	2.18251	.47948	2.08560	.50113	1.99550	.52316	1.91147	23
38	.45854	2.18084	.47984	2.08405	.50149	1.99406	.52353	1.91012	22
39	.45889	2.17916	.48019	2.08250	.50185	1.99261	.52390	1.90876	21
40	.45924	2.17749	.48055	2.08094	.50222	1.99116	.52427	1.90741	20
41	.45960	2.17582	.48091	2.07939	.50258	1.98972	.52464	1.90607	19
42	.45995	2.17416	.48127	2.07785	.50295	1.98828	.52501	1.90472	18
43	.46030	2.17249	.48163	2.07630	.50331	1.98684	.52538	1.90337	17
44	.46065	2.17083	.48198	2.07476	.50368	1.98540	.52575	1.90203	16
45	.46101	2.16917	.48234	2.07321	.50404	1.98396	.52613	1.90069	15
46	.46136	2.16751	.48270	2.07167	.50441	1.98253	.52650	1.89935	14
47	.46171	2.16585	.48306	2.07014	.50477	1.98110	.52687	1.89801	13
48	.46206	2.16420	.48342	2.06860	.50514	1.97966	.52724	1.89667	12
49	.46242	2.16255	.48378	2.06706	.50550	1.97823	.52761	1.89533	11
50	.46277	2.16090	.48414	2.06553	.50587	1.97681	.52798	1.89400	10
51	.46312	2.15925	.48450	2.06400	.50623	1.97538	.52836	1.89266	9
52	.46348	2.15760	.48486	2.06247	.50660	1.97395	.52873	1.89133	8
53	.46383	2.15596	.48521	2.06094	.50696	1.97253	.52910	1.89000	7
54	.46418	2.15432	.48557	2.05942	.50733	1.97111	.52947	1.88867	6
55	.46454	2.15268	.48593	2.05790	.50769	1.96969	.52985	1.88734	5
56	.46489	2.15104	.48629	2.05637	.50806	1.96827	.53022	1.88602	4
57	.46525	2.14940	.48665	2.05485	.50843	1.96685	.53059	1.88469	3
58	.46560	2.14777	.48701	2.05333	.50879	1.96544	.53096	1.88337	2
59	.46595	2.14614	.48737	2.05182	.50916	1.96402	.53134	1.88205	1
60	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	65°		64°		63°		62°		

TABLE XXVIII.—NATURAL TANGENTS AND COTANGENTS.

′	28°		29°		30°		31°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.53171	1.88073	.55431	1.80405	.57735	1.73205	.60086	1.66428	60
1	.53208	1.87941	.55469	1.80281	.57774	1.73089	.60126	1.66318	59
2	.53246	1.87809	.55507	1.80158	.57813	1.72973	.60165	1.66209	58
3	.53283	1.87677	.55545	1.80034	.57851	1.72857	.60205	1.66099	57
4	.53320	1.87546	.55583	1.79911	.57890	1.72741	.60245	1.65990	56
5	.53358	1.87415	.55621	1.79788	.57929	1.72625	.60284	1.65881	55
6	.53395	1.87283	.55659	1.79665	.57968	1.72509	.60324	1.65772	54
7	.53432	1.87152	.55697	1.79542	.58007	1.72393	.60364	1.65663	53
8	.53470	1.87021	.55736	1.79419	.58046	1.72278	.60403	1.65554	52
9	.53507	1.86891	.55774	1.79296	.58085	1.72163	.60443	1.65445	51
10	.53545	1.86760	.55812	1.79174	.58124	1.72047	.60483	1.65337	50
11	.53582	1.86630	.55850	1.79051	.58162	1.71932	.60522	1.65228	49
12	.53620	1.86499	.55888	1.78929	.58201	1.71817	.60562	1.65120	48
13	.53657	1.86369	.55926	1.78807	.58240	1.71702	.60602	1.65011	47
14	.53694	1.86239	.55964	1.78685	.58279	1.71588	.60642	1.64903	46
15	.53732	1.86109	.56003	1.78563	.58318	1.71473	.60681	1.64795	45
16	.53769	1.85979	.56041	1.78441	.58357	1.71358	.60721	1.64687	44
17	.53807	1.85850	.56079	1.78319	.58396	1.71244	.60761	1.64579	43
18	.53844	1.85720	.56117	1.78198	.58435	1.71129	.60801	1.64471	42
19	.53882	1.85591	.56156	1.78077	.58474	1.71015	.60841	1.64363	41
20	.53920	1.85462	.56194	1.77955	.58513	1.70901	.60881	1.64256	40
21	.53957	1.85333	.56232	1.77834	.58552	1.70787	.60921	1.64148	39
22	.53995	1.85204	.56270	1.77713	.58591	1.70673	.60960	1.64041	38
23	.54032	1.85075	.56309	1.77592	.58631	1.70560	.61000	1.63934	37
24	.54070	1.84946	.56347	1.77471	.58670	1.70446	.61040	1.63826	36
25	.54107	1.84818	.56385	1.77351	.58709	1.70332	.61080	1.63719	35
26	.54145	1.84689	.56424	1.77230	.58748	1.70219	.61120	1.63612	34
27	.54183	1.84561	.56462	1.77110	.58787	1.70106	.61160	1.63505	33
28	.54220	1.84433	.56501	1.76990	.58826	1.69992	.61200	1.63398	32
29	.54258	1.84305	.56539	1.76869	.58865	1.69879	.61240	1.63292	31
30	.54296	1.84177	.56577	1.76749	.58905	1.69766	.61280	1.63185	30
31	.54333	1.84049	.56616	1.76629	.58944	1.69653	.61320	1.63079	29
32	.54371	1.83922	.56654	1.76510	.58983	1.69541	.61360	1.62972	28
33	.54409	1.83794	.56693	1.76390	.59022	1.69428	.61400	1.62866	27
34	.54446	1.83667	.56731	1.76271	.59061	1.69316	.61440	1.62760	26
35	.54484	1.83540	.56769	1.76151	.59101	1.69203	.61480	1.62654	25
36	.54522	1.83413	.56808	1.76032	.59140	1.69091	.61520	1.62548	24
37	.54560	1.83286	.56846	1.75913	.59179	1.68979	.61561	1.62442	23
38	.54597	1.83159	.56885	1.75794	.59218	1.68866	.61601	1.62336	22
39	.54635	1.83033	.56923	1.75675	.59258	1.68754	.61641	1.62230	21
40	.54673	1.82906	.56962	1.75556	.59297	1.68643	.61681	1.62125	20
41	.54711	1.82780	.57000	1.75437	.59336	1.68531	.61721	1.62019	19
42	.54748	1.82654	.57039	1.75319	.59376	1.68419	.61761	1.61914	18
43	.54786	1.82528	.57078	1.75200	.59415	1.68308	.61801	1.61808	17
44	.54824	1.82402	.57116	1.75082	.59454	1.68196	.61842	1.61703	16
45	.54862	1.82276	.57155	1.74964	.59494	1.68085	.61882	1.61598	15
46	.54900	1.82150	.57193	1.74846	.59533	1.67974	.61922	1.61493	14
47	.54938	1.82025	.57232	1.74728	.59573	1.67863	.61962	1.61388	13
48	.54975	1.81899	.57271	1.74610	.59612	1.67752	.62003	1.61283	12
49	.55013	1.81774	.57309	1.74492	.59651	1.67641	.62043	1.61179	11
50	.55051	1.81649	.57348	1.74375	.59691	1.67530	.62083	1.61074	10
51	.55089	1.81524	.57386	1.74257	.59730	1.67419	.62124	1.60970	9
52	.55127	1.81399	.57425	1.74140	.59770	1.67309	.62164	1.60865	8
53	.55165	1.81274	.57464	1.74022	.59809	1.67198	.62204	1.60761	7
54	.55203	1.81150	.57503	1.73905	.59849	1.67088	.62245	1.60657	6
55	.55241	1.81025	.57541	1.73788	.59888	1.66978	.62285	1.60553	5
56	.55279	1.80901	.57580	1.73671	.59928	1.66867	.62325	1.60449	4
57	.55317	1.80777	.57619	1.73555	.59967	1.66757	.62366	1.60345	3
58	.55355	1.80653	.57657	1.73438	.60007	1.66647	.62406	1.60241	2
59	.55393	1.80529	.57696	1.73321	.60046	1.66538	.62446	1.60137	
60	.55431	1.80405	.57735	1.73205	.60086	1.66428	.62487	1.60033	
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	
	61°		60°		59°		58°		

TABLE XXVIII.—NATURAL TANGENTS AND COTANGENTS.

′	24°		25°		26°		27°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.44523	2.24604	.46631	2.14451	.48773	2.05030	.50953	1.96261	60
1	.44558	2.24428	.46666	2.14288	.48809	2.04879	.50989	1.96120	59
2	.44593	2.24252	.46702	2.14125	.48845	2.04728	.51026	1.95979	58
3	.44627	2.24077	.46737	2.13963	.48881	2.04577	.51063	1.95838	57
4	.44662	2.23902	.46772	2.13801	.48917	2.04426	.51099	1.95698	56
5	.44697	2.23727	.46808	2.13639	.48953	2.04276	.51136	1.95557	55
6	.44732	2.23553	.46843	2.13477	.48989	2.04125	.51173	1.95417	54
7	.44767	2.23378	.46879	2.13316	.49026	2.03975	.51209	1.95277	53
8	.44802	2.23204	.46914	2.13154	.49062	2.03825	.51246	1.95137	52
9	.44837	2.23030	.46950	2.12993	.49098	2.03675	.51283	1.94997	51
10	.44872	2.22857	.46985	2.12832	.49134	2.03526	.51319	1.94858	50
11	.44907	2.22683	.47021	2.12671	.49170	2.03376	.51356	1.94718	49
12	.44942	2.22510	.47056	2.12511	.49206	2.03227	.51393	1.94579	48
13	.44977	2.22337	.47092	2.12350	.49242	2.03078	.51430	1.94440	47
14	.45012	2.22164	.47128	2.12190	.49278	2.02929	.51467	1.94301	46
15	.45047	2.21992	.47163	2.12030	.49315	2.02780	.51503	1.94162	45
16	.45082	2.21819	.47199	2.11871	.49351	2.02631	.51540	1.94023	44
17	.45117	2.21647	.47234	2.11711	.49387	2.02483	.51577	1.93885	43
18	.45152	2.21475	.47270	2.11552	.49423	2.02335	.51614	1.93746	42
19	.45187	2.21304	.47305	2.11392	.49459	2.02187	.51651	1.93608	41
20	.45222	2.21132	.47341	2.11233	.49495	2.02039	.51688	1.93470	40
21	.45257	2.20961	.47377	2.11075	.49532	2.01891	.51724	1.93332	39
22	.45292	2.20790	.47412	2.10916	.49568	2.01743	.51761	1.93195	38
23	.45327	2.20619	.47448	2.10758	.49604	2.01596	.51798	1.93057	37
24	.45362	2.20449	.47483	2.10600	.49640	2.01449	.51835	1.92920	36
25	.45397	2.20278	.47519	2.10442	.49677	2.01302	.51872	1.92782	35
26	.45432	2.20108	.47555	2.10284	.49713	2.01155	.51909	1.92645	34
27	.45467	2.19938	.47590	2.10126	.49749	2.01008	.51946	1.92508	33
28	.45502	2.19769	.47626	2.09969	.49786	2.00862	.51983	1.92371	32
29	.45538	2.19599	.47662	2.09811	.49822	2.00715	.52020	1.92235	31
30	.45573	2.19430	.47698	2.09654	.49858	2.00569	.52057	1.92098	30
31	.45608	2.19261	.47733	2.09498	.49894	2.00423	.52094	1.91962	29
32	.45643	2.19092	.47769	2.09341	.49931	2.00277	.52131	1.91826	28
33	.45678	2.18923	.47805	2.09184	.49967	2.00131	.52168	1.91690	27
34	.45713	2.18755	.47840	2.09028	.50004	1.99986	.52205	1.91554	26
35	.45748	2.18587	.47876	2.08872	.50040	1.99841	.52242	1.91418	25
36	.45784	2.18419	.47912	2.08716	.50076	1.99695	.52279	1.91282	24
37	.45819	2.18251	.47948	2.08560	.50113	1.99550	.52316	1.91147	23
38	.45854	2.18084	.47984	2.08405	.50149	1.99406	.52353	1.91012	22
39	.45889	2.17916	.48019	2.08250	.50185	1.99261	.52390	1.90876	21
40	.45924	2.17749	.48055	2.08094	.50222	1.99116	.52427	1.90741	20
41	.45960	2.17582	.48091	2.07939	.50258	1.98972	.52464	1.90607	19
42	.45995	2.17416	.48127	2.07785	.50295	1.98828	.52501	1.90472	18
43	.46030	2.17249	.48163	2.07630	.50331	1.98684	.52538	1.90337	17
44	.46065	2.17083	.48198	2.07476	.50368	1.98540	.52575	1.90203	16
45	.46101	2.16917	.48234	2.07321	.50404	1.98396	.52613	1.90069	15
46	.46136	2.16751	.48270	2.07167	.50441	1.98253	.52650	1.89935	14
47	.46171	2.16585	.48306	2.07014	.50477	1.98110	.52687	1.89801	13
48	.46206	2.16420	.48342	2.06860	.50514	1.97966	.52724	1.89667	12
49	.46242	2.16255	.48378	2.06706	.50550	1.97823	.52761	1.89533	11
50	.46277	2.16090	.48414	2.06553	.50587	1.97681	.52798	1.89400	10
51	.46312	2.15925	.48450	2.06400	.50623	1.97538	.52836	1.89266	9
52	.46348	2.15760	.48486	2.06247	.50660	1.97395	.52873	1.89133	8
53	.46383	2.15596	.48521	2.06094	.50696	1.97253	.52910	1.89000	7
54	.46418	2.15432	.48557	2.05942	.50733	1.97111	.52947	1.88867	6
55	.46454	2.15268	.48593	2.05790	.50769	1.96969	.52985	1.88734	5
56	.46489	2.15104	.48629	2.05637	.50806	1.96827	.53022	1.88602	4
57	.46525	2.14940	.48665	2.05485	.50843	1.96685	.53059	1.88469	3
58	.46560	2.14777	.48701	2.05333	.50879	1.96544	.53096	1.88337	2
59	.46595	2.14614	.48737	2.05182	.50916	1.96402	.53134	1.88205	1
60	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	65°		64°		63°		62°		

TABLE XXVIII.—NATURAL TANGENTS AND COTANGENTS.

′	28°		29°		30°		31°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.53171	1.88073	.55431	1.80405	.57735	1.73205	.60086	1.66428	60
1	.53208	1.87941	.55469	1.80281	.57774	1.73089	.60126	1.66318	59
2	.53246	1.87809	.55507	1.80158	.57813	1.72973	.60165	1.66209	58
3	.53283	1.87677	.55545	1.80034	.57851	1.72857	.60205	1.66099	57
4	.53320	1.87546	.55583	1.79911	.57890	1.72741	.60245	1.65990	56
5	.53358	1.87415	.55621	1.79788	.57929	1.72625	.60284	1.65881	55
6	.53395	1.87283	.55659	1.79665	.57968	1.72509	.60324	1.65772	54
7	.53432	1.87152	.55697	1.79542	.58007	1.72393	.60364	1.65663	53
8	.53470	1.87021	.55736	1.79419	.58046	1.72278	.60403	1.65554	52
9	.53507	1.86891	.55774	1.79296	.58085	1.72163	.60443	1.65445	51
10	.53545	1.86760	.55812	1.79174	.58124	1.72047	.60483	1.65337	50
11	.53582	1.86630	.55850	1.79051	.58162	1.71932	.60522	1.65228	49
12	.53620	1.86499	.55888	1.78929	.58201	1.71817	.60562	1.65120	48
13	.53657	1.86369	.55926	1.78807	.58240	1.71702	.60602	1.65011	47
14	.53694	1.86239	.55964	1.78685	.58279	1.71588	.60642	1.64903	46
15	.53732	1.86109	.56003	1.78563	.58318	1.71473	.60681	1.64795	45
16	.53769	1.85979	.56041	1.78441	.58357	1.71358	.60721	1.64687	44
17	.53807	1.85850	.56079	1.78319	.58396	1.71244	.60761	1.64579	43
18	.53844	1.85720	.56117	1.78198	.58435	1.71129	.60801	1.64471	42
19	.53882	1.85591	.56156	1.78077	.58474	1.71015	.60841	1.64363	41
20	.53920	1.85462	.56194	1.77955	.58513	1.70901	.60881	1.64256	40
21	.53957	1.85333	.56232	1.77834	.58552	1.70787	.60921	1.64148	39
22	.53995	1.85204	.56270	1.77713	.58591	1.70673	.60960	1.64041	38
23	.54032	1.85075	.56309	1.77592	.58631	1.70560	.61000	1.63934	37
24	.54070	1.84946	.56347	1.77471	.58670	1.70446	.61040	1.63826	36
25	.54107	1.84818	.56385	1.77351	.58709	1.70332	.61080	1.63719	35
26	.54145	1.84689	.56424	1.77230	.58748	1.70219	.61120	1.63612	34
27	.54183	1.84561	.56462	1.77110	.58787	1.70106	.61160	1.63505	33
28	.54220	1.84433	.56501	1.76990	.58826	1.69992	.61200	1.63398	32
29	.54258	1.84305	.56539	1.76869	.58865	1.69879	.61240	1.63292	31
30	.54296	1.84177	.56577	1.76749	.58905	1.69766	.61280	1.63185	30
31	.54333	1.84049	.56616	1.76629	.58944	1.69653	.61320	1.63079	29
32	.54371	1.83922	.56654	1.76510	.58983	1.69541	.61360	1.62972	28
33	.54409	1.83794	.56693	1.76390	.59022	1.69428	.61400	1.62866	27
34	.54446	1.83667	.56731	1.76271	.59061	1.69316	.61440	1.62760	26
35	.54484	1.83540	.56769	1.76151	.59101	1.69203	.61480	1.62654	25
36	.54522	1.83413	.56808	1.76032	.59140	1.69091	.61520	1.62548	24
37	.54560	1.83286	.56846	1.75913	.59179	1.68979	.61561	1.62442	23
38	.54597	1.83159	.56885	1.75794	.59218	1.68866	.61601	1.62336	22
39	.54635	1.83033	.56923	1.75675	.59258	1.68754	.61641	1.62230	21
40	.54673	1.82906	.56962	1.75556	.59297	1.68643	.61681	1.62125	20
41	.54711	1.82780	.57000	1.75437	.59336	1.68531	.61721	1.62019	19
42	.54748	1.82654	.57039	1.75319	.59376	1.68419	.61761	1.61914	18
43	.54786	1.82528	.57078	1.75200	.59415	1.68308	.61801	1.61808	17
44	.54824	1.82402	.57116	1.75082	.59454	1.68196	.61842	1.61703	16
45	.54862	1.82276	.57155	1.74964	.59494	1.68085	.61882	1.61598	15
46	.54900	1.82150	.57193	1.74846	.59533	1.67974	.61922	1.61493	14
47	.54938	1.82025	.57232	1.74728	.59573	1.67863	.61962	1.61388	13
48	.54975	1.81899	.57271	1.74610	.59612	1.67752	.62003	1.61283	12
49	.55013	1.81774	.57309	1.74492	.59651	1.67641	.62043	1.61179	11
50	.55051	1.81649	.57348	1.74375	.59691	1.67530	.62083	1.61074	10
51	.55089	1.81524	.57386	1.74257	.59730	1.67419	.62124	1.60970	9
52	.55127	1.81399	.57425	1.74140	.59770	1.67309	.62164	1.60865	8
53	.55165	1.81274	.57464	1.74022	.59809	1.67198	.62204	1.60761	7
54	.55203	1.81150	.57503	1.73905	.59849	1.67088	.62245	1.60657	6
55	.55241	1.81025	.57541	1.73788	.59888	1.66978	.62285	1.60553	5
56	.55279	1.80901	.57580	1.73671	.59928	1.66867	.62325	1.60449	4
57	.55317	1.80777	.57619	1.73555	.59967	1.66757	.62366	1.60345	3
58	.55355	1.80653	.57657	1.73438	.60007	1.66647	.62406	1.60241	2
59	.55393	1.80529	.57696	1.73321	.60046	1.66538	.62446	1.60137	1
60	.55431	1.80405	.57735	1.73205	.60086	1.66428	.62487	1.60033	
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	
	61°		60°		59°		58°		

TABLE XXVIII.—NATURAL TANGENTS AND COTANGENTS.

′	32°		33°		34°		35°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	60
1	.62527	1.59930	.64982	1.53888	.67493	1.48163	.70064	1.42726	59
2	.62568	1.59826	.65024	1.53791	.67536	1 48070	.70107	1.42638	58
3	.62608	1.59723	.65065	1.53693	.67578	1.47977	.70151	1.42550	57
4	.62649	1.59620	.65106	1.53595	.67620	1.47885	.70194	1.42462	56
5	.62689	1.59517	.65148	1.53497	.67663	1.47792	.70238	1.42374	55
6	.62730	1.59414	.65189	1.53400	.67705	1.47699	.70281	1.42286	54
7	.62770	1.59311	.65231	1.53302	.67748	1.47607	.70325	1.42198	53
8	.62811	1.59208	.65272	1.53205	.67790	1.47514	.70368	1.42110	52
9	.62852	1.59105	.65314	1.53107	.67832	1.47422	.70412	1.42022	51
10	.62892	1.59002	.65355	1.53010	.67875	1.47330	.70455	1.41934	50
11	.62933	1.58900	.65397	1.52913	.67917	1.47238	.70499	1.41847	49
12	.62973	1.58797	.65438	1.52816	.67960	1.47146	.70542	1.41759	48
13	.63014	1.58695	.65480	1.52719	.68002	1.47053	.70586	1.41672	47
14	.63055	1.58593	.65521	1.52622	.68045	1.46962	.70629	1.41584	46
15	.63095	1.58490	.65563	1.52525	.68088	1.46870	.70673	1.41497	45
16	.63136	1.58388	.65604	1.52429	.68130	1.46778	.70717	1.41409	44
17	.63177	1.58286	.65646	1.52332	.68173	1.46686	.70760	1.41322	43
18	.63217	1.58184	.65688	1.52235	.68215	1.46595	.70804	1.41235	42
19	.63258	1.58083	.65729	1.52139	.68258	1.46503	.70848	1.41148	41
20	.63299	1.57981	.65771	1.52043	.68301	1.46411	.70891	1.41061	40
21	.63340	1.57879	.65813	1.51946	.68343	1.46320	.70935	1.40974	39
22	.63380	1.57778	.65854	1.51850	.68386	1.46229	.70979	1.40887	38
23	.63421	1.57676	.65896	1.51754	.68429	1.46137	.71023	1.40800	37
24	.63462	1.57575	.65938	1.51658	.68471	1.46046	.71066	1.40714	36
25	.63503	1.57474	.65980	1.51562	.68514	1.45955	.71110	1.40627	35
26	.63544	1.57372	.66021	1.51466	.68557	1.45864	.71154	1.40540	34
27	.63584	1.57271	.66063	1.51370	.68600	1.45773	.71198	1.40454	33
28	.63625	1.57170	.66105	1.51275	.68642	1.45682	.71242	1.40367	32
29	.63666	1.57069	.66147	1.51179	.68685	1.45592	.71285	1.40281	31
30	.63707	1.56969	.66189	1.51084	.68728	1.45501	.71329	1.40195	30
31	.63748	1.56868	.66230	1.50988	.68771	1.45410	.71373	1.40109	29
32	.63789	1.56767	.66272	1.50893	.68814	1.45320	.71417	1.40022	28
33	.63830	1.56667	.66314	1.50797	.68857	1.45229	.71461	1.39936	27
34	.63871	1.56566	.66356	1.50702	.68900	1.45139	.71505	1.39850	26
35	.63912	1.56466	.66398	1.50607	.68942	1.45049	.71549	1.39764	25
36	.63953	1.56366	.66440	1.50512	.68985	1.44958	.71593	1.39679	24
37	.63994	1.56265	.66482	1.50417	.69028	1.44868	.71637	1.39593	23
38	.64035	1.56165	.66524	1.50322	.69071	1.44778	.71681	1.39507	22
39	.64076	1.56065	.66566	1.50228	.69114	1.44688	.71725	1.39421	21
40	.64117	1.55966	.66608	1.50133	.69157	1.44598	.71769	1.39336	20
41	.64158	1.55866	.66650	1.50038	.69200	1.44508	.71813	1.39250	19
42	.64199	1.55766	.66692	1.49944	.69243	1.44418	.71857	1.39165	18
43	.64240	1.55666	.66734	1.49849	.69286	1.44329	.71901	1.39079	17
44	.64281	1.55567	.66776	1.49755	.69329	1.44239	.71946	1.38994	16
45	.64322	1.55467	.66818	1.49661	.69372	1.44149	.71990	1.38909	15
46	.64363	1.55368	.66860	1.49566	.69416	1.44060	.72034	1.38824	14
47	.64404	1.55269	.66902	1.49472	.69459	1.43970	.72078	1.38738	13
48	.64446	1.55170	.66944	1.49378	.69502	1.43881	.72122	1.38653	12
49	.64487	1.55071	.66986	1.49284	.69545	1.43792	.72167	1.38568	11
50	.64528	1.54972	.67028	1.49190	.69588	1.43703	.72211	1.38484	10
51	.64569	1.54873	.67071	1.49097	.69631	1.43614	.72255	1.38399	9
52	.64610	1.54774	.67113	1.49003	.69675	1.43525	.72299	1.38314	8
53	.64652	1.54675	.67155	1.48909	.69718	1.43436	.72344	1.38229	7
54	.64693	1.54576	.67197	1.48816	.69761	1.43347	.72388	1.38145	6
55	.64734	1.54478	.67239	1.48722	.69804	1.43258	.72432	1.38060	5
56	.64775	1.54379	.67282	1.48629	.69847	1.43169	.72477	1.37976	4
57	.64817	1.54281	.67324	1.48536	.69891	1.43080	.72521	1.37891	3
58	.64858	1.54183	.67366	1.48442	.69934	1.42992	.72565	1.37807	2
59	.64899	1.54085	.67409	1.48349	.69977	1.42903	.72610	1.37722	1
60	.64941	1.53986	.67451	1.48256	.70021	1.42815	.72654	1.37638	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	57°		56°		55°		54°		

TABLE XXVIII.—NATURAL TANGENTS AND COTANGENTS.

′	36°		37°		38°		39°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.73457	1.36134	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.74312	1.34568	.77057	1.29775	.79877	1.25193	82776	1.20808	23
38	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
′	*Cotang*	*Tang*	Cotang	Tang	Cotang	Tang	Cotang	Tang	
	53°		52°		51°		50°		

TABLE XXVIII.—NATURAL TANGENTS AND COTANGENTS.

′	40°		41°		42°		43°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	33
28	.85308	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	24
37	.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	0
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	
	49°		48°		47°		46°		

TABLE XXVIII—NATURAL TANGENTS AND COTANGENTS.

′	44°		′	′	44°		′	′	44°		′
	Tang	Cotang			Tang	Cotang			Tang	Cotang	
0	.96569	1.03553	60	20	.97700	1.02355	40	40	.98843	1.01170	20
1	.96625	1.03493	59	21	.97756	1.02295	39	41	.98901	1.01112	19
2	.96681	1.03433	58	22	.97813	1.02236	38	42	.98958	1.01053	18
3	.96738	1.03372	57	23	.97870	1.02176	37	43	.99016	1.00994	17
4	.96794	1.03312	56	24	.97927	1.02117	36	44	.99073	1.00935	16
5	.96850	1.03252	55	25	.97984	1.02057	35	45	.99131	1.00876	15
6	.96907	1.03192	54	26	.98041	1.01998	34	46	.99189	1.00818	14
7	.96963	1.03132	53	27	.98098	1.01939	33	47	.99247	1.00759	13
8	.97020	1.03072	52	28	.98155	1.01879	32	48	.99304	1.00701	12
9	.97076	1.03012	51	29	.98213	1.01820	31	49	.99362	1.00642	11
10	.97133	1.02952	50	30	.98270	1.01761	30	50	.99420	1.00583	10
11	.97189	1.02892	49	31	.98327	1.01702	29	51	.99478	1.00525	9
12	.97246	1.02832	48	32	.98384	1.01642	28	52	.99536	1.00467	8
13	.97302	1.02772	47	33	.98441	1.01583	27	53	.99594	1.00408	7
14	.97359	1.02713	46	34	.98499	1.01524	26	54	.99652	1.00350	6
15	.97416	1.02653	45	35	.98556	1.01465	25	55	.99710	1.00291	5
16	.97472	1.02593	44	36	.98613	1.01406	24	56	.99768	1.00233	4
17	.97529	1.02533	43	37	.98671	1.01347	23	57	.99826	1.00175	3
18	.97586	1.02474	42	38	.98728	1.01288	22	58	.99884	1.00116	2
19	.97643	1.02414	41	39	.98786	1.01229	21	59	.99942	1.00058	1
20	.97700	1.02355	40	40	.98843	1.01170	20	60	1.00000	1.00000	0
′	Cotang	Tang	′	′	Cotang	Tang	′	′	Cotang	Tang	′
	45°				45°				45°		

ABLE XXIX.—NATURAL VERSE

'	0°		1°		2°			
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.
0	.00000	.00000	.00015	.00015	.00061	.00061	.00137	.00137
1	.00000	.00000	.00016	.00016	.00062	.00062	.00139	.00139
2	.00000	.00000	.00016	.00016	.00063	.00063	.00140	.00140
3	.00000	.00000	.00017	.00017	.00064	.00064	.00142	.00142
4	.00000	.00000	.00017	.00017	.00065	.00065	.00143	.00143
5	.00000	.00000	.00018	.00018	.00066	.00066	.00145	.00145
6	.00000	.00000	.00018	.00018	.00067	.00067	.00146	.0014
7	.00000	.00000	.00019	.00019	.00068	.00068	.00148	.0014
8	.00000	.00000	.00020	.00020	.00069	.00069	.00150	.001
9	.00000	.00000	.00020	.00020	.00070	.00070	.00151	.00
10	.00000	.00000	.00021	.00021	.00071	.90072	.00153	.0
11	.00001	.00001	.00021	.00021	.00073	.00073	.00154	
12	.00001	.00001	.00022	.00022	.00074	.00074	.00156	
13	.00001	.00001	.00023	.00023	.00075	.00075	.00158	
14	.00001	.00001	.00023	.00023	.00076	.00076	.00159	
15	.00001	.00001	.00024	.00024	.00077	.00077	.00161	
16	.00001	.00001	.00024	.00024	.00078	.00078	.00162	
17	.00001	.00001	.00025	.00025	.00079	.00079	.00164	
18	.00001	.00001	.00026	.00026	.00081	.00081	.0016	
19	.00002	.00002	.00026	.00026	.00082	.00082	.0016	
20	.00002	.00002	.00027	.00027	.00083	.00083	.001	
21	.00002	.00002	.00028	.00028	.00084	.00084	.001	
22	.00002	.00002	.00028	.00028	.00085	.00085	.00	
23	.00002	.00002	.00029	.00029	.00087	.00087	.00	
24	.00002	.00002	.00030	.00030	.00088	.00088	.0	
25	.00003	.00003	.00031	.00031	.00089	.00089	.0	
26	.00003	.00003	.00031	.00031	.00090	.00090	.	
27	.00003	.00003	.00032	.00032	.00091	.00091	.	
28	.00003	.00003	.00033	.00033	.00093	.00093		
29	.00004	.00004	.00034	.00034	.00094	.00094		
30	.00004	.00004	.00034	.00034	.00095	.00095		
31	.00004	.00004	.00035	.00035	.00096	.00097		
32	.00004	.00004	.00036	.00036	.00098	.00098		
33	.00005	.00005	.00037	.00037	.00099	.00099		
34	.00005	.00005	.00037	.00037	.00100	.00100		
35	.00005	.00005	.00038	.00038	.00102	.00102		
36	.00005	.00005	.00039	.00039	.00103	.00103		
37	.00006	.00006	.00040	.00040	.00104	.00104		
38	.00006	.00006	.00041	.00041	.00106	.00106		
39	.00006	.00006	.00041	.00041	.00107	.00107		
40	.00007	.00007	.00042	.00042	.00108	.00108		
41	.00007	.00007	.00043	.00043	.00110	.00110		
42	.00007	.00007	.00044	.00044	.00111	.00111		
43	.00008	.00008	.00045	.00045	.00112	.00113		
44	.00008	.00008	.00046	.00046	.00114	.00114		
45	.00009	.00009	.00047	.00047	.00115	.00115		
46	.00009	.00009	.00048	.00048	.00117	.00117		
47	.00009	.00009	.00048	.00048	.00118	.00118		
48	.00010	.00010	.00049	.00049	.00119	.00120		
49	.00010	.00010	.00050	.00050	.00121	.00121		
50	.00011	.00011	.00051	.00051	.00122	.0012		
51	.00011	.00011	.00052	.00052	.00124	.0012		
52	.00011	.00011	.00053	.00053	.00125	.0012		
53	.00012	.00012	.00054	.00054	.00127	.0012		
54	.00012	.00012	.00055	.00055	.00128	.0012		
55	.00013	.00013	.00056	.00056	.00130	[illegible]		
56	.00013	.00013	.00057	.00057	.00131	.00		
57	.00014	.00014	.00058	.00058	.00133	.0		
58	.00014	.00014	.00059	.00059	.00134	.		
59	.00015	.00015	.00060	.00060	.00136			
60	.00015	.00015	.00061	.00061	.00137			

TABLE XXIX.—NATURAL VERSED SINES AND EXTERNAL SECANTS

′	4°		5°		6°		7°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	.00244	.00244	.00381	.00382	.00548	.00551	.00745	.00751	0
1	.00246	.00246	.00383	.00385	.00551	.00554	.00749	.00755	1
2	.00248	.00248	.00386	.00387	.00554	.00557	.00752	.00758	2
3	.00250	.00250	.00388	.00390	.00557	.00560	.00756	.00762	3
4	.00252	.00252	.00391	.00392	.00560	.00563	.00760	.00765	4
5	.00254	.00254	.00393	.00395	.00563	.00566	.00763	.00769	5
6	.00256	.00257	.00396	.00397	.00566	.00569	.00767	.00773	6
7	.00258	.00259	.00398	.00400	.00569	.00573	.00770	.00776	7
8	.00260	.00261	.00401	.00403	.00572	.00576	.00774	.00780	8
9	.00262	.00263	.00404	.00405	.00576	.00579	.00778	.00784	9
10	.00264	.00265	.00406	.00408	.00579	.00582	.00781	.00787	10
11	.00266	.00267	.00409	.00411	.00582	.00585	.00785	.00791	11
12	.00269	.00269	.00412	.00413	.00585	.00588	.00789	.00795	12
13	.00271	.00271	.00414	.00416	.00588	.00592	.00792	.00799	13
14	.00273	.00274	.00417	.00419	.00591	.00595	.00796	.00802	14
15	.00275	.00276	.00420	.00421	.00594	.00598	.00800	.00806	15
16	.00277	.00278	.00422	.00424	.00598	.00601	.00803	.00810	16
17	.00279	.00280	.00425	.00427	.00601	.00604	.00807	.00813	17
18	.00281	.00282	.00428	.00429	.00604	.00608	.00811	.00817	18
19	.00284	.00284	.00430	.00432	.00607	.00611	.00814	.00821	19
20	.00286	.00287	.00433	.00435	.00610	.00614	.00818	.00825	20
21	.00288	.00289	.00436	.00438	.00614	.00617	.00822	.00828	21
22	.00290	.00291	.00438	.00440	.00617	.00621	.00825	.00832	22
23	.00293	.00293	.00441	.00443	.00620	.00624	.00829	.00836	23
24	.00295	.00296	.00444	.00446	.00623	.00627	.00833	.00840	24
25	.00297	.00298	.00447	.00449	.00626	.00630	.00837	.00844	25
26	.00299	.00300	.00449	.00451	.00630	.00634	.00840	.00848	26
27	.00301	.00302	.00452	.00454	.00633	.00637	.00844	.00851	27
28	.00304	.00305	.00455	.00457	.00636	.00640	.00848	.00855	28
29	.00306	.00307	.00458	.00460	.00640	.00644	.00852	.00859	29
30	.00308	.00309	.00460	.00463	.00643	.00647	.00856	.00863	30
31	.00311	.00312	.00463	.00465	.00646	.00650	.00859	.00867	31
32	.00313	.00314	.00466	.00468	.00649	.00654	.00863	.00871	32
33	.00315	.00316	.00469	.00471	.00653	.00657	.00867	.00875	33
34	.00317	.00318	.00472	.00474	.00656	.00660	.00871	.00878	34
35	.00320	.00321	.00474	.00477	.00659	.00664	.00875	.00882	35
36	.00322	.00323	.00477	.00480	.00663	.00667	.00878	.00886	36
37	.00324	.00326	.00480	.00482	.00666	.00671	.00882	.00890	37
38	.00327	.00328	.00483	.00485	.00669	.00674	.00886	.00894	38
39	.00329	.00330	.00486	.00488	.00673	.00677	.00890	.00898	39
40	.00332	.00333	.00489	.00491	.00676	.00681	.00894	.00902	40
41	.00334	.00335	.00492	.00494	.00680	.00684	.00898	.00906	41
42	.00336	.00337	.00494	.00497	.00683	.00688	.00902	.00910	42
43	.00339	.00340	.00497	.00500	.00686	.00691	.00906	.00914	43
44	.00341	.00342	.00500	.00503	.00690	.00695	.00909	.00918	44
45	.00343	.00345	.00503	.00506	.00693	.00698	.00913	.00922	45
46	.00346	.00347	.00506	.00509	.00697	.00701	.00917	.00926	46
47	.00348	.00350	.00509	.00512	.00700	.00705	.00921	.00930	47
48	.00351	.00352	.00512	.00515	.00703	.00708	.00925	.00934	48
49	.00353	.00354	.00515	.00518	.00707	.00712	.00929	.00938	49
50	.00356	.00357	.00518	.00521	.00710	.00715	.00933	.00942	50
51	.00358	.00359	.00521	.00524	.00714	.00719	.00937	.00946	51
52	.00361	.00362	.00524	.00527	.00717	.00722	.00941	.00950	52
53	.00363	.00364	.00527	.00530	.00721	.00726	.00945	.00954	53
54	.00365	.00367	.00530	.00533	.00724	.00730	.00949	.00958	54
55	.00368	.00369	.00533	.00536	.00728	.00733	.00953	.00962	55
56	.00370	.00372	.00536	.00539	.00731	.00737	.00957	.00966	56
57	.00373	.00374	.00539	.00542	.00735	.00740	.00961	.00970	
58	.00375	.00377	.00542	.00545	.00738	.00744	.00965	.00975	
59	.00378	.00379	.00545	.00548	.00742	.00747	.00969	.00979	
60	.00381	.00382	.00548	.00551	.00745	.00751	.00973	.0098	

TABLE XXIX.—NATURAL VERSED SINES AND EXTERNAL SECA

′	8°		9°		10°		11°	
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.
0	.00973	.00983	.01231	.01247	.01519	.01543	.01837	.01872
1	.00977	.00987	.01236	.01251	.01524	.01548	.01843	.01877
2	.00981	.00991	.01240	.01256	.01529	.01553	.01848	.01883
3	.00985	.00995	.01245	.01261	.01534	.01558	.01854	.01889
4	.00989	.00999	.01249	.01265	.01540	.01564	.01860	.01895
5	.00994	.01004	.01254	.01270	.01545	.01569	.01865	.01901
6	.00998	.01008	.01259	.01275	.01550	.01574	.01871	.01906
7	.01002	.01012	.01263	.01279	.01555	.01579	.01876	.01912
8	.01006	.01016	.01268	.01284	.01560	.01585	.01882	.01918
9	.01010	.01020	.01272	.01289	.01565	.01590	.01888	.01924
10	.01014	.01024	.01277	.01294	.01570	.01595	.01893	.01930
11	.01018	.01029	.01282	.01298	.01575	.01601	.01899	.01936
12	.01022	.01033	.01286	.01303	.01580	.01606	.01904	.01941
13	.01027	.01037	.01291	.01308	.01586	.01611	.01910	.01947
14	.01031	.01041	.01296	.01313	.01591	.01616	.01916	.01953
15	.01035	.01046	.01300	.01318	.01596	.01622	.01921	.01959
16	.01039	.01050	.01305	.01322	.01601	.01627	.01927	.01965
17	.01043	.01054	.01310	.01327	.01606	.01633	.01933	.01971
18	.01047	.01059	.01314	.01332	.01612	.01638	.01939	.01977
19	.01052	.01063	.01319	.01337	.01617	.01643	.01944	.01983
20	.01056	.01067	.01324	.01342	.01622	.01649	.01950	.01989
21	.01060	.01071	.01329	.01346	.01627	.01654	.01956	.01995
22	.01064	.01076	.01333	.01351	.01632	.01659	.01961	.02001
23	.01069	.01080	.01338	.01356	.01638	.01665	.01967	.02007
24	.01073	.01084	.01343	.01361	.01643	.01670	.01973	.02013
25	.01077	.01089	.01348	.01366	.01648	.01676	.01979	.02019
26	.01081	.01093	.01352	.01371	.01653	.01681	.01984	.02025
27	.01086	.01097	.01357	.01376	.01659	.01687	.01990	.02031
28	.01090	.01102	.01362	.01381	.01664	.01692	.01996	.02037
29	.01094	.01106	.01367	.01386	.01669	.01698	.02002	.02043
30	.01098	.01111	.01371	.01391	.01675	.01703	.02008	.02049
31	.01103	.01115	.01376	.01395	.01680	.01709	.02013	.02055
32	.01107	.01119	.01381	.01400	.01685	.01714	.02019	.02061
33	.01111	.01124	.01386	.01405	.01690	.01720	.02025	.02067
34	.01116	.01128	.01391	.01410	.01696	.01725	.02031	.02073
35	.01120	.01133	.01396	.01415	.01701	.01731	.02037	.02079
36	.01124	.01137	.01400	.01420	.01706	.01736	.02042	.02085
37	.01129	.01142	.01405	.01425	.01712	.01742	.02048	.02091
38	.01133	.01146	.01410	.01430	.01717	.01747	.02054	.02097
39	.01137	.01151	.01415	.01435	.01723	.01753	.02060	.02103
40	.01142	.01155	.01420	.01440	.01728	.01758	.02066	.02110
41	.01146	.01160	.01425	.01445	.01733	.01764	.02072	.02116
42	.01151	.01164	.01430	.01450	.01739	.01769	.02078	.02122
43	.01155	.01169	.01435	.01455	.01744	.01775	.02084	.02128
44	.01159	.01173	.01439	.01461	.01750	.01781	.02090	.02134
45	.01164	.01178	.01444	.01466	.01755	.01786	.02095	.02140
46	.01168	.01182	.01449	.01471	.01760	.01792	.02101	.02146
47	.01173	.01187	.01454	.01476	.01766	.01798	.02107	.02153
48	.01177	.01191	.01459	.01481	.01771	.01803	.02113	.02159
49	.01182	.01196	.01464	.01486	.01777	.01809	.02119	.02165
50	.01186	.01200	.01469	.01491	.01782	.01815	.02125	.02171
51	.01191	.01205	.01474	.01496	.01788	.01820	.02131	.02178
52	.01195	.01209	.01479	.01501	.01793	.01826	.02137	.02184
53	.01200	.01214	.01484	.01506	.01799	.01832	.02143	.02190
54	.01204	.01219	.01489	.01512	.01804	.01837	.02149	.02196
55	.01209	.01223	.01494	.01517	.01810	.01843	.02155	.02203
56	.01213	.01228	.01499	.01522	.01815	.01849	.02161	.02209
57	.01218	.01233	.01504	.01527	.01821	.01854	.02167	.0221
58	.01222	.01237	.01509	.01532	.01826	.01860	.02173	.022
59	.01227	.01242	.01514	.01537	.01832	.01866	.02179	.02
60	.01231	.01247	.01519	.01543	.01837	.01872	.02185	.0

BLE XXIX.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

′	12° Vers.	12° Ex. sec.	13° Vers.	13° Ex. sec.	14° Vers.	14° Ex. sec.	15° Vers.	15° Ex. sec.	′
0	.02185	.02234	.02563	.02630	.02970	.03061	.03407	.03528	0
1	.02191	.02240	.02570	.02637	.02977	.03069	.03415	.03536	1
2	.02197	.02247	.02576	.02644	.02985	.03076	.03422	.03544	2
3	.02203	.02253	.02583	.02651	.02992	.03084	.03430	.03552	3
4	.02210	.02259	.02589	.02658	.02999	.03091	.03438	.03560	4
5	.02216	.02266	.02596	.02665	.03006	.03099	.03445	.03568	5
6	.02222	.02272	.02602	.02672	.03013	.03106	.03453	.03576	6
7	.02228	.02279	.02609	.02679	.03020	.03114	.03460	.03584	7
8	.02234	.02285	.02616	.02686	.03027	.03121	.03468	.03592	8
9	.02240	.02291	.02622	.02693	.03034	.03129	.03476	.03601	9
10	.02246	.02298	.02629	.02700	.03041	.03137	.03483	.03609	10
11	.02252	.02304	.02635	.02707	.03048	.03144	.03491	.03617	11
12	.02258	.02311	.02642	.02714	.03055	.03152	.03498	.03625	12
13	.02265	.02317	.02649	.02721	.03063	.03159	.03506	.03633	13
14	.02271	.02323	.02655	.02728	.03070	.03167	.03514	.03642	14
15	.02277	.02330	.02662	.02735	.03077	.03175	.03521	.03650	15
16	.02283	.02336	.02669	.02742	.03084	.03182	.03529	.03658	16
17	.02289	.02343	.02675	.02749	.03091	.03190	.03537	.03666	17
18	.02295	.02349	.02682	.02756	.03098	.03198	.03544	.03674	18
19	.02302	.02356	.02689	.02763	.03106	.03205	.03552	.03683	19
20	.02308	.02362	.02696	.02770	.03113	.03213	.03560	.03691	20
21	.02314	.02369	.02702	.02777	.03120	.03221	.03567	.03699	21
22	.02320	.02375	.02709	.02784	.03127	.03228	.03575	.03708	22
23	.02327	.02382	.02716	.02791	.03134	.03236	.03583	.03716	23
24	.02333	.02388	.02722	.02799	.03142	.03244	.03590	.03724	24
25	.02339	.02395	.02729	.02806	.03149	.03251	.03598	.03732	25
26	.02345	.02402	.02736	.02813	.03156	.03259	.03606	.03741	26
27	.02352	.02408	.02743	.02820	.03163	.03267	.03614	.03749	27
28	.02358	.02415	.02749	.02827	.03171	.03275	.03621	.03758	28
29	.02364	.02421	.02756	.02834	.03178	.03282	.03629	.03766	29
30	.02370	.02428	.02763	.02842	.03185	.03290	.03637	.03774	30
31	.02377	.02435	.02770	.02849	.03193	.03298	.03645	.03783	31
32	.02383	.02441	.02777	.02856	.03200	.03306	.03653	.03791	32
33	.02389	.02448	.02783	.02863	.03207	.03313	.03660	.03799	33
34	.02396	.02454	.02790	.02870	.03214	.03321	.03668	.03808	34
35	.02402	.02461	.02797	.02878	.03222	.03329	.03676	.03816	35
36	.02408	.02468	.02804	.02885	.03229	.03337	.03684	.03825	36
37	.02415	.02474	.02811	.02892	.03236	.03345	.03692	.03833	37
38	.02421	.02481	.02818	.02899	.03244	.03353	.03699	.03842	38
39	.02427	.02488	.02824	.02907	.03251	.03360	.03707	.03850	39
40	.02434	.02494	.02831	.02914	.03258	.03368	.03715	.03858	40
41	.02440	.02501	.02838	.02921	.03266	.03376	.03723	.03867	41
42	.02447	.02508	.02845	.02928	.03273	.03384	.03731	.03875	42
43	.02453	.02515	.02852	.02936	.03281	.03392	.03739	.03884	43
44	.02459	.02521	.02859	.02943	.03288	.03400	.03747	.03892	44
45	.02466	.02528	.02866	.02950	.03295	.03408	.03754	.03901	45
46	.02472	.02535	.02873	.02958	.03303	.03416	.03762	.03909	46
47	.02479	.02542	.02880	.02965	.03310	.03424	.03770	.03918	47
48	.02485	.02548	.02887	.02972	.03318	.03432	.03778	.03927	48
49	.02492	.02555	.02894	.02980	.03325	.03439	.03786	.03935	49
50	.02498	.02562	.02900	.02987	.03333	.03447	.03794	.03944	50
51	.02504	.02569	.02907	.02994	.03340	.03455	.03802	.03952	51
52	.02511	.02576	.02914	.03002	.03347	.03463	.03810	.03961	52
53	.02517	.02582	.02921	.03009	.03355	.03471	.03818	.03969	53
54	.02524	.02589	.02928	.03017	.03362	.03479	.03826	.03978	54
55	.02530	.02596	.02935	.03024	.03370	.03487	.03834	.03987	55
56	.02537	.02603	.02942	.03032	.03377	.03495	.03842	.03995	56
57	.02543	.02610	.02949	.03039	.03385	.03503	.03850	.04004	57
58	.02550	.02617	.02956	.03046	.03392	.03512	.03858	.04013	58
9	.02556	.02624	.02963	.03054	.03400	.03520	.03866	.04021	59
	.02563	.02630	.02970	.03061	.03407	.03528	.03874	.04030	6

TABLE XXIX.—NATURAL VERSED SINES AND EXTERNAL SECAN

′	16°		17°		18°		19°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	.03874	.04030	.04370	.04569	.04894	.05146	.05448	.05762	0
1	.03882	.04039	.04378	.04578	.04903	.05156	.05458	.05773	1
2	.03890	.04047	.04387	.04588	.04912	.05166	.05467	.05783	2
3	.03898	.04056	.04395	.04597	.04921	.05176	.05477	.05794	3
4	.03906	.04065	.04404	.04606	.04930	.05186	.05486	.05805	4
5	.03914	.04073	.04412	.04616	.04939	.05196	.05496	.05815	5
6	.03922	.04082	.04421	.04625	.04948	.05206	.05505	.05826	6
7	.03930	.04091	.04429	.04635	.04957	.05216	.05515	.05836	7
8	.03938	.04100	.04438	.04644	.04967	.05226	.05524	.05847	8
9	.03946	.04108	.04446	.04653	.04976	.05236	.05534	.05858	9
10	.03954	.04117	.04455	.04663	.04985	.05246	.05543	.05869	10
11	.03963	.04126	.04464	.04672	.04994	.05256	.05553	.05879	11
12	.03971	.04135	.04472	.04682	.05003	.05266	.05562	.05890	12
13	.03979	.04144	.04481	.04691	.05012	.05276	.05572	.05901	13
14	.03987	.04152	.04489	.04700	.05021	.05286	.05582	.05911	14
15	.03995	.04161	.04498	.04710	.05030	.05297	.05591	.05922	15
16	.04003	.04170	.04507	.04719	.05039	.05307	.05601	.05933	16
17	.04011	.04179	.04515	.04729	.05048	.05317	.05610	.05944	17
18	.04019	.04188	.04524	.04738	.05057	.05327	.05620	.05955	18
19	.04028	.04197	.04533	.04748	.05067	.05337	.05630	.05965	19
20	.04036	.04206	.04541	.04757	.05076	.05347	.05639	.05976	20
21	.04044	.04214	.04550	.04767	.05085	.05357	.05649	.05987	21
22	.04052	.04223	.04559	.04776	.05094	.05367	.05658	.05998	22
23	.04060	.04232	.04567	.04786	.05103	.05378	.05668	.06009	23
24	.04069	.04241	.04576	.04795	.05112	.05388	.05678	.06020	24
25	.04077	.04250	.04585	.04805	.05122	.05398	.05687	.06030	25
26	.04085	.04259	.04593	.04815	.05131	.05408	.05697	.06041	26
27	.04093	.04268	.04602	.04824	.05140	.05418	.05707	.06052	27
28	.04102	.04277	.04611	.04834	.05149	.05429	.05716	.06063	28
29	.04110	.04286	.04620	.04843	.05158	.05439	.05726	.06074	29
30	.04118	.04295	.04628	.04853	.05168	.05449	.05736	.06085	30
31	.04126	.04304	.04637	.04863	.05177	.05460	.05746	.06096	31
32	.04135	.04313	.04646	.04872	.05186	.05470	.05755	.06107	32
33	.04143	.04322	.04655	.04882	.05195	.05480	.05765	.06118	33
34	.04151	.04331	.04663	.04891	.05205	.05490	.05775	.06129	34
35	.04159	.04340	.04672	.04901	.05214	.05501	.05785	.06140	35
36	.04168	.04349	.04681	.04911	.05223	.05511	.05794	.06151	36
37	.04176	.04358	.04690	.04920	.05232	.05521	.05804	.06162	37
38	.04184	.04367	.04699	.04930	.05242	.05532	.05814	.06173	38
39	.04193	.04376	.04707	.04940	.05251	.05542	.05824	.06184	39
40	.04201	.04385	.04716	.04950	.05260	.05552	.05833	.06195	40
41	.04209	.04394	.04725	.04959	.05270	.05563	.05843	.06206	41
42	.04218	.04403	.04734	.04969	.05279	.05573	.05853	.06217	42
43	.04226	.04413	.04743	.04979	.05288	.05584	.05863	.06228	43
44	.04234	.04422	.04752	.04989	.05298	.05594	.05873	.06239	44
45	.04243	.04431	.04760	.04998	.05307	.05604	.05882	.06250	45
46	.04251	.04440	.04769	.05008	.05316	.05615	.05892	.06261	46
47	.04260	.04449	.04778	.05018	.05326	.05625	.05902	.06272	47
48	.04268	.04458	.04787	.05028	.05335	.05636	.05912	.06283	48
49	.04276	.04468	.04796	.05038	.05344	.05646	.05922	.06295	49
50	.04285	.04477	.04805	.05047	.05354	.05657	.05932	.06306	50
51	.04293	.04486	.04814	.05057	.05363	.05667	.05942	.06317	51
52	.04302	.04495	.04823	.05067	.05373	.05678	.05951	.06328	52
53	.04310	.04504	.04832	.05077	.05382	.05688	.05961	.06339	53
54	.04319	.04514	.04841	.05087	.05391	.05699	.05971	.06350	54
55	.04327	.04523	.04850	.05097	.05401	.05709	.05981	.06362	55
56	.04336	.04532	.04858	.05107	.05410	.05720	.05991	.06373	56
57	.04344	.04541	.04867	.05116	.05420	.05730	.06001	.06384	57
58	.04353	.04551	.04876	.05126	.05429	.05741	.06011	.06395	58
59	.04361	.04560	.04885	.05136	.05439	.05751	.06021	.06407	
60	.04370	.04569	.04894	.05146	.05448	.05762	.06031	.06418	

TABLE XXIX.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

′	20°		21°		22°		23°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	.06031	.06418	.06642	.07115	.07282	.07853	.07950	.08636	0
1	.06041	.06429	.06652	.07126	.07293	.07866	.07961	.08649	1
2	.06051	.06440	.06663	.07138	.07303	.07879	.07972	.08663	2
3	.06061	.06452	.06673	.07150	.07314	.07892	.07984	.08676	3
4	.06071	.06463	.06684	.07162	.07325	.07904	.07995	.08690	4
5	.06081	.06474	.06694	.07174	.07336	.07917	.08006	.08703	5
6	.06091	.06486	.06705	.07186	.07347	.07930	.08018	.08717	6
7	.06101	.06497	.06715	.07199	.07358	.07943	.08029	.08730	7
8	.06111	.06508	.06726	.07211	.07369	.07955	.08041	.08744	8
9	.06121	.06520	.06736	.07223	.07380	.07968	.08052	.08757	9
10	.06131	.06531	.06747	.07235	.07391	.07981	.08064	.08771	10
11	.06141	.06542	.06757	.07247	.07402	.07994	.08075	.08784	11
12	.06151	.06554	.06768	.07259	.07413	.08006	.08086	.08798	12
13	.06161	.06565	.06778	.07271	.07424	.08019	.08098	.08811	13
14	.06171	.06577	.06789	.07283	.07435	.08032	.08109	.08825	14
15	.06181	.06588	.06799	.07295	.07446	.08045	.08121	.08839	15
16	.06191	.06600	.06810	.07307	.07457	.08058	.08132	.08852	16
17	.06201	.06611	.06820	.07320	.07468	.08071	.08144	.08866	17
18	.06211	.06622	.06831	.07332	.07479	.08084	.08155	.08880	18
19	.06221	.06634	.06841	.07344	.07490	.08097	.08167	.08893	19
20	.06231	.06645	.06852	.07356	.07501	.08109	.08178	.08907	20
21	.06241	.06657	.06863	.07368	.07512	.08122	.08190	.08921	21
22	.06252	.06668	.06873	.07380	.07523	.08135	.08201	.08934	22
23	.06262	.06680	.06884	.07393	.07534	.08148	.08213	.08948	23
24	.06272	.06691	.06894	.07405	.07545	.08161	.08225	.08962	24
25	.06282	.06703	.06905	.07417	.07556	.08174	.08236	.08975	25
26	.06292	.06715	.06916	.07429	.07568	.08187	.08248	.08989	26
27	.06302	.06726	.06926	.07442	.07579	.08200	.08259	.09003	27
28	.06312	.06738	.06937	.07454	.07590	.08213	.08271	.09017	28
29	.06323	.06749	.06948	.07466	.07601	.08226	.08282	.09030	29
30	.06333	.06761	.06958	.07479	.07612	.08239	.08294	.09044	30
31	.06343	.06773	.06969	.07491	.07623	.08252	.08306	.09058	31
32	.06353	.06784	.06980	.07503	.07634	.08265	.08317	.09072	32
33	.06363	.06796	.06990	.07516	.07645	.08278	.08329	.09086	33
34	.06374	.06807	.07001	.07528	.07657	.08291	.08340	.09099	34
35	.06384	.06819	.07012	.07540	.07668	.08305	.08352	.09113	35
36	.06394	.06831	.07022	.07553	.07679	.08318	.08364	.09127	36
37	.06404	.06843	.07033	.07565	.07690	.08331	.08375	.09141	37
38	.06415	.06854	.07044	.07578	.07701	.08344	.08387	.09155	38
39	.06425	.06866	.07055	.07590	.07713	.08357	.08399	.09169	39
40	.06435	.06878	.07065	.07602	.07724	.08370	.08410	.09183	40
41	.06445	.06889	.07076	.07615	.07735	.08383	.08422	.09197	41
42	.06456	.06901	.07087	.07627	.07746	.08397	.08434	.09211	42
43	.06466	.06913	.07098	.07640	.07757	.08410	.08445	.09224	43
44	.06476	.06925	.07108	.07652	.07769	.08423	.08457	.09238	44
45	.06486	.06936	.07119	.07665	.07780	.08436	.08469	.09252	45
46	.06497	.06948	.07130	.07677	.07791	.08449	.08481	.09266	46
47	.06507	.06960	.07141	.07690	.07802	.08463	.08492	.09280	47
48	.06517	.06972	.07151	.07702	.07814	.08476	.08504	.09294	48
49	.06528	.06984	.07162	.07715	.07825	.08489	.08516	.09308	49
50	.06538	.06995	.07173	.07727	.07836	.08503	.08528	.09323	50
51	.06548	.07007	.07184	.07740	.07848	.08516	.08539	.09337	51
52	.06559	.07019	.07195	.07752	.07859	.08529	.08551	.09351	52
53	.06569	.07031	.07206	.07765	.07870	.08542	.08563	.09365	53
54	.06580	.07043	.07216	.07778	.07881	.08556	.08575	.09379	54
55	.06590	.07055	.07227	.07790	.07893	.08569	.08586	.09393	55
56	.06600	.07067	.07238	.07803	.07904	.08582	.08598	.09407	56
57	.06611	.07079	.07249	.07816	.07915	.08596	.08610	.09421	57
58	.06621	.07091	.07260	.07828	.07927	.08609	.08622	.09435	58
59	.06632	.07103	.07271	.07841	.07938	.08622	.08634	.09449	59
60	.06642	.07115	.07282	.07853	.07950	.08636	.08645	.09464	60

TABLE XXIX.—NATURAL VERSED SINES AND EXTERNAL SECANTS

′	24°		25°		26°		27°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	.08645	.09464	.09369	.10338	.10121	.11260	.10899	.12233	0
1	.08657	.09478	.09382	.10353	.10133	.11276	.10913	.12249	1
2	.08669	.09492	.09394	.10368	.10146	.11292	.10926	.12266	2
3	.08681	.09506	.09406	.10383	.10159	.11308	.10939	.12283	3
4	.08693	.09520	.09418	.10398	.10172	.11323	.10952	.12299	4
5	.08705	.09535	.09431	.10413	.10184	.11339	.10965	.12316	5
6	.08717	.09549	.09443	.10428	.10197	.11355	.10979	.12333	6
7	.08728	.09563	.09455	.10443	.10210	.11371	.10992	.12349	7
8	.08740	.09577	.09468	.10458	.10223	.11387	.11005	.12366	8
9	.08752	.09592	.09480	.10473	.10236	.11403	.11019	.12383	9
10	.08764	.09606	.09493	.10488	.10248	.11419	.11032	.12400	10
11	.08776	.09620	.09505	.10503	.10261	.11435	.11045	.12416	11
12	.08788	.09635	.09517	.10518	.10274	.11451	.11058	.12433	12
13	.08800	.09649	.09530	.10533	.10287	.11467	.11072	.12450	13
14	.08812	.09663	.09542	.10549	.10300	.11483	.11085	.12467	14
15	.08824	.09678	.09554	.10564	.10313	.11499	.11098	.12484	15
16	.08836	.09692	.09567	.10579	.10326	.11515	.11112	.12501	16
17	.08848	.09707	.09579	.10594	.10338	.11531	.11125	.12518	17
18	.08860	.09721	.09592	.10609	.10351	.11547	.11138	.12534	18
19	.08872	.09735	.09604	.10625	.10364	.11563	.11152	.12551	19
20	.08884	.09750	.09617	.10640	.10377	.11579	.11165	.12568	20
21	.08896	.09764	.09629	.10655	.10390	.11595	.11178	.12585	21
22	.08908	.09779	.09642	.10670	.10403	.11611	.11192	.12602	22
23	.08920	.09793	.09654	.10686	.10416	.11627	.11205	.12619	23
24	.08932	.09808	.09666	.10701	.10429	.11643	.11218	.12636	24
25	.08944	.09822	.09679	.10716	.10442	.11659	.11232	.12653	25
26	.08956	.09837	.09691	.10731	.10455	.11675	.11245	.12670	26
27	.08968	.09851	.09704	.10747	.10468	.11691	.11259	.12687	27
28	.08980	.09866	.09716	.10762	.10481	.11708	.11272	.12704	28
29	.08992	.09880	.09729	.10777	.10494	.11724	.11285	.12721	29
30	.09004	.09895	.09741	.10793	.10507	.11740	.11299	.12738	30
31	.09016	.09909	.09754	.10808	.10520	.11756	.11312	.12755	31
32	.09028	.09924	.09767	.10824	.10533	.11772	.11326	.12772	32
33	.09040	.09939	.09779	.10839	.10546	.11789	.11339	.12789	33
34	.09052	.09953	.09792	.10854	.10559	.11805	.11353	.12807	34
35	.09064	.09968	.09804	.10870	.10572	.11821	.11366	.12824	35
36	.09076	.09982	.09817	.10885	.10585	.11838	.11380	.12841	36
37	.09089	.09997	.09829	.10901	.10598	.11854	.11393	.12858	37
38	.09101	.10012	.09842	.10916	.10611	.11870	.11407	.12875	38
39	.09113	.10026	.09854	.10932	.10624	.11886	.11420	.12892	39
40	.09125	.10041	.09867	.10947	.10637	.11903	.11434	.12910	40
41	.09137	.10055	.09880	.10963	.10650	.11919	.11447	.12927	41
42	.09149	.10071	.09892	.10978	.10663	.11936	.11461	.12944	42
43	.09161	.10085	.09905	.10994	.10676	.11952	.11474	.12961	43
44	.09174	.10100	.09918	.11009	.10689	.11968	.11488	.12979	44
45	.09186	.10115	.09930	.11025	.10702	.11985	.11501	.12996	45
46	.09198	.10130	.09943	.11041	.10715	.12001	.11515	.13013	46
47	.09210	.10144	.09955	.11056	.10728	.12018	.11528	.13031	47
48	.09222	.10159	.09968	.11072	.10741	.12034	.11542	.13048	48
49	.09234	.10174	.09981	.11087	.10755	.12051	.11555	.13065	49
50	.09247	.10189	.09993	.11103	.10768	.12067	.11569	.13083	50
51	.09259	.10204	.10006	.11119	.10781	.12084	.11583	.13100	51
52	.09271	.10218	.10019	.11134	.10794	.12100	.11596	.13117	52
53	.09283	.10233	.10032	.11150	.10807	.12117	.11610	.13135	53
54	.09296	.10248	.10044	.11166	.10820	.12133	.11623	.13152	54
55	.09308	.10263	.10057	.11181	.10833	.12150	.11637	.13170	55
56	.09320	.10278	.10070	.11197	.10847	.12166	.11651	.13187	56
57	.09332	.10293	.10082	.11213	.10860	.12183	.11664	.13205	57
58	.09345	.10308	.10095	.11229	.10873	.12199	.11678	.13222	58
59	.09357	.10323	.10108	.11244	.10886	.12216	.11692	.13240	59
60	.09369	.10338	.10121	.11260	.10899	.12233	.11705	.13257	60

TABLE XXIX.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

′	28°		29°		30°		31°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	.11705	.13257	.12538	.14335	.13397	.15470	.14283	.16663	0
1	.11719	.13275	.12552	.14354	.13412	.15489	.14298	.16684	1
2	.11733	.13292	.12566	.14372	.13427	.15509	.14313	.16704	2
3	.11746	.13310	.12580	.14391	.13441	.15528	.14328	.16725	3
4	.11760	.13327	.12595	.14409	.13456	.15548	.14343	.16745	4
5	.11774	.13345	.12609	.14428	.13470	.15567	.14358	.16766	5
6	.11787	.13362	.12623	.14446	.13485	.15587	.14373	.16786	6
7	.11801	.13380	.12637	.14465	.13499	.15606	.14388	.16806	7
8	.11815	.13398	.12651	.14483	.13514	.15626	.14403	.16827	8
9	.11828	.13415	.12665	.14502	.13529	.15645	.14418	.16848	9
10	.11842	.13433	.12679	.14521	.13543	.15665	.14433	.16868	10
11	.11856	.13451	.12694	.14539	.13558	.15684	.14449	.16889	11
12	.11870	.13468	.12708	.14558	.13573	.15704	.14464	.16909	12
13	.11883	.13486	.12722	.14576	.13587	.15724	.14479	.16930	13
14	.11897	.13504	.12736	.14595	.13602	.15743	.14494	.16950	14
15	.11911	.13521	.12750	.14614	.13616	.15763	.14509	.16971	15
16	.11925	.13539	.12765	.14632	.13631	.15782	.14524	.16992	16
17	.11938	.13557	.12779	.14651	.13646	.15802	.14539	.17012	17
18	.11952	.13575	.12793	.14670	.13660	.15822	.14554	.17033	18
19	.11966	.13593	.12807	.14689	.13675	.15841	.14569	.17054	19
20	.11980	.13610	.12822	.14707	.13690	.15861	.14584	.17075	20
21	.11994	.13628	.12836	.14726	.13705	.15881	.14599	.17095	21
22	.12007	.13646	.12850	.14745	.13719	.15901	.14615	.17116	22
23	.12021	.13664	.12864	.14764	.13734	.15920	.14630	.17137	23
24	.12035	.13682	.12879	.14782	.13749	.15940	.14645	.17158	24
25	.12049	.13700	.12893	.14801	.13763	.15960	.14660	.17178	25
26	.12063	.13718	.12907	.14820	.13778	.15980	.14675	.17199	26
27	.12077	.13735	.12921	.14839	.13793	.16000	.14690	.17220	27
28	.12091	.13753	.12936	.14858	.13808	.16019	.14706	.17241	28
29	.12104	.13771	.12950	.14877	.13822	.16039	.14721	.17262	29
30	.12118	.13789	.12964	.14896	.13837	.16059	.14736	.17283	30
31	.12132	.13807	.12979	.14914	.13852	.16079	.14751	.17304	31
32	.12146	.13825	.12993	.14933	.13867	.16099	.14766	.17325	32
33	.12160	.13843	.13007	.14952	.13881	.16119	.14782	.17346	33
34	.12174	.13861	.13022	.14971	.13896	.16139	.14797	.17367	34
35	.12188	.13879	.13036	.14990	.13911	.16159	.14812	.17388	35
36	.12202	.13897	.13051	.15009	.13926	.16179	.14827	.17409	36
37	.12216	.13916	.13065	.15028	.13941	.16199	.14843	.17430	37
38	.12230	.13934	.13079	.15047	.13955	.16219	.14858	.17451	38
39	.12244	.13952	.13094	.15066	.13970	.16239	.14873	.17472	39
40	.12257	.13970	.13108	.15085	.13985	.16259	.14888	.17493	40
41	.12271	.13988	.13122	.15105	.14000	.16279	.14904	.17514	41
42	.12285	.14006	.13137	.15124	.14015	.16299	.14919	.17535	42
43	.12299	.14024	.13151	.15143	.14030	.16319	.14934	.17556	43
44	.12313	.14042	.13166	.15162	.14044	.16339	.14949	.17577	44
45	.12327	.14061	.13180	.15181	.14059	.16359	.14965	.17598	45
46	.12341	.14079	.13195	.15200	.14074	.16380	.14980	.17620	46
47	.12355	.14097	.13209	.15219	.14089	.16400	.14995	.17641	47
48	.12369	.14115	.13223	.15239	.14104	.16420	.15011	.17662	48
49	.12383	.14134	.13238	.15258	.14119	.16440	.15026	.17683	49
50	.12397	.14152	.13252	.15277	.14134	.16460	.15041	.17704	50
51	.12411	.14170	.13267	.15296	.14149	.16481	.15057	.17726	51
52	.12425	.14188	.13281	.15315	.14164	.16501	.15072	.17747	52
53	.12439	.14207	.13296	.15335	.14179	.16521	.15087	.17768	53
54	.12454	.14225	.13310	.15354	.14194	.16541	.15103	.17790	54
55	.12468	.14243	.13325	.15373	.14208	.16562	.15118	.17811	55
56	.12482	.14262	.13339	.15393	.14223	.16582	.15134	.17832	56
57	.12496	.14280	.13354	.15412	.14238	.16602	.15149	.17854	57
58	.12510	.14299	.13368	.15431	.14253	.16623	.15164	.17875	58
59	.12524	.14317	.13383	.15451	.14268	.16643	.15180	.17896	59
60	.12538	.14335	.13397	.15470	.14283	.16663	.15195	.17918	60

′	32°		33°		34°		35°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	.15195	.17918	.16133	.19236	.17096	.20622	.18085	.22077	0
1	.15211	.17939	.16149	.19259	.17113	.20645	.18101	.22102	1
2	.15226	.17961	.16165	.19281	.17129	.20669	.18118	.22127	2
3	.15241	.17982	.16181	.19304	.17145	.20693	.18135	.22152	3
4	.15257	.18004	.16196	.19327	.17161	.20717	.18152	.22177	4
5	.15272	.18025	.16212	.19349	.17178	.20740	.18168	.22202	5
6	.15288	.18047	.16228	.19372	.17194	.20764	.18185	.22227	6
7	.15303	.18068	.16244	.19394	.17210	.20788	.18202	.22252	7
8	.15319	.18090	.16260	.19417	.17227	.20812	.18218	.22277	8
9	.15334	.18111	.16276	.19440	.17243	.20836	.18235	.22302	9
10	.15350	.18133	.16292	.19463	.17259	.20859	.18252	.22327	10
11	.15365	.18155	.16308	.19485	.17276	.20883	.18269	.22352	11
12	.15381	.18176	.16324	.19508	.17292	.20907	.18286	.22377	12
13	.15396	.18198	.16340	.19531	.17308	.20931	.18302	.22402	13
14	.15412	.18220	.16355	.19554	.17325	.20955	.18319	.22428	14
15	.15427	.18241	.16371	.19576	.17341	.20979	.18336	.22453	15
16	.15443	.18263	.16387	.19599	.17357	.21003	.18353	.22478	16
17	.15458	.18285	.16403	.19622	.17374	.21027	.18369	.22503	17
18	.15474	.18307	.16419	.19645	.17390	.21051	.18386	.22528	18
19	.15489	.18328	.16435	.19668	.17407	.21075	.18403	.22554	19
20	.15505	.18350	.16451	.19691	.17423	.21099	.18420	.22579	20
21	.15520	.18372	.16467	.19713	.17439	.21123	.18437	.22604	21
22	.15536	.18394	.16483	.19736	.17456	.21147	.18454	.22629	22
23	.15552	.18416	.16499	.19759	.17472	.21171	.18470	.22655	23
24	.15567	.18437	.16515	.19782	.17489	.21195	.18487	.22680	24
25	.15583	.18459	.16531	.19805	.17505	.21220	.18504	.22706	25
26	.15598	.18481	.16547	.19828	.17522	.21244	.18521	.22731	26
27	.15614	.18503	.16563	.19851	.17538	.21268	.18538	.22756	27
28	.15630	.18525	.16579	.19874	.17554	.21292	.18555	.22782	28
29	.15645	.18547	.16595	.19897	.17571	.21316	.18572	.22807	29
30	.15661	.18569	.16611	.19920	.17587	.21341	.18588	.22833	30
31	.15676	.18591	.16627	.19944	.17604	.21365	.18605	.22858	31
32	.15692	.18613	.16644	.19967	.17620	.21389	.18622	.22884	32
33	.15708	.18635	.16660	.19990	.17637	.21414	.18639	.22909	33
34	.15723	.18657	.16676	.20013	.17653	.21438	.18656	.22935	34
35	.15739	.18679	.16692	.20036	.17670	.21462	.18673	.22960	35
36	.15755	.18701	.16708	.20059	.17686	.21487	.18690	.22986	36
37	.15770	.18723	.16724	.20083	.17703	.21511	.18707	.23012	37
38	.15786	.18745	.16740	.20106	.17719	.21535	.18724	.23037	38
39	.15802	.18767	.16756	.20129	.17736	.21560	.18741	.23063	39
40	.15818	.18790	.16772	.20152	.17752	.21584	.18758	.23089	40
41	.15833	.18812	.16788	.20176	.17769	.21609	.18775	.23114	41
42	.15849	.18834	.16805	.20199	.17786	.21633	.18792	.23140	42
43	.15865	.18856	.16821	.20222	.17802	.21658	.18809	.23166	43
44	.15880	.18878	.16837	.20246	.17819	.21682	.18826	.23192	44
45	.15896	.18901	.16853	.20269	.17835	.21707	.18843	.23217	45
46	.15912	.18923	.16869	.20292	.17852	.21731	.18860	.23243	46
47	.15928	.18945	.16885	.20316	.17868	.21756	.18877	.23269	47
48	.15943	.18967	.16902	.20339	.17885	.21781	.18894	.23295	48
49	.15959	.18990	.16918	.20363	.17902	.21805	.18911	.23321	49
50	.15975	.19012	.16934	.20386	.17918	.21830	.18928	.23347	50
51	.15991	.19034	.16950	.20410	.17935	.21855	.18945	.23373	51
52	.16006	.19057	.16966	.20433	.17952	.21879	.18962	.23399	52
53	.16022	.19079	.16983	.20457	.17968	.21904	.18979	.23424	53
54	.16038	.19102	.16999	.20480	.17985	.21929	.18996	.23450	54
55	.16054	.19124	.17015	.20504	.18001	.21953	.19013	.23476	55
56	.16070	.19146	.17031	.20527	.18018	.21978	.19030	.23502	56
57	.16085	.19169	.17047	.20551	.18035	.22003	.19047	.23529	57
58	.16101	.19191	.17064	.20575	.18051	.22028	.19064	.23555	58
9	.16117	.19214	.17080	.20598	.18068	.22053	.19081	.23581	
	.16133	.19236	.17096	.20622	.18085	.22077	.19098	.23607	

ABLE XXIX.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

′	36°		37°		38°		39°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	.19098	.23607	.20136	.25214	.21199	.26902	.22285	.28676	0
1	.19115	.23633	.20154	.25241	.21217	.26931	.22304	.28706	1
2	.19133	.23659	.20171	.25269	.21235	.26960	.22322	.28737	2
3	.19150	.23685	.20189	.25296	.21253	.26988	.22340	.28767	3
4	.19167	.23711	.20207	.25324	.21271	.27017	.22359	.28797	4
5	.19184	.23738	.20224	.25351	.21289	.27046	.22377	.28828	5
6	.19201	.23764	.20242	.25379	.21307	.27075	.22395	.28858	6
7	.19218	.23790	.20259	.25406	.21324	.27104	.22414	.28889	7
8	.19235	.23816	.20277	.25434	.21342	.27133	.22432	.28919	8
9	.19252	.23843	.20294	.25462	.21360	.27162	.22450	.28950	9
10	.19270	.23869	.20312	.25489	.21378	.27191	.22469	.28980	10
11	.19287	.23895	.20329	.25517	.21396	.27221	.22487	.29011	11
12	.19304	.23922	.20347	.25545	.21414	.27250	.22506	.29042	12
13	.19321	.23948	.20365	.25572	.21432	.27279	.22524	.29072	13
14	.19338	.23975	.20382	.25600	.21450	.27308	.22542	.29103	14
15	.19356	.24001	.20400	.25628	.21468	.27337	.22561	.29133	15
16	.19373	.24028	.20417	.25656	.21486	.27366	.22579	.29164	16
17	.19390	.24054	.20435	.25683	.21504	.27396	.22598	.29195	17
18	.19407	.24081	.20453	.25711	.21522	.27425	.22616	.29226	18
19	.19424	.24107	.20470	.25739	.21540	.27454	.22634	.29256	19
20	.19442	.24134	.20488	.25767	.21558	.27483	.22653	.29287	20
21	.19459	.24160	.20506	.25795	.21576	.27513	.22671	.29318	21
22	.19476	.24187	.20523	.25823	.21595	.27542	.22690	.29349	22
23	.19493	.24213	.20541	.25851	.21613	.27572	.22708	.29380	23
24	.19511	.24240	.20559	.25879	.21631	.27601	.22727	.29411	24
25	.19528	.24267	.20576	.25907	.21649	.27630	.22745	.29442	25
26	.19545	.24293	.20594	.25935	.21667	.27660	.22764	.29473	26
27	.19562	.24320	.20612	.25963	.21685	.27689	.22782	.29504	27
28	.19580	.24347	.20629	.25991	.21703	.27719	.22801	.29535	28
29	.19597	.24373	.20647	.26019	.21721	.27748	.22819	.29566	29
30	.19614	.24400	.20665	.26047	.21739	.27778	.22838	.29597	30
31	.19632	.24427	.20682	.26075	.21757	.27807	.22856	.29628	31
32	.19649	.24454	.20700	.26104	.21775	.27837	.22875	.29659	32
33	.19666	.24481	.20718	.26132	.21794	.27867	.22893	.29690	33
34	.19684	.24508	.20736	.26160	.21812	.27896	.22912	.29721	34
35	.19701	.24534	.20753	.26188	.21830	.27926	.22930	.29752	35
36	.19718	.24561	.20771	.26216	.21848	.27956	.22949	.29784	36
37	.19736	.24588	.20789	.26245	.21866	.27985	.22967	.29815	37
38	.19753	.24615	.20807	.26273	.21884	.28015	.22986	.29846	38
39	.19770	.24642	.20824	.26301	.21902	.28045	.23004	.29877	39
40	.19788	.24669	.20842	.26330	.21921	.28075	.23023	.29909	40
41	.19805	.24696	.20860	.26358	.21939	.28105	.23041	.29940	41
42	.19822	.24723	.20878	.26387	.21957	.28134	.23060	.29971	42
43	.19840	.24750	.20895	.26415	.21975	.28164	.23079	.30003	43
44	.19857	.24777	.20913	.26443	.21993	.28194	.23097	.30034	44
45	.19875	.24804	.20931	.26472	.22012	.28224	.23116	.30066	45
46	.19892	.24832	.20949	.26500	.22030	.28254	.23134	.30097	46
47	.19909	.24859	.20967	.26529	.22048	.28284	.23153	.30129	47
48	.19927	.24886	.20985	.26557	.22066	.28314	.23172	.30160	48
49	.19944	.24913	.21002	.26586	.22084	.28344	.23190	.30192	49
50	.19962	.24940	.21020	.26615	.22103	.28374	.23209	.30223	50
51	.19979	.24967	.21038	.26643	.22121	.28404	.23228	.30255	51
52	.19997	.24995	.21056	.26672	.22139	.28434	.23246	.30287	52
53	.20014	.25022	.21074	.26701	.22157	.28464	.23265	.30318	53
54	.20032	.25049	.21092	.26729	.22176	.28495	.23283	.30350	54
55	.20049	.25077	.21109	.26758	.22194	.28525	.23302	.30382	55
56	.20066	.25104	.21127	.26787	.22212	.28555	.23321	.30413	56
57	.20084	.25131	.21145	.26815	.22231	.28585	.23339	.30445	57
8	.20101	.25159	.21163	.26844	.22249	.28615	.23358	.30477	58
	.20119	.25186	.21181	.26873	.22267	.28646	.23377	.30509	
	.20136	.25214	.21199	.26902	.22285	.28676	.23396	.3054	

TABLE XXIX.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

′	48°		49°		50°		51°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	.33087	.49448	.34394	.52425	.35721	.55572	.37068	.58902	0
1	.33109	.49496	.34416	.52476	.35744	.55626	.37091	.58959	1
2	.33130	.49544	.34438	.52527	.35766	.55680	.37113	.59016	2
3	.33152	.49593	.34460	.52579	.35788	.55734	.37136	.59073	3
4	.33173	.49641	.34482	.52630	.35810	.55789	.37158	.59130	4
5	.33195	.49690	.34504	.52681	.35833	.55843	.37181	.59188	5
6	.33217	.49738	.34526	.52732	.35855	.55897	.37204	.59245	6
7	.33238	.49787	.34548	.52784	.35877	.55951	.37226	.59302	7
8	.33260	.49835	.34570	.52835	.35900	.56005	.37249	.59360	8
9	.33282	.49884	.34592	.52886	.35922	.56060	.37272	.59418	9
10	.33303	.49933	.34614	.52938	.35944	.56114	.37294	.59475	10
11	.33325	.49981	.34636	.52989	.35967	.56169	.37317	.59533	11
12	.33347	.50030	.34658	.53041	.35989	.56223	.37340	.59590	12
13	.33368	.50079	.34680	.53092	.36011	.56278	.37362	.59648	13
14	.33390	.50128	.34702	.53144	.36034	.56332	.37385	.59706	14
15	.33412	.50177	.34724	.53196	.36056	.56387	.37408	.59764	15
16	.33434	.50226	.34746	.53247	.36078	.56442	.37430	.59822	16
17	.33455	.50275	.34768	.53299	.36101	.56497	.37453	.59880	17
18	.33477	.50324	.34790	.53351	.36123	.56551	.37476	.59938	18
19	.33499	.50373	.34812	.53403	.36146	.56606	.37498	.59996	19
20	.33520	.50422	.34834	.53455	.36168	.56661	.37521	.60054	20
21	.33542	.50471	.34856	.53507	.36190	.56716	.37544	.60112	21
22	.33564	.50521	.34878	.53559	.36213	.56771	.37567	.60171	22
23	.33586	.50570	.34900	.53611	.36235	.56826	.37589	.60229	23
24	.33607	.50619	.34923	.53663	.36258	.56881	.37612	.60287	24
25	.33629	.50669	.34945	.53715	.36280	.56937	.37635	.60346	25
26	.33651	.50718	.34967	.53768	.36302	.56992	.37658	.60404	26
27	.33673	.50767	.34989	.53820	.36325	.57047	.37680	.60463	27
28	.33694	.50817	.35011	.53872	.36347	.57103	.37703	.60521	28
29	.33716	.50866	.35033	.53924	.36370	.57158	.37726	.60580	29
30	.33738	.50916	.35055	.53977	.36392	.57213	.37749	.60639	30
31	.33760	.50966	.35077	.54029	.36415	.57269	.37771	.60698	31
32	.33782	.51015	.35099	.54082	.36437	.57324	.37794	.60756	32
33	.33803	.51065	.35122	.54134	.36460	.57380	.37817	.60815	33
34	.33825	.51115	.35144	.54187	.36482	.57436	.37840	.60874	34
35	.33847	.51165	.35166	.54240	.36504	.57491	.37862	.60933	35
36	.33869	.51215	.35188	.54292	.36527	.57547	.37885	.60992	36
37	.33891	.51265	.35210	.54345	.36549	.57603	.37908	.61051	37
38	.33912	.51314	.35232	.54398	.36572	.57659	.37931	.61111	38
39	.33934	.51364	.35254	.54451	.36594	.57715	.37954	.61170	39
40	.33956	.51415	.35277	.54504	.36617	.57771	.37976	.61229	40
41	.33978	.51465	.35299	.54557	.36639	.57827	.37999	.61288	41
42	.34000	.51515	.35321	.54610	.36662	.57883	.38022	.61348	42
43	.34022	.51565	.35343	.54663	.36684	.57939	.38045	.61407	43
44	.34044	.51615	.35365	.54716	.36707	.57995	.38068	.61467	44
45	.34065	.51665	.35388	.54769	.36729	.58051	.38091	.61526	45
46	.34087	.51716	.35410	.54822	.36752	.58108	.38113	.61586	46
47	.34109	.51766	.35432	.54876	.36775	.58164	.38136	.61646	47
48	.34131	.51817	.35454	.54929	.36797	.58221	.38159	.61705	48
49	.34153	.51867	.35476	.54982	.36820	.58277	.38182	.61765	49
50	.34175	.51918	.35499	.55036	.36842	.58333	.38205	.61825	50
51	.34197	.51968	.35521	.55089	.36865	.58390	.38228	.61885	51
52	.34219	.52019	.35543	.55143	.36887	.58447	.38251	.61945	52
53	.34241	.52069	.35565	.55196	.36910	.58503	.38274	.62005	53
54	.34262	.52120	.35588	.55250	.36932	.58560	.38296	.62065	54
55	.34284	.52171	.35610	.55303	.36955	.58617	.38319	.62125	55
56	.34306	.52222	.35632	.55357	.36978	.58674	.38342	.62185	56
57	.34328	.52273	.35654	.55411	.37000	.58731	.38365	.62246	57
58	.34350	.52323	.35677	.55465	.37023	.58788	.38388	.62306	58
59	.34372	.52374	.35699	.55518	.37045	.58845	.38411	.62366	59
60	.34394	.52425	.35721	.55572	.37068	.58902	.38434	.62427	60

TABLE XXIX.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

′	52°		53°		54°		55°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	.38434	.62427	.39819	.66164	.41221	.70130	.42642	.74345	0
1	.38457	.62487	.39842	.66228	.41245	.70198	.42666	.74417	1
2	.38480	.62548	.39865	.66292	.41269	.70267	.42690	.74490	2
3	.38503	.62609	.39888	.66357	.41292	.70335	.42714	.74562	3
4	.38526	.62669	.39911	.66421	.41316	.70403	.42738	.74635	4
5	.38549	.62730	.39935	.66486	.41339	.70472	.42762	.74708	5
6	.38571	.62791	.39958	.66550	.41363	.70540	.42785	.74781	6
7	.38594	.62852	.39981	.66615	.41386	.70609	.42809	.74854	7
8	.38617	.62913	.40005	.66679	.41410	.70677	.42833	.74927	8
9	.38640	.62974	.40028	.66744	.41433	.70746	.42857	.75000	9
10	.38663	.63035	.40051	.66809	.41457	.70815	.42881	.75073	10
11	.38686	.63096	.40074	.66873	.41481	.70884	.42905	.75146	11
12	.38709	.63157	.40098	.66938	.41504	.70953	.42929	.75219	12
13	.38732	.63218	.40121	.67003	.41528	.71022	.42953	.75293	13
14	.38755	.63279	.40144	.67068	.41551	.71091	.42976	.75366	14
15	.38778	.63341	.40168	.67133	.41575	.71160	.43000	.75440	15
16	.38801	.63402	.40191	.67199	.41599	.71229	.43024	.75513	16
17	.38824	.63464	.40214	.67264	.41622	.71298	.43048	.75587	17
18	.38847	.63525	.40237	.67329	.41646	.71368	.43072	.75661	18
19	.38870	.63587	.40261	.67394	.41670	.71437	.43096	.75734	19
20	.38893	.63648	.40284	.67460	.41693	.71506	.43120	.75808	20
21	.38916	.63710	.40307	.67525	.41717	.71576	.43144	.75882	21
22	.38939	.63772	.40331	.67591	.41740	.71646	.43168	.75956	22
23	.38962	.63834	.40354	.67656	.41764	.71715	.43192	.76031	23
24	.38985	.63895	.40378	.67722	.41788	.71785	.43216	.76105	24
25	.39009	.63957	.40401	.67788	.41811	.71855	.43240	.76179	25
26	.39032	.64019	.40424	.67853	.41835	.71925	.43264	.76253	26
27	.39055	.64081	.40448	.67919	.41859	.71995	.43287	.76328	27
28	.39078	.64144	.40471	.67985	.41882	.72065	.43311	.76402	28
29	.39101	.64206	.40494	.68051	.41906	.72135	.43335	.76477	29
30	.39124	.64268	.40518	.68117	.41930	.72205	.43359	.76552	30
31	.39147	.64330	.40541	.68183	.41953	.72275	.43383	.76626	31
32	.39170	.64393	.40565	.68250	.41977	.72346	.43407	.76701	32
33	.39193	.64455	.40588	.68316	.42001	.72416	.43431	.76776	33
34	.39216	.64518	.40611	.68382	.42024	.72487	.43455	.76851	34
35	.39239	.64580	.40635	.68449	.42048	.72557	.43479	.76926	35
36	.39262	.64643	.40658	.68515	.42072	.72628	.43503	.77001	36
37	.39286	.64705	.40682	.68582	.42096	.72698	.43527	.77077	37
38	.39309	.64768	.40705	.68648	.42119	.72769	.43551	.77152	38
39	.39332	.64831	.40728	.68715	.42143	.72840	.43575	.77227	39
40	.39355	.64894	.40752	.68782	.42167	.72911	.43599	.77303	40
41	.39378	.64957	.40775	.68848	.42191	.72982	.43623	.77378	41
42	.39401	.65020	.40799	.68915	.42214	.73053	.43647	.77454	42
43	.39424	.65083	.40822	.68982	.42238	.73124	.43671	.77530	43
44	.39447	.65146	.40846	.69049	.42262	.73195	.43695	.77606	44
45	.39471	.65209	.40869	.69116	.42285	.73267	.43720	.77681	45
46	.39494	.65272	.40893	.69183	.42309	.73338	.43744	.77757	46
47	.39517	.65336	.40916	.69250	.42333	.73409	.43768	.77833	47
48	.39540	.65399	.40939	.69318	.42357	.73481	.43792	.77910	48
49	.39563	.65462	.40963	.69385	.42381	.73552	.43816	.77986	49
50	.39586	.65526	.40986	.69452	.42404	.73624	.43840	.78062	50
51	.39610	.65589	.41010	.69520	.42428	.73696	.43864	.78138	51
52	.39633	.65653	.41033	.69587	.42452	.73768	.43888	.78215	52
53	.39656	.65717	.41057	.69655	.42476	.73840	.43912	.78291	53
54	.39679	.65780	.41080	.69723	.42499	.73911	.43936	.78368	54
55	.39702	.65844	.41104	.69790	.42523	.73983	.43960	.78445	55
56	.39726	.65908	.41127	.69858	.42547	.74056	.43984	.78521	56
57	.39749	.65972	.41151	.69926	.42571	.74128	.44008	.78598	57
58	.39772	.66036	.41174	.69994	.42595	.74200	.44032	.78675	58
59	.39795	.66100	.41198	.70062	.42619	.74272	.44057	.78752	59
60	.39819	.66164	.41221	.70130	.42642	.74345	.44081	.78829	60

TABLE XXIX.—NATURAL VERSED SINES AND EXTERNAL SECANTS

′	56° Vers.	56° Ex. sec.	57° Vers.	57° Ex. sec.	58° Vers.	58° Ex. sec.	59° Vers.	59° Ex. sec.	′
0	.44081	.78829	.45536	.83608	.47008	.88708	.48496	.94160	0
1	.44105	.78906	.45560	.83690	.47033	.88796	.48521	.94254	1
2	.44129	.78984	.45585	.83773	.47057	.88884	.48546	.94349	2
3	.44153	.79061	.45609	.83855	.47082	.88972	.48571	.94443	3
4	.44177	.79138	.45634	.83938	.47107	.89060	.48596	.94537	4
5	.44201	.79216	.45658	.84020	.47131	.89148	.48621	.94632	5
6	.44225	.79293	.45683	.84103	.47156	.89237	.48646	.94726	6
7	.44250	.79371	.45707	.84186	.47181	.89325	.48671	.94821	7
8	.44274	.79449	.45731	.84269	.47206	.89414	.48696	.94916	8
9	.44298	.79527	.45756	.84352	.47230	.89503	.48721	.95011	9
10	.44322	.79604	.45780	.84435	.47255	.89591	.48746	.95106	10
11	.44346	.79682	.45805	.84518	.47280	.89680	.48771	.95201	11
12	.44370	.79761	.45829	.84601	.47304	.89769	.48796	.95296	12
13	.44395	.79839	.45854	.84685	.47329	.89858	.48821	.95392	13
14	.44419	.79917	.45878	.84768	.47354	.89948	.48846	.95487	14
15	.44443	.79995	.45903	.84852	.47379	.90037	.48871	.95583	15
16	.44467	.80074	.45927	.84935	.47403	.90126	.48896	.95678	16
17	.44491	.80152	.45951	.85019	.47428	.90216	.48921	.95774	17
18	.44516	.80231	.45976	.85103	.47453	.90305	.48946	.95870	18
19	.44540	.80309	.46000	.85187	.47478	.90395	.48971	.95966	19
20	.44564	.80388	.46025	.85271	.47502	.90485	.48996	.96062	20
21	.44588	.80467	.46049	.85355	.47527	.90575	.49021	.96158	21
22	.44612	.80546	.46074	.85439	.47552	.90665	.49046	.96255	22
23	.44637	.80625	.46098	.85523	.47577	.90755	.49071	.96351	23
24	.44661	.80704	.46123	.85608	.47601	.90845	.49096	.96448	24
25	.44685	.80783	.46147	.85692	.47626	.90935	.49121	.96544	25
26	.44709	.80862	.46172	.85777	.47651	.91026	.49146	.96641	26
27	.44734	.80942	.46196	.85861	.47676	.91116	.49171	.96738	27
28	.44758	.81021	.46221	.85946	.47701	.91207	.49196	.96835	28
29	.44782	.81101	.46246	.86031	.47725	.91297	.49221	.96932	29
30	.44806	.81180	.46270	.86116	.47750	.91388	.49246	.97029	30
31	.44831	.81260	.46295	.86201	.47775	.91479	.49271	.97127	31
32	.44855	.81340	.46319	.86286	.47800	.91570	.49296	.97224	32
33	.44879	.81419	.46344	.86371	.47825	.91661	.49321	.97322	33
34	.44903	.81499	.46368	.86457	.47849	.91752	.49346	.97420	34
35	.44928	.81579	.46393	.86542	.47874	.91844	.49372	.97517	35
36	.44952	.81659	.46417	.86627	.47899	.91935	.49397	.97615	36
37	.44976	.81740	.46442	.86713	.47924	.92027	.49422	.97713	37
38	.45001	.81820	.46466	.86799	.47949	.92118	.49447	.97811	38
39	.45025	.81900	.46491	.86885	.47974	.92210	.49472	.97910	39
40	.45049	.81981	.46516	.86990	.47998	.92302	.49497	.98008	40
41	.45073	.82061	.46540	.87056	.48023	.92394	.49522	.98107	41
42	.45098	.82142	.46565	.87142	.48048	.92486	.49547	.98205	42
43	.45122	.82222	.46589	.87229	.48073	.92578	.49572	.98304	43
44	.45146	.82303	.46614	.87315	.48098	.92670	.49597	.98403	44
45	.45171	.82384	.46639	.87401	.48123	.92762	.49623	.98502	45
46	.45195	.82465	.46663	.87488	.48148	.92855	.49648	.98601	46
47	.45219	.82546	.46688	.87574	.48172	.92947	.49673	.98700	47
48	.45244	.82627	.46712	.87661	.48197	.93040	.49698	.98799	48
49	.45268	.82709	.46737	.87748	.48222	.93133	.49723	.98899	49
50	.45292	.82790	.46762	.87834	.48247	.93226	.49748	.98998	50
51	.45317	.82871	.46786	.87921	.48272	.93319	.49773	.99098	51
52	.45341	.82953	.46811	.88008	.48297	.93412	.49798	.99198	52
53	.45365	.83034	.46836	.88095	.48322	.93505	.49824	.99298	53
54	.45390	.83116	.46860	.88183	.48347	.93598	.49849	.99398	54
55	.45414	.83198	.46885	.88270	.48372	.93692	.49874	.99498	55
56	.45439	.83280	.46909	.88357	.48396	.93785	.49899	.99598	56
57	.45463	.83362	.46934	.88445	.48421	.93879	.49924	.99698	57
58	.45487	.83444	.46959	.88532	.48446	.93973	.49950	.99799	58
59	.45512	.83526	.46983	.88620	.48471	.94066	.49975	.99899	59
60	.45536	.83608	.47008	.88708	.48496	.94160	.50000	1.00000	60

TABLE XXIX.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

′	60°		61°		62°		63°		′
	Vers.	Ex se	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	.50000	1.00000	.51519	1.06267	.53053	1.13005	.54601	1.20269	0
1	.50025	1.00101	.51544	1.06375	.53079	1.13122	.54627	1.20395	1
2	.50050	1.00202	.51570	1.06483	.53104	1.13239	.54653	1.20521	2
3	.50076	1.00303	.51595	1.06592	.53130	1.13356	.54679	1.20647	3
4	.50101	1.00404	.51621	1.06701	.53156	1.13473	.54705	1.20773	4
5	.50126	1.00505	.51646	1.06809	.53181	1.13590	.54731	1.20900	5
6	.50151	1.00607	.51672	1.06918	.53207	1.13707	.54757	1.21026	6
7	.50176	1.00708	.51697	1.07027	.53233	1.13825	.54782	1.21153	7
8	.50202	1.00810	.51723	1.07137	.53258	1.13942	.54808	1.21280	8
9	.50227	1.00912	.51748	1.07246	.53284	1.14060	.54834	1.21407	9
10	.50252	1.01014	.51774	1.07356	.53310	1.14178	.54860	1.21535	10
11	.50277	1.01116	.51799	1.07465	.53336	1.14296	.54886	1.21662	11
12	.50303	1.01218	.51825	1.07575	.53361	1.14414	.54912	1.21790	12
13	.50328	1.01320	.51850	1.07685	.53387	1.14533	.54938	1.21918	13
14	.50353	1.01422	.51876	1.07795	.53413	1.14651	.54964	1.22045	14
15	.50378	1.01525	.51901	1.07905	.53439	1.14770	.54990	1.22174	15
16	.50404	1.01628	.51927	1.08015	.53464	1.14889	.55016	1.22302	16
17	.50429	1.01730	.51952	1.08126	.53490	1.15008	.55042	1.22430	17
18	.50454	1.01833	.51978	1.08236	.53516	1.15127	.55068	1.22559	18
19	.50479	1.01936	.52003	1.08347	.53542	1.15246	.55094	1.22688	19
20	.50505	1.02039	.52029	1.08458	.53567	1.15366	.55120	1.22817	20
21	.50530	1.02143	.52054	1.08569	.53593	1.15485	.55146	1.22946	21
22	.50555	1.02246	.52080	1.08680	.53619	1.15605	.55172	1.23075	22
23	.50581	1.02349	.52105	1.08791	.53645	1.15725	.55198	1.23205	23
24	.50606	1.02453	.52131	1.08903	.53670	1.15845	.55224	1.23334	24
25	.50631	1.02557	.52156	1.09014	.53696	1.15965	.55250	1.23464	25
26	.50656	1.02661	.52182	1.09126	.53722	1.16085	.55276	1.23594	26
27	.50682	1.02765	.52207	1.09238	.53748	1.16206	.55302	1.23724	27
28	.50707	1.02869	.52233	1.09350	.53774	1.16326	.55328	1.23855	28
29	.50732	1.02973	.52259	1.09462	.53799	1.16447	.55354	1.23985	29
30	.50758	1.03077	.52284	1.09574	.53825	1.16568	.55380	1.24116	30
31	.50783	1.03182	.52310	1.09686	.53851	1.16689	.55406	1.24247	31
32	.50808	1.03286	.52335	1.09799	.53877	1.16810	.55432	1.24378	32
33	.50834	1.03391	.52361	1.09911	.53903	1.16932	.55458	1.24509	33
34	.50859	1.03496	.52386	1.10024	.53928	1.17053	.55484	1.24640	34
35	.50884	1.03601	.52412	1.10137	.53954	1.17175	.55510	1.24772	35
36	.50910	1.03706	.52438	1.10250	.53980	1.17297	.55536	1.24903	36
37	.50935	1.03811	.52463	1.10363	.54006	1.17419	.55563	1.25035	37
38	.50960	1.03916	.52489	1.10477	.54032	1.17541	.55589	1 25167	38
39	.50986	1.04022	.52514	1.10590	.54058	1.17663	.55615	1.25300	39
40	.51011	1.04128	.52540	1.10704	.54083	1.17786	.55641	1.25432	40
41	.51036	1.04233	.52566	1.10817	.54109	1.17909	.55667	1.25565	41
42	.51062	1.04339	.52591	1.10931	.54135	1.18031	.55693	1.25697	42
43	.51087	1.04445	.52617	1.11045	.54161	1.18154	.55719	1.25830	43
44	.51113	1.04551	.52642	1.11159	.54187	1.18277	.55745	1.25963	44
45	.51138	1.04658	.52668	1.11274	.54213	1.18401	.55771	1.26097	45
46	.51163	1.04764	.52694	1.11388	.54238	1.18524	.55797	1.26230	46
47	.51189	1.04870	.52719	1.11503	.54264	1.18648	.55823	1.26364	47
48	.51214	1.04977	.52745	1.11617	.54290	1.18772	.55849	1.26498	48
49	.51239	1.05084	.52771	1.11732	.54316	1.18895	.55876	1.26632	49
50	.51265	1.05191	.52796	1.11847	.54342	1.19019	.55902	1.26766	50
51	.51290	1.05298	.52822	1.11963	.54368	1.19144	.55928	1.26900	51
52	.51316	1.05405	.52848	1.12078	.54394	1.19268	.55954	1.27035	52
53	.51341	1.05512	.52873	1.12193	.54420	1.19393	.55980	1.27169	53
54	.51366	1.05619	.52899	1.12309	.54446	1.19517	.56006	1.27304	54
55	.51392	1.05727	.52924	1.12425	.54471	1.19642	.56032	1.27439	55
56	.51417	1.05835	.52950	1.12540	.54497	1.19767	.56058	1.27574	56
57	.51443	*1.05942*	.52976	1.12657	.54523	1.19892	.56084	1.27710	57
58	*.51468*	*1.06050*	.53001	1.12773	.54549	1.20018	.56111	1.27845	
59	*.51494*	*1.06158*	.53027	1.12889	.54575	1.20143	.56137	1.27981	
60	*.51519*	*1.06267*	.53053	1.13005	.54601	1.20269	.56163	1.28117	

TABLE XXIX.—NATURAL VERSED SINES AND EXTERNAL SEC

′	64°		65°		66°		67°	
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex.
0	.56163	1.28117	.57738	1.36620	.59326	1.45859	.60927	1.55
1	.56189	1.28253	.57765	1.36768	.59353	1.46020	.60954	1.56
2	.56215	1.28390	.57791	1.36916	.59379	1.46181	.60980	1.56
3	.56241	1.28526	.57817	1.37064	.59406	1.46342	.61007	1.56
4	.56267	1.28663	.57844	1.37212	.59433	1.46504	.61034	1.56
5	.56294	1.28800	.57870	1.37361	.59459	1.46665	.61061	1.56
6	.56320	1.28937	.57896	1.37509	.59486	1.46827	.61088	1.56
7	.56346	1.29074	.57923	1.37658	.59512	1.46989	.61114	1.57
8	.56372	1.29211	.57949	1.37808	.59539	1.47152	.61141	1.57
9	.56398	1.29349	.57976	1.37957	.59566	1.47314	.61168	1.57
10	.56425	1.29487	.58002	1.38107	.59592	1.47477	.61195	1.57
11	.56451	1.29625	.58028	1.38256	.59619	1.47640	.61222	1.5
12	.56477	1.29763	.58055	1.38406	.59645	1.47804	.61248	1.5
13	.56503	1.29901	.58081	1.38556	.59672	1.47967	.61275	1.5
14	.56529	1.30040	.58108	1.38707	.59699	1.48131	.61302	1.5
15	.56555	1.30179	.58134	1.38857	.59725	1.48295	.61329	1.5
16	.56582	1.30318	.58160	1.39008	.59752	1.48459	.61356	1.5
17	.56608	1.30457	.58187	1.39159	.59779	1.48624	.61383	1.5
18	.56634	1.30596	.58213	1.39311	.59805	1.48789	.61409	1.5
19	.56660	1.30735	.58240	1.39462	.59832	1.48954	.61436	1.5
20	.56687	1.30875	.58266	1.39614	.59859	1.49119	.61463	1.5
21	.56713	1.31015	.58293	1.39766	.59885	1.49284	.61490	1.
22	.56739	1.31155	.58319	1.39918	.59912	1.49450	.61517	1.
23	.56765	1.31295	.58345	1.40070	.59938	1.49616	.61544	1.
24	.56791	1.31436	.58372	1.40222	.59965	1.49782	.61570	1.
25	.56818	1.31576	.58398	1.40375	.59992	1.49948	.61597	1.
26	.56844	1.31717	.58425	1.40528	.60018	1.50115	.61624	1.
27	.56870	1.31858	.58451	1.40681	.60045	1.50282	.61651	1.
28	.56896	1.31999	.58478	1.40835	.60072	1.50449	.61678	1.
29	.56923	1.32140	.58504	1.40988	.60098	1.50617	.61705	1.
30	.56949	1.32282	.58531	1.41142	.60125	1.50784	.61732	1.
31	.56975	1.32424	.58557	1.41296	.60152	1.50952	.61759	1.
32	.57001	1.32566	.58584	1.41450	.60178	1.51120	.61785	1.
33	.57028	1.32708	.58610	1.41605	.60205	1.51289	.61812	1.
34	.57054	1.32850	.58637	1.41760	.60232	1.51457	.61839	1.
35	.57080	1.32993	.58663	1.41914	.60259	1.51626	.61866	1.
36	.57106	1.33135	.58690	1.42070	.60285	1.51795	.61893	1.
37	.57133	1.33278	.58716	1.42225	.60312	1.51965	.61920	1.
38	.57159	1.33422	.58743	1.42380	.60339	1.52134	.61947	1.
39	.57185	1.33565	.58769	1.42536	.60365	1.52304	.61974	1.
40	.57212	1.33708	.58796	1.42692	.60392	1.52474	.62001	1.
41	.57238	1.33852	.58822	1.42848	.60419	1.52645	.62027	1
42	.57264	1.33996	.58849	1.43005	.60445	1.52815	.62054	1
43	.57291	1.34140	.58875	1.43162	.60472	1.52986	.62081	1
44	.57317	1.34284	.58902	1.43318	.60499	1.53157	.62108	1
45	.57343	1.34429	.58928	1.43476	.60526	1.53329	.62135	1
46	.57369	1.34573	.58955	1.43633	.60552	1.53500	.62162	1
47	.57396	1.34718	.58981	1.43790	.60579	1.53672	.62189	1
48	.57422	1.34863	.59008	1.43948	.60606	1.53845	.62216	1
49	.57448	1.35009	.59034	1.44106	.60633	1.54017	.62243	1
50	.57475	1.35154	.59061	1.44264	.60659	1.54190	.62270	1
51	.57501	1.35300	.59087	1.44423	.60686	1.54363	.62297	1
52	.57527	1.35446	.59114	1.44582	.60713	1.54536	.62324	1
53	.57554	1.35592	.59140	1.44741	.60740	1.54709	.62351	1
54	.57580	1.35738	.59167	1.44900	.60766	1.54883	.62378	1
55	.57606	1.35885	.59194	1.45059	.60793	1.55057	.62405	1
56	.57633	1.36031	.59220	1.45219	.60820	1.55231	.62431	1
57	.57659	1.36178	.59247	1.45378	.60847	1.55405	.62458	
58	.57685	1.36325	.59273	1.45539	.60873	1.55580	.62485	
59	.57712	1.36473	.59300	1.45699	.60900	1.55755	.62512	
60	.57738	1.36620	.59326	1.45859	.60927	1.55930	.62539	

TABLE XXIX.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

′	68°		69°		70°		71°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	.62539	1.66947	.64163	1.79043	.65798	1.92380	.67443	2.07155	0
1	.62566	1.67139	.64190	1.79254	.65825	1.92614	.67471	2.07415	1
2	.62593	1.67332	.64218	1.79466	.65853	1.92849	.67498	2.07675	2
3	.62620	1.67525	.64245	1.79679	.65880	1.93083	.67526	2.07936	3
4	.62647	1.67718	.64272	1.79891	.65907	1.93318	.67553	2.08197	4
5	.62674	1.67911	.64299	1.80104	.65935	1.93554	.67 81	2.08459	5
6	.62701	1.68105	.64326	1.80318	.65962	1.93790	.67608	2.08721	6
7	.62728	1.68299	.64353	1.80531	.65989	1.94026	.67636	2.08983	7
8	.62755	1.68494	.64381	1.80746	.66017	1.94263	.67663	2.09246	8
9	.62782	1.68689	.64408	1.80960	.66044	1.94500	.67691	2.09510	9
10	.62809	1.68884	.64435	1.81175	.66071	1.94737	.67718	2.09774	10
11	.62836	1.69079	.64462	1.81390	.66099	1.94975	.67746	2.10038	11
12	.62863	1.69275	.64489	1.81605	.66126	1.95213	.67773	2.10303	12
13	.62890	1.69471	.64517	1.81821	.66154	1.95452	.67801	2.10568	13
14	.62917	1.69667	.64544	1.82037	.66181	1.95691	.67829	2.10834	14
15	.62944	1.69864	.64571	1.82254	.66208	1.95931	.67856	2.11101	15
16	.62971	1.70061	.64598	1.82471	.66236	1.96171	.67884	2.11367	16
17	.62998	1.70258	.64625	1.82688	.66263	1.96411	.67911	2.11635	17
18	.63025	1.70455	.64653	1.82906	.66290	1.96652	.67939	2.11903	18
19	.63052	1.70653	.64680	1.83124	.66318	1.96893	.67966	2.12171	19
20	.63079	1.70851	.64707	1.83342	.66345	1.97135	.67994	2.12440	20
21	.63106	1.71050	.64734	1.83561	.66373	1.97377	.68021	2.12709	21
22	.63133	1.71249	.64761	1.83780	.66400	1.97619	.68049	2.12979	22
23	.63161	1.71448	.64789	1.83999	.66427	1.97862	.68077	2.13249	23
24	.63188	1.71647	.64816	1.84219	.66455	1.98106	.68104	2.13520	24
25	.63215	1.71847	.64843	1.84439	.66482	1.98349	.68132	2.13791	25
26	.63242	1.72047	.64870	1.84659	.66510	1.98594	.68159	2.14063	26
27	.63269	1.72247	.64898	1.84880	.66537	1.98838	.68187	2.14335	27
28	.63296	1.72448	.64925	1.85102	.66564	1.99083	.68214	2.14608	28
29	.63323	1.72649	.64952	1.85323	.66592	1.99329	.68242	2.14881	29
30	.63350	1.72850	.64979	1.85545	.66619	1.99574	.68270	2.15155	30
31	.63377	1.73052	.65007	1.85767	.66647	1.99821	.68297	2.15429	31
32	.63404	1.73254	.65034	1.85990	.66674	2.00067	.68325	2.15704	32
33	.63431	1.73456	.65061	1.86213	.66702	2.00315	.68352	2.15979	33
34	.63458	1.73659	.65088	1.86437	.66729	2.00562	.68380	2.16255	34
35	.63485	1.73862	.65116	1.86661	.66756	2.00810	.68408	2.16531	35
36	.63512	1.74065	.65143	1.86885	.66784	2.01059	.68435	2.16808	36
37	.63539	1.74269	.65170	1.87109	.66811	2.01308	.68463	2.17085	37
38	.63566	1.74473	.65197	1.87334	.66839	2.01557	.68490	2.17363	38
39	.63594	1.74677	.65225	1.87560	.66866	2.01807	.68518	2.17641	39
40	.63621	1.74881	.65252	1.87785	.66894	2.02057	.68546	2.17920	40
41	.63648	1.75086	.65279	1.88011	.66921	2.02308	.68573	2.18199	41
42	.63675	1.75292	.65306	1.88238	.66949	2.02559	.68601	2.18479	42
43	.63702	1.75497	.65334	1.88465	.66976	2.02810	.68628	2.18759	43
44	.63729	1.75703	.65361	1.88692	.67003	2.03062	.68656	2.19040	44
45	.63756	1.75909	.65388	1.88920	.67031	2.03315	.68684	2.19322	45
46	.63783	1.76116	.65416	1.89148	.67058	2.03568	.68711	2.19604	46
47	.63810	1.76323	.65443	1.89376	.67086	2.03821	.68739	2.19886	47
48	.63838	1.76530	.65470	1.89605	.67113	2.04075	.68767	2.20169	48
49	.63865	1.76737	.65497	1.89834	.67141	2.04329	.68794	2.20453	49
50	.63892	1.76945	.65525	1.90063	.67168	2.04584	.68822	2.20737	50
51	.63919	1.77154	.65552	1.90293	.67196	2.04839	.68849	2.21021	51
52	.63946	1.77362	.65579	1.90524	.67223	2.05094	.68877	2.21306	52
53	.63973	1.77571	.65607	1.90754	.67251	2.05350	.68905	2.21592	53
54	.64000	1.77780	.65634	1.90986	.67278	2.05607	.68932	2.21878	54
55	.64027	1.77990	.65661	1.91217	.67306	2.05864	.68960	2.22165	55
56	.64055	1.78200	.65689	1.91449	.67333	2.06121	.68988	2.22452	56
57	.64082	1.78410	.65716	1.91681	.67361	2.06379	.69015	2.22740	
58	.64109	1.78621	.65743	1.91914	.67388	2.06637	.69043	2.23028	
59	.64136	1.78832	.65771	1.92147	.67416	2.06896	.69071	2.2331	
60	.64163	1.79043	.65798	1.92380	.67443	2.07155	.69098	2.236	

TABLE XXIX.—NATURAL VERSED SINES AND EXTERNAL SECAN

′	72°		73°		74°		75°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	.69098	2.23607	.70763	2.42030	.72436	2.62796	.74118	2.86370	0
1	.69126	2.23897	.70791	2.42356	.72464	2.63164	.74146	2.86790	1
2	.69154	2.24187	.70818	2.42683	.72492	2.63533	.74174	2.87211	2
3	.69181	2.24478	.70846	2.43010	.72520	2.63903	.74202	2.87633	3
4	.69209	2.24770	.70874	2.43337	.72548	2.64274	.74231	2.88056	4
5	.69237	2.25062	.70902	2.43666	.72576	2.64645	.74259	2.88479	5
6	.69264	2.25355	.70930	2.43995	.72604	2.65018	.74287	2.88904	6
7	.69292	2.25648	.70958	2.44324	.72632	2.65391	.74315	2.89330	7
8	.69320	2.25942	.70985	2.44655	.72660	2.65765	.74343	2.89756	8
9	.69347	2.26237	.71013	2.44986	.72688	2.66140	.74371	2.90184	9
10	.69375	2.26531	.71041	2.45317	.72716	2.66515	.74399	2.90613	10
11	.69403	2.26827	.71069	2.45650	.72744	2.66892	.74427	2.91042	11
12	.69430	2.27123	.71097	2.45983	.72772	2.67269	.74455	2.91473	12
13	.69458	2.27420	.71125	2.46316	.72800	2.67647	.74484	2.91904	13
14	.69486	2.27717	.71153	2.46651	.72828	2.68025	.74512	2.92337	14
15	.69514	2.28015	.71180	2.46986	.72856	2.68405	.74540	2.92770	15
16	.69541	2.28313	.71208	2.47321	.72884	2.68785	.74568	2.93204	16
17	.69569	2.28612	.71236	2.47658	.72912	2.69167	.74596	2.93640	17
18	.69597	2.28912	.71264	2.47995	.72940	2.69549	.74624	2.94076	18
19	.69624	2.29212	.71292	2.48333	.72968	2.69931	.74652	2.94514	19
20	.69652	2.29512	.71320	2.48671	.72996	2.70315	.74680	2.94952	20
21	.69680	2.29814	.71348	2.49010	.73024	2.70700	.74709	2.95392	21
22	.69708	2.30115	.71375	2.49350	.73052	2.71085	.74737	2.95832	22
23	.69735	2.30418	.71403	2.49691	.73080	2.71471	.74765	2.96274	23
24	.69763	2.30721	.71431	2.50032	.73108	2.71858	.74793	2.96716	24
25	.69791	2.31024	.71459	2.50374	.73136	2.72246	.74821	2.97160	25
26	.69818	2.31328	.71487	2.50716	.73164	2.72635	.74849	2.97604	26
27	.69846	2.31633	.71515	2.51060	.73192	2.73024	.74878	2.98050	27
28	.69874	2.31939	.71543	2.51404	.73220	2.73414	.74906	2.98497	28
29	.69902	2.32244	.71571	2.51748	.73248	2.73806	.74934	2.98944	29
30	.69929	2.32551	.71598	2.52094	.73276	2.74198	.74962	2.99393	30
31	.69957	2.32858	.71626	2.52440	.73304	2.74591	.74990	2.99843	31
32	.69985	2.33166	.71654	2.52787	.73332	2.74984	.75018	3.00293	32
33	.70013	2.33474	.71682	2.53134	.73360	2.75379	.75047	3.00745	33
34	.70040	2.33783	.71710	2.53482	.73388	2.75775	.75075	3.01198	34
35	.70068	2.34092	.71738	2.53831	.73416	2.76171	.75103	3.01652	35
36	.70096	2.34403	.71766	2.54181	.73444	2.76568	.75131	3.02107	36
37	.70124	2.34713	.71794	2.54531	.73472	2.76966	.75159	3.02563	37
38	.70151	2.35025	.71822	2.54883	.73500	2.77365	.75187	3.03020	38
39	.70179	2.35336	.71850	2.55235	.73529	2.77765	.75216	3.03479	39
40	.70207	2.35649	.71877	2.55587	.73557	2.78166	.75244	3.03938	40
41	.70235	2.35962	.71905	2.55940	.73585	2.78568	.75272	3.04398	41
42	.70263	2.36276	.71933	2.56294	.73613	2.78970	.75300	3.04860	42
43	.70290	2.36590	.71961	2.56649	.73641	2.79374	.75328	3.05322	43
44	.70318	2.36905	.71989	2.57005	.73669	2.79778	.75356	3.05786	44
45	.70346	2.37221	.72017	2.57361	.73697	2.80183	.75385	3.06251	45
46	.70374	2.37537	.72045	2.57718	.73725	2.80589	.75413	3.06717	46
47	.70401	2.37854	.72073	2.58076	.73753	2.80996	.75441	3.07184	47
48	.70429	2.38171	.72101	2.58434	.73781	2.81404	.75469	3.07652	48
49	.70457	2.38489	.72129	2.58794	.73809	2.81813	.75497	3.08121	49
50	.70485	2.38808	.72157	2.59154	.73837	2.82223	.75526	3.08591	50
51	.70513	2.39128	.72185	2.59514	.73865	2.82633	.75554	3.09063	51
52	.70540	2.39448	.72213	2.59876	.73893	2.83045	.75582	3.09535	52
53	.70568	2.39768	.72241	2.60238	.73921	2.83457	.75610	3.10009	53
54	.70596	2.40089	.72269	2.60601	.73950	2.83871	.75639	3.10484	54
55	.70624	2.40411	.72296	2.60965	.73978	2.84285	.75667	3.10960	55
56	.70652	2.40734	.72324	2.61330	.74006	2.84700	.75695	3.11437	56
57	.70679	2.41057	.72352	2.61695	.74034	2.85116	.75723	3.11915	
58	.70707	2.41381	.72380	2.62061	.74062	2.85533	.75751	3.12394	
59	.70735	2.41705	.72408	2.62428	.74090	2.85951	.75780	3.12875	
60	.70763	2.42030	.72436	2.62796	.74118	2.86370	.75808	3.13357	

ABLE XXIX.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

′	76°		77°		78°		79°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	.75808	3.13357	.77505	3.44541	.79209	3.80973	.80919	4.24084	0
1	.75836	3.13839	.77533	3.45102	.79237	3.81633	.80948	4.24870	1
2	.75864	3.14323	.77562	3.45664	.79266	3.82294	.80976	4.25658	2
3	.75892	3.14809	.77500	3.46228	.79294	3.82956	.81005	4.26448	3
4	.75921	3.15295	.77618	3.46793	.79323	3.83621	.81033	4.27241	4
5	.75949	3.15782	.77647	3.47360	.79351	3.84288	.81062	4.28036	5
6	.75977	3.16271	.77675	3.47928	.79380	3.84956	.81090	4.28833	6
7	.76005	3.16761	.77703	3.48498	.79408	3.85627	.81119	4.29634	7
8	.76034	3.17252	.77732	3.49069	.79437	3.86299	.81148	4.30436	8
9	.76062	3.17744	.77760	3.49642	.79465	3.86973	.81176	4.31241	9
10	.76090	3.18238	.77788	3.50216	.79493	3.87649	.81205	4.32049	10
11	.76118	3.18733	.77817	3.50791	.79522	3.88327	.81233	4.32859	11
12	.76147	3.19228	.77845	3.51368	.79550	3.89007	.81262	4.33671	12
13	.76175	3.19725	.77874	3.51947	.79579	3.89689	.81290	4.34486	13
14	.76203	3.20224	.77902	3.52527	.79607	3.90373	.81319	4.35304	14
15	.76231	3.20723	.77930	3.53109	.79636	3.91058	.81348	4.36124	15
16	.76260	3.21224	.77959	3.53692	.79664	3.91746	.81376	4.36947	16
17	.76288	3.21726	.77987	3.54277	.79693	3.92436	.81405	4.37772	17
18	.76316	3.22229	.78015	3.54863	.79721	3.93128	.81433	4.38600	18
19	.76344	3.22734	.78044	3.55451	.79750	3.93821	.81462	4.39430	19
20	.76373	3.23239	.78072	3.56041	.79778	3.94517	.81491	4.40263	20
21	.76401	3.23746	.78101	3.56632	.79807	3.95215	.81519	4.41099	21
22	.76429	3.24255	.78129	3.57224	.79835	3.95914	.81548	4.41937	22
23	.76458	3.24764	.78157	3.57819	.79864	3.96616	.81576	4.42778	23
24	.76486	3.25275	.78186	3.58414	.79892	3.97320	.81605	4.43622	24
25	.76514	3.25787	.78214	3.59012	.79921	3.98025	.81633	4.44468	25
26	.76542	3.26300	.78242	3.59611	.79949	3.98733	.81662	4.45317	26
27	.76571	3.26814	.78271	3.60211	.79978	3.99443	.81691	4.46169	27
28	.76599	3.27330	.78299	3.60813	.80006	4.00155	.81719	4.47023	28
29	.76627	3.27847	.78328	3.61417	.80035	4.00869	.81748	4.47881	29
30	.76655	3.28366	.78356	3.62023	.80063	4.01585	.81776	4.48740	30
31	.76684	3.28885	.78384	3.62630	.80092	4.02303	.81805	4.49603	31
32	.76712	3.29406	.78413	3.63238	.80120	4.03024	.81834	4.50468	32
33	.76740	3.29929	.78441	3.63849	.80149	4.03746	.81862	4.51337	33
34	.76769	3.30452	.78470	3.64461	.80177	4.04471	.81891	4.52208	34
35	.76797	3.30977	.78498	3.65074	.80206	4.05197	.81919	4.53081	35
36	.76825	3.31503	.78526	3.65690	.80234	4.05926	.81948	4.53958	36
37	.76854	3.32031	.78555	3.66307	.80263	4.06657	.81977	4.54837	37
38	.76882	3.32560	.78583	3.66925	.80291	4.07390	.82005	4.55720	38
39	.76910	3.33090	.78612	3.67545	.80320	4.08125	.82034	4.56605	39
40	.76938	3.33622	.78640	3.68167	.80348	4.08863	.82063	4.57493	40
41	.76967	3.34154	.78669	3.68791	.80377	4.09602	.82091	4.58383	41
42	.76995	3.34689	.78697	3.69417	.80405	4.10344	.82120	4.59277	42
43	.77023	3.35224	.78725	3.70044	.80434	4.11088	.82148	4.60174	43
44	.77052	3.35761	.78754	3.70673	.80462	4.11835	.82177	4.61073	44
45	.77080	3.36299	.78782	3.71303	.80491	4.12583	.82206	4.61976	45
46	.77108	3.36839	.78811	3.71935	.80520	4.13334	.82234	4.62881	46
47	.77137	3.37380	.78839	3.72569	.80548	4.14087	.82263	4.63790	47
48	.77165	3.37923	.78868	3.73205	.80577	4.14842	.82202	4.64701	48
49	.77193	3.38466	.78896	3.73843	.80605	4.15599	.82320	4.65616	49
50	.77222	3.39012	.78024	3.74482	.80634	4.16359	.82349	4.66533	50
51	.77250	3.39558	.78953	3.75123	.80662	4.17121	.82377	4.67454	51
52	.77278	3.40106	.78981	3.75766	.80691	4.17886	.82406	4.68377	52
53	.77307	3.40656	.79010	3.76411	.80719	4.18652	.82435	4.69304	53
54	.77335	3.41206	.79038	3.77057	.80748	4.19421	.82463	4.70234	54
55	.77363	3.41759	.79067	3.77705	.80776	4.20193	.82492	4.71166	55
56	.77392	3.42312	.79095	3.78355	.80805	4.20966	.82521	4.72102	56
7	*.77420*	*3.42867*	.79123	3.79007	.80833	4.21742	.82549	4.73041	57
	.77448	*3.43424*	.79152	3.79661	.80862	4.22521	.82578	4.73983	
	.77477	*3.43982*	.79180	3.80316	.80891	4.23301	.82607	4.74929	
	.77505	*3.44541*	.79209	3.80973	.80919	4.24084	.82635	4.75877	

′	80°		81°		82°		83°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	.82635	4.75877	.84357	5.39245	.86083	6.18530	.87813	7.20551	0
1	.82664	4.76829	.84385	5.40422	.86112	6.20020	.87842	7.22500	1
2	.82692	4.77784	.84414	5.41602	.86140	6.21517	.87871	7.24457	2
3	.82721	4.78742	.84443	5.42787	.86169	6.23019	.87900	7.26425	3
4	.82750	4.79703	.84471	5.43977	.86198	6.24529	.87929	7.28402	4
5	.82778	4.80667	.84500	5.45171	.86227	6.26044	.87957	7.30388	5
6	.82807	4.81635	.84529	5.46369	.86256	6.27566	.87986	7.32384	6
7	.82836	4.82606	.84558	5.47572	.86284	6.29095	.88015	7.34390	7
8	.82864	4.83581	.84586	5.48779	.86313	6.30630	.88044	7.36405	8
9	.82893	4.84558	.84615	5.49991	.86342	6.32171	.88073	7.38431	9
10	.82922	4.85539	.84644	5.51208	.86371	6.33719	.88102	7.40466	10
11	.82950	4.86524	.84673	5.52429	.86400	6.35274	.88131	7.42511	11
12	.82979	4.87511	.84701	5.53655	.86428	6.36835	.88160	7.44566	12
13	.83008	4.88502	.84730	5.54886	.86457	6.38403	.88188	7.46632	13
14	.83036	4.89497	.84759	5.56121	.86486	6.39978	.88217	7.48707	14
15	.83065	4.90495	.84788	5.57361	.86515	6.41560	.88246	7.50793	15
16	.83094	4.91496	.84816	5.58606	.86544	6.43148	.88275	7.52889	16
17	.83122	4.92501	.84845	5.59855	.86573	6.44743	.88304	7.54996	17
18	.83151	4.93509	.84874	5.61110	.86601	6.46346	.88333	7.57113	18
19	.83180	4.94521	.84903	5.62369	.86630	6.47955	.88362	7.59241	19
20	.83208	4.95536	.84931	5.63633	.86659	6.49571	.88391	7.61379	20
21	.83237	4.96555	.84960	5.64902	.86688	6.51194	.88420	7.63528	21
22	.83266	4.97577	.84989	5.66176	.86717	6.52825	.88448	7.65688	22
23	.83294	4.98603	.85018	5.67454	.86746	6.54462	.88477	7.67859	23
24	.83323	4.99633	.85046	5.68738	.86774	6.56107	.88506	7.70041	24
25	.83352	5.00666	.85075	5.70027	.86803	6.57759	.88535	7.72234	25
26	.83380	5.01703	.85104	5.71321	.86832	6.59418	.88564	7.74438	26
27	.83409	5.02743	.85133	5.72620	.86861	6.61085	.88593	7.76653	27
28	.83438	5.03787	.85162	5.73924	.86890	6.62759	.88622	7.78880	28
29	.83467	5.04834	.85190	5.75233	.86919	6.64441	.88651	7.81118	29
30	.83495	5.05886	.85219	5.76547	.86947	6.66130	.88680	7.83367	30
31	.83524	5.06941	.85248	5.77866	.86976	6.67826	.88709	7.85628	31
32	.83553	5.08000	.85277	5.79191	.87005	6.69530	.88737	7.87901	32
33	.83581	5.09062	.85305	5.80521	.87034	6.71242	.88766	7.90186	33
34	.83610	5.10129	.85334	5.81856	.87063	6.72962	.88795	7.92482	34
35	.83639	5.11199	.85363	5.83196	.87092	6.74689	.88824	7.94791	35
36	.83667	5.12273	.85392	5.84542	.87120	6.76424	.88853	7.97111	36
37	.83696	5.13350	.85420	5.85893	.87149	6.78167	.88882	7.99444	37
38	.83725	5.14432	.85449	5.87250	.87178	6.79918	.88911	8.01788	38
39	.83754	5.15517	.85478	5.88612	.87207	6.81677	.88940	8.04146	39
40	.83782	5.16607	.85507	5.89979	.87236	6.83443	.88969	8.06515	40
41	.83811	5.17700	.85536	5.91352	.87265	6.85218	.88998	8.08897	41
42	.83840	5.18797	.85564	5.92731	.87294	6.87001	.89027	8.11292	42
43	.83868	5.19898	.85593	5.94115	.87322	6.88792	.89055	8.13699	43
44	.83897	5.21004	.85622	5.95505	.87351	6.90592	.89084	8.16120	44
45	.83926	5.22113	.85651	5.96900	.87380	6.92400	.89113	8.18553	45
46	.83954	5.23226	.85680	5.98301	.87409	6.94216	.89142	8.20999	46
47	.83983	5.24343	.85708	5.99708	.87438	6.96040	.89171	8.23459	47
48	.84012	5.25464	.85737	6.01120	.87467	6.97873	.89200	8.25931	48
49	.84041	5.26590	.85766	6.02538	.87496	6.99714	.89229	8.28417	49
50	.84069	5.27719	.85795	6.03962	.87524	7.01565	.89258	8.30917	50
51	.84098	5.28853	.85823	6.05392	.87553	7.03423	.89287	8.33430	51
52	.84127	5.29991	.85852	6.06828	.87582	7.05291	.89316	8.35957	52
53	.84155	5.31133	.85881	6.08269	.87611	7.07167	.89345	8.38497	53
54	.84184	5.32279	.85910	6.09717	.87640	7.09052	.89374	8.41052	54
55	.84213	5.33429	.85939	6.11171	.87669	7.10946	.89403	8.43620	55
56	.84242	5.34584	.85967	6.12630	.87698	7.12849	.89431	8.46208	56
57	.84270	5.35743	.85996	6.14096	.87726	7.14760	.89460	8.48806	57
58	.84299	5.36906	.86025	6.15568	.87755	7.16681	.89489	8.51411	
59	.84328	5.38073	.86054	6.17046	.87784	7.18612	.89518	8.54037	
60	.84357	5.39245	.86083	6.18530	.87813	7.20551	.89547	8.5667	

TABLE XXIX.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

′	84°		85°		86°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	.89547	8.56677	.91284	10.47371	.93024	13.33559	0
1	.89576	8.59382	.91313	10.51199	.93053	13.39547	1
2	.89605	8.62002	.91342	10.55052	.93082	13.45586	2
3	.89634	8.64687	.91371	10.58932	.93111	13.51676	3
4	.89663	8.67387	.91400	10.62837	.93140	13.57817	4
5	.89692	8.70103	.91429	10.66769	.93169	13.64011	5
6	.89721	8.72833	.91458	10.70728	.93198	13.70258	6
7	.89750	8.75579	.91487	10.74714	.93227	13.76558	7
8	.89779	8.78341	.91516	10.78727	.93257	13.82913	8
9	.89808	8.81119	.91545	10.82768	.93286	13.89323	9
10	.89836	8.83912	.91574	10.86837	.93315	13.95788	10
11	.89865	8.86722	.91603	10.90934	.93344	14.02310	11
12	.89894	8.89547	.91632	10.95060	.93373	14.08890	12
13	.89923	8.92389	.91661	10.99214	.93402	14.15527	13
14	.89952	8.95248	.91690	11.03397	.93431	14.22223	14
15	.89981	8.98123	.91719	11.07610	.93460	14.28979	15
16	.90010	9.01015	.91748	11.11852	.93489	14.35795	16
17	.90039	9.03923	.91777	11.16125	.93518	14.42672	17
18	.90068	9.06849	.91806	11.20427	.93547	14.49611	18
19	.90097	9.09792	.91835	11.24761	.93576	14.56614	19
20	.90126	9.12752	.91864	11.29125	.93605	14.63679	20
21	.90155	9.15730	.91893	11.33521	.93634	14.70810	21
22	.90184	9.18725	.91922	11.37948	.93663	14.78005	22
23	.90213	9.21739	.91951	11.42408	.93692	14.85268	23
24	.90242	9.24770	.91980	11.46900	.93721	14.92597	24
25	.90271	9.27819	.92009	11.51424	.93750	14.99995	25
26	.90300	9.30887	.92038	11.55982	.93779	15.07462	26
27	.90329	9.33973	.92067	11.60572	.93803	15.14999	27
28	.90358	9.37077	.92096	11.65197	.93837	15.22607	28
29	.90386	9.40201	.92125	11.69856	.93866	15.30287	29
30	.90415	9.43343	.92154	11.74550	.93895	15.38041	30
31	.90444	9.46505	.92183	11.79278	.93924	15.45869	31
32	.90473	9.49685	.92212	11.84042	.93953	15.53772	32
33	.90502	9.52886	.92241	11.88841	.93982	15.61751	33
34	.90531	9.56106	.92270	11.93677	.94011	15.69808	34
35	.90560	9.59346	.92299	11.98549	.94040	15.77944	35
36	.90589	9.62605	.92328	12.03458	.94069	15.86159	36
37	.90618	9.65885	.92357	12.08040	.94098	15.94456	37
38	.90647	9.69186	.92386	12.13388	.94127	16.02835	38
39	.90676	9.72507	.92415	12.18411	.94156	16.11297	39
40	.90705	9.75849	.92444	12.23472	.94186	16.19843	40
41	.90734	9.79212	.92473	12.28572	.94215	16.28476	41
42	.90763	9.82596	.92502	12.33712	.94244	16.37196	42
43	.90792	9.86001	.92531	12.38891	.94273	16.46005	43
44	.90821	9.89428	.92560	12.44112	.94302	16.54903	44
45	.90850	9 92877	.92589	12.49373	.94331	16.63893	45
46	.90879	9.96348	.92618	12.54676	.94360	16.72975	46
47	.90908	9.99841	.92647	12.60021	.94389	16.82152	47
48	.90937	10.03356	.92676	12.65408	.94418	16.91424	48
49	.90966	10.06894	.92705	12.70838	.94447	17.00794	49
50	.90995	10.10455	.92734	12.76312	.94476	17.10262	50
51	.91024	10.14039	.92763	12.81829	.94505	17.19830	51
52	.91053	10.17646	.92792	12.87391	.94534	17.29501	52
53	.91082	10.21277	.92821	12.92999	.94563	17.39274	53
54	.91111	10.24932	.92850	12.98651	.94592	17.49153	54
55	.91140	10.28610	.92879	13 04350	.94621	17.59139	55
56	.91169	10.32313	.92908	13.10096	.94650	17.69233	56
57	.91197	10.36040	.92937	13.15889	.94679	17.79438	57
58	.91226	10.39792	.92966	13.21730	.94708	17.89755	58
59	.91255	10.43569	.92995	13.27620	.94737	18.00185	
60	.91284	10.47371	.93024	13.33559	.94766	18.10732	

TABLE XXIX.—NATURAL VERSED SINES AND EXTERNAL SECAN

′	87°		88°		89°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	.94766	18.10732	.96510	27.65371	.98255	56.29869	0
1	.94795	18.21397	.96539	27.89440	.98284	57.26976	1
2	.94825	18.32182	.96568	28.13917	.98313	58.27431	2
3	.94854	18.43088	.96597	28.38812	.98342	59.31411	3
4	.94883	18.54119	.96626	28.64137	.98371	60.39105	4
5	.94912	18.65275	.96655	28.89903	.98400	61.50715	5
6	.94941	18.76560	.96684	29.16120	.98429	62.66460	6
7	.94970	18.87976	.96714	29.42802	.98458	63.86572	7
8	.94999	18.99524	.96743	29.69960	.98487	65.11304	8
9	.95028	19.11208	.96772	29.97607	.98517	66.40927	9
10	.95057	19.23028	.96801	30.25758	.98546	67.75736	10
11	.95086	19.34989	.96830	30.54425	.98575	69.16047	11
12	.95115	19.47093	.96859	30.83623	.98604	70.62285	12
13	.95144	19.59341	.96888	31.13366	.98633	72.14583	13
14	.95173	19.71737	.96917	31.43671	.98662	73.73586	14
15	.95202	19.84283	.96946	31.74554	.98691	75.39655	15
16	.95231	19.96982	.96975	32.06030	.98720	77.13274	16
17	.95260	20.09838	.97004	32.38118	.98749	78.94968	17
18	.95289	20.22852	.97033	32.70835	.98778	80.85315	18
19	.95318	20.36027	.97062	33.04199	.98807	82.84947	19
20	.95347	20.49368	.97092	33.38232	.98836	84.94561	20
21	.95377	20.62876	.97121	33.72952	.98866	87.14924	21
22	.95406	20.76555	.97150	34.08380	.98895	89.46886	22
23	.95435	20.90400	.97179	34.44539	.98924	91.91387	23
24	.95464	21.04440	.97208	34.81452	.98953	94.49471	24
25	.95493	21.18653	.97237	35.19141	.98982	97.22303	25
26	.95522	21.33050	.97266	35.57633	.99011	100.1119	26
27	.95551	21.47635	.97295	35.96953	.99040	103.1757	27
28	.95580	21.62413	.97324	36.37127	.99069	106.4311	28
29	.95609	21.77386	.97353	36.78185	.99098	109.8966	29
30	.95638	21.92559	.97382	37.20155	.99127	113.5930	30
31	.95667	22.07935	.97411	37.63068	.99156	117.5444	31
32	.95696	22.23520	.97440	38.06957	.99186	121.7780	32
33	.95725	22.39316	.97470	38.51855	.99215	126.3253	33
34	.95754	22.55329	.97499	38.97797	.99244	131.2223	34
35	.95783	22.71563	.97528	39.44820	.99273	136.5111	35
36	.95812	22.88022	.97557	39.92963	.99302	142.2406	36
37	.95842	23.04712	.97586	40.42266	.99331	148.4684	37
38	.95871	23.21637	.97615	40.92772	.99360	155.2623	38
39	.95900	23.38802	.97644	41.44525	.99389	162.7033	39
40	.95929	23.56212	.97673	41.97571	.99418	170.8883	40
41	.95958	23.73873	.97702	42.51961	.99447	179.9350	41
42	.95987	23.91790	.97731	43.07746	.99476	189.9868	42
43	.96016	24.09969	.97760	43.64980	.99505	201.2212	43
44	.96045	24.28414	.97789	44.23720	.99535	213.8600	44
45	.96074	24.47134	.97819	44.84026	.99564	228.1839	45
46	.96103	24.66132	.97848	45.45963	.99593	244.5540	46
47	.96132	24.85417	.97877	46.09596	.99622	263.4427	47
48	.96161	25.04994	.97906	46.74997	.99651	285.4795	48
49	.96190	25.24869	.97935	47.42241	.99680	311.5230	49
50	.96219	25.45051	.97964	48.11406	.99709	342.7752	50
51	.96248	25.65546	.97993	48.82576	.99738	380.9723	51
52	.96277	25.86360	.98022	49.55840	.99767	428.7187	52
53	.96307	26.07503	.98051	50.31290	.99796	490.1070	53
54	.96336	26.28981	.98080	51.09027	.99825	571.9581	54
55	.96365	26.50804	.98109	51.89156	.99855	686.5496	55
56	.96394	26.72978	.98138	52.71790	.99884	858.4269	56
57	.96423	26.95513	.98168	53.57046	.99913	1144.916	57
58	.96452	27.18417	.98197	54.45053	.99942	1717.874	58
59	.96481	27.41700	.98226	55.35946	.99971	3436.747	59
60	.96510	27.65371	.98255	56.29869	1.00000	Infinite	60

BLE XXX.—CUBIC YARDS PER 100 FEET. SLOPES ¼ : 1.

th	Base 12	Base 14	Base 16	Base 18	Base 22	Base 24	Base 26	Base 28
	45	53	60	68	82	90	97	105
	93	107	122	137	167	181	196	211
	142	163	186	208	253	275	297	319
	193	222	252	281	341	370	400	430
	245	282	319	356	431	468	505	542
	300	344	389	433	522	567	611	656
	356	408	460	512	616	668	719	771
	415	474	533	593	711	770	830	889
	475	542	608	675	808	875	942	1008
	537	611	685	759	907	981	1056	1130
	601	682	764	845	1008	1090	1171	1253
	667	756	844	933	1111	1200	1289	1378
	734	831	926	1023	1216	1312	1408	1505
	804	907	1010	1115	1322	1426	1530	1633
	875	986	1096	1208	1431	1542	1653	1764
	948	1067	1184	1304	1541	1659	1778	1896
	1023	1149	1274	1401	1653	1779	1905	2031
	1100	1233	1366	1500	1767	1900	2033	2167
	1179	1319	1460	1601	1882	2023	2164	2305
	1259	1407	1555	1704	2000	2148	2296	2444
	1342	1497	1653	1808	2119	2275	2431	2586
	1426	1589	1752	1915	2241	2404	2567	2730
	1512	1682	1853	2023	2364	2534	2705	2875
	1600	1778	1955	2133	2489	2667	2844	3022
	1690	1875	2060	2245	2616	2801	2986	3171
	1781	1974	2166	2359	2744	2937	3130	3322
	1875	2075	2274	2475	2875	3075	3275	3475
	1970	2178	2384	2593	3007	3215	3422	3630
	2068	2282	2496	2712	3142	3356	3571	3786
	2167	2389	2610	2833	3278	3500	3722	3944
	2268	2497	2726	2956	3416	3645	3875	4105
	2370	2607	2844	3081	3556	3793	4030	4267
	2475	2719	2964	3208	3697	3942	4186	4431
	2581	2833	3085	3337	3841	4093	4344	4596
	2690	2949	3208	3468	3986	4245	4505	4764
	2800	3067	3333	3600	4133	4400	4667	4933
	2912	3186	3460	3734	4282	4556	4831	5105
	3026	3307	3589	3870	4433	4715	4996	5278
	3142	3431	3719	4008	4586	4875	5164	5453
	3259	3556	3852	4148	4741	5037	5333	5630
	3379	3682	3986	4290	4897	5201	5505	5808
	3500	3811	4122	4433	5056	5367	5678	5989
	3623	3942	4260	4579	5216	5534	5853	6171
	3748	4074	4400	4726	5378	5704	6030	6356
	3875	4208	4541	4875	5542	5875	6208	6542
	4004	4344	4684	5026	5707	6048	6389	6730
	4134	4482	4830	5179	5875	6223	6571	6919
	4267	4622	4978	5333	6044	6400	6756	7111
	4401	4764	5127	5490	6216	6579	6942	7305
	4537	4907	5278	5648	6389	6759	7130	7500
	4675	5053	5430	5808	6564	6942	7319	7697
	4815	5200	5584	5970	6741	7126	7511	7896
	4956	5349	5741	6134	6919	7312	7705	8097
	5100	5500	5900	6300	7100	7500	7900	8300
	5245	5653	6060	6468	7282	7690	8097	8505
	5393	5807	6222	6637	7467	7881	8296	8711
	5542	5964	6386	6808	7653	8075	8497	8919
	5693	6122	6552	6981	7841	8270	8700	9130
	5845	6282	6719	7156	8031	8468	8905	9342
	6000	6444	6889	7333	8222	8667	9111	9556

ABLE XXX.—CUBIC YARDS

epth d	Base 12	Base 14	Base 16	Base 18	Base 22	Base 24	26	2
1	46	54	61	69	83	91	98	10
2	96	111	126	141	170	185	200	215
3	150	172	194	217	261	283	306	3
4	207	237	267	296	356	385	415	4
5	269	306	343	380	454	491	528	5
6	333	378	422	467	556	600	644	
7	402	454	506	557	661	713	765	
8	474	533	593	652	770	830	889	
9	550	617	683	750	883	950	1017	
10	630	704	778	852	1000	1074	1148	
11	713	794	876	957	1120	1202	1283	
12	800	889	978	1067	1244	1333	1422	
13	891	987	1083	1180	1372	1469	156	
14	985	1089	1193	1296	1504	1607	171	
15	1083	1194	1306	1417	1639	1750	18	
16	1185	1304	1422	1541	1779	1896	2	
17	1291	1417	1543	1669	1920	2046		
18	1400	1533	1667	1800	2067	2200		
19	1513	1654	1794	1935	2217	2357		
20	1630	1778	1926	2074	2370	2519		
21	1750	1906	2061	2217	2528	2683		
22	1874	2037	2200	2363	2689	2852		
23	2002	2172	2343	2513	2854	3024		
24	2133	2311	2489	2667	3022	3200		
25	2269	2454	2639	2824	3194	3380		
26	2407	2600	2793	2985	3370	3563		
27	2550	2750	2950	3150	3550	3750		
28	2696	2904	3111	3319	3733	3941		
29	2846	3061	3276	3491	3920	413		
30	3000	3222	3444	3667	4111	433		
31	3157	3387	3617	3846	4306	45		
32	3319	3556	3793	4030	4504	47		
33	3483	3728	3972	4217	4706	49		
34	3652	3904	4156	4407	4911	51		
35	3824	4083	4343	4602	5120	5		
36	4000	4267	4533	4800	5333	5		
37	4180	4454	4728	5002	5550	5		
38	4363	4644	4926	5207	5770	6		
39	4550	4839	5128	5417	5994	6		
40	4741	5037	5333	5630	6222			
41	4935	5239	5543	5846	6454			
42	5133	5444	5756	6067	6689			
43	5335	5654	5972	6291	6928			
44	5541	5867	6193	6519	7170			
45	5750	6083	6417	6750	7417			
46	5963	6304	6644	6985	7667			
47	6180	6528	6876	7224	7920			
48	6400	6756	7111	7467	8178			
49	6624	6987	7350	7713	8439			
50	6852	7222	7593	7963	8704			
51	7083	7461	7839	8217	8972			
52	7319	7704	8089	8474	9244			
53	7557	7950	8343	8735	9520			
54	7800	8200	8600	9000	9800			
55	8046	8454	8861	9269	10083			
56	8296	8711	9126	9541	10370			
57	8550	8972	9394	9817	10661			
58	8807	9237	9667	10096	1095			
59	9069	9506	9943	10380	112			
60	9333	9778	10222	10667	11			

TABLE XXX.—CUBIC YARDS PER 100 FEET. SLOPES 1 : 1.

Depth d	Base 12	Base 14	Base 16	Base 18	Base 20	Base 28	Base 30	Base 32
1	48	56	63	70	78	107	115	122
2	104	119	133	148	163	222	237	252
3	167	189	211	233	256	344	367	389
4	237	267	296	326	356	474	504	533
5	315	352	389	426	463	611	648	685
6	400	444	489	533	578	756	800	844
7	493	544	596	648	700	907	959	1011
8	593	652	711	770	830	1067	1126	1185
9	700	767	833	900	967	1233	1300	1367
10	815	889	963	1037	1111	1407	1481	1556
11	937	1019	1100	1181	1263	1589	1670	1752
12	1067	1156	1244	1333	1422	1778	1867	1956
13	1204	1300	1396	1493	1589	1974	2070	2167
14	1348	1452	1556	1659	1763	2178	2281	2385
15	1500	1611	1722	1833	1944	2389	2500	2611
16	1659	1778	1896	2015	2133	2607	2726	2844
17	1826	1952	2078	2204	2330	2833	2959	3085
18	2000	2133	2267	2400	2533	3067	3200	3333
19	2181	2322	2463	2604	2744	3307	3448	3589
20	2370	2519	2667	2815	2963	3556	3704	3852
21	2567	2722	2878	3033	3189	3811	3967	4122
22	2770	2933	3096	3259	3422	4074	4237	4444
23	2981	3152	3322	3493	3663	4344	4515	4685
24	3200	3378	3556	3733	3911	4622	4800	4978
25	3426	3611	3796	3981	4167	4907	5093	5278
26	3659	3852	4044	4237	4430	5200	5393	5585
27	3900	4100	4300	4500	4700	5500	5700	5900
28	4148	4356	4563	4770	4978	5807	6015	6222
29	4404	4619	4833	5048	5263	6122	6337	6552
30	4667	4889	5111	5333	5556	6444	6667	6889
31	4937	5167	5396	5626	5856	6774	7004	7233
32	5215	5452	5689	5926	6163	7111	7348	7585
33	5500	5744	5989	6233	6478	7456	7700	7944
34	5793	6044	6296	6548	6800	7807	8059	8311
35	6093	6352	6611	6870	7130	8167	8426	8685
36	6400	6667	6933	7200	7467	8533	8800	9067
37	6715	6989	7263	7537	7811	8907	9181	9456
38	7037	7319	7600	7881	8163	9289	9570	9852
39	7367	7656	7944	8233	8522	9678	9967	10256
40	7704	8000	8296	8593	8889	10074	10370	10667
41	8048	8352	8656	8959	9263	10478	10781	11085
42	8400	8711	9022	9333	9644	10889	11200	11511
43	8759	9078	9396	9715	10033	11307	11626	11944
44	9126	9452	9778	10104	10430	11733	12059	12385
45	9500	9833	10167	10500	10833	12167	12500	12833
46	9881	10222	10563	10904	11244	12607	12948	13289
47	10270	10619	10967	11315	11663	13056	13404	13752
48	10667	11022	11378	11733	12089	13511	13867	14222
49	11070	11433	11796	12159	12522	13974	14337	14700
50	11481	11852	12222	12593	12963	14444	14815	15185
51	11900	12278	12656	13033	13411	14922	15300	15678
52	12326	12711	13096	13481	13867	15407	15793	16178
53	12759	13152	13544	13937	14330	15900	16293	16685
54	13200	13600	14000	14400	14800	16400	16800	17200
55	13648	14056	14463	14870	15278	16907	17315	17722
56	14104	14519	14933	15348	15763	17422	17837	18252
57	14567	14989	15411	15833	16256	17944	18367	18789
58	15037	15467	15896	16326	16756	18474	18904	19333
59	15515	15952	16389	16826	17263	19011	19448	19885
60	16000	16444	16889	17333	17778	19556	20000	20444

TABLE XXX.—CUBIC YARDS PER 100 FEET. SLOPES

Depth d	Base 12	Base 14	Base 16	Base 18	Base 20	Base 28	Base 30
1	49	56	64	71	79	108	116
2	107	122	137	152	167	226	241
3	175	197	219	242	264	353	375
4	252	281	311	341	370	489	519
5	338	375	412	449	486	634	671
6	433	478	522	567	611	789	833
7	538	590	642	694	745	953	1005
8	652	711	770	830	889	1126	1185
9	775	842	908	975	1042	1308	1375
10	907	981	1056	1130	1204	1500	1574
11	1049	1131	1212	1294	1375	1701	1782
12	1200	1289	1378	1467	1556	1911	2000
13	1360	1456	1553	1649	1745	2131	2227
14	1530	1633	1737	1841	1944	2359	2463
15	1708	1819	1931	2042	2153	2597	2708
16	1896	2015	2133	2252	2370	2844	2963
17	2094	2219	2345	2471	2597	3101	3227
18	2300	2433	2567	2700	2833	3367	3500
19	2516	2656	2797	2938	3079	3642	3782
20	2741	2889	3037	3185	3333	3926	4074
21	2975	3131	3286	3442	3597	4220	4375
22	3219	3381	3544	3707	3870	4522	4685
23	3471	3642	3812	3982	4153	4834	5005
24	3733	3911	4089	4267	4444	5156	5333
25	4005	4190	4375	4560	4745	5486	5671
26	4285	4478	4670	4863	5056	5826	6019
27	4575	4775	4975	5175	5375	6175	6375
28	4874	5081	5289	5496	5704	6533	6741
29	5182	5397	5612	5827	6042	6901	7116
30	5500	5722	5944	6167	6389	7278	7500
31	5827	6056	6286	6516	6745	7664	7894
32	6163	6400	6637	6874	7111	8059	8296
33	6508	6753	6997	7242	7486	8464	8708
34	6863	7115	7367	7619	7870	8878	9130
35	7227	7486	7745	8005	8264	9301	9560
36	7600	7867	8133	8400	8667	9733	10000
37	7982	8256	8531	8805	9079	10175	10449
38	8374	8656	8937	9219	9500	10626	10907
39	8775	9064	9353	9642	9931	11086	11375
40	9185	9481	9778	10074	10370	11556	11852
41	9605	9908	10212	10516	10819	12034	12338
42	10033	10344	10656	10967	11278	12522	12833
43	10471	10790	11108	11427	11745	13020	13338
44	10919	11244	11570	11896	12222	13526	13852
45	11375	11708	12042	12375	12708	14042	1437
46	11841	12181	12522	12863	13204	14567	1490
47	12316	12664	13012	13360	13708	15101	1544
48	12800	13156	13511	13867	14222	15644	1600
49	13294	13656	14019	14382	14745	16197	1656
50	13796	14167	14537	14907	15278	16759	171
51	14308	14686	15064	15442	15819	17331	177
52	14830	15215	15600	15985	16370	17911	182
53	15360	15753	16145	16538	16931	18501	188
54	15900	16300	16700	17100	17500	19100	195
55	16449	16856	17264	17671	18079	19708	201
56	17007	17422	17837	18252	18667	20326	207
57	*17575*	17997	18419	18442	19264	20953	21
58	*18152*	18581	19011	19441	19870	21589	22
59	*18738*	19175	19612	20049	20486	22234	2
60	*19333*	19778	20222	20667	21111	22889	

TABLE XXX.—CUBIC YARDS PER 100 FEET. SLOPES 1½ : 1.

Depth d	Base 12	Base 14	Base 16	Base 18	Base 20	Base 28	Base 30	Base 32
1	50	57	65	72	80	109	117	124
2	111	126	141	156	170	230	244	259
3	183	206	228	250	272	361	383	406
4	267	296	326	356	385	504	533	563
5	361	398	435	472	509	657	694	731
6	467	511	556	600	644	822	867	911
7	583	635	687	739	791	998	1050	1102
8	711	770	830	889	948	1185	1244	1304
9	850	917	983	1050	1116	1383	1450	1517
10	1000	1074	1148	1222	1296	1593	1667	1741
11	1161	1243	1324	1406	1487	1813	1894	1976
12	1333	1422	1511	1600	1689	2044	2133	2222
13	1517	1613	1709	1806	1902	2287	2383	2480
14	1711	1815	1919	2022	2126	2541	2644	2748
15	1917	2028	2139	2250	2361	2806	2917	3028
16	2133	2252	2370	2489	2607	3081	3200	3319
17	2361	2487	2613	2739	2865	3369	3494	3620
18	2600	2733	2867	3000	3133	3667	3800	3933
19	2850	2991	3131	3272	3413	3976	4117	4257
20	3111	3259	3407	3556	3704	4296	4444	4592
21	3383	3539	3694	3850	4005	4628	4783	4939
22	3667	3830	3993	4156	4318	4970	5133	5296
23	3961	4131	4302	4472	4642	5324	5494	5665
24	4267	4444	4622	4800	4978	5689	5867	6044
25	4583	4769	4954	5139	5324	6065	6250	6435
26	4911	5104	5296	5489	5681	6452	6644	6837
27	5250	5450	5650	5850	6050	6850	7050	7250
28	5600	5807	6015	6222	6430	7259	7467	7674
29	5961	6176	6391	6606	6820	7680	7894	8109
30	6333	6556	6778	7000	7222	8111	8333	8555
31	6717	6946	7176	7406	7635	8554	8783	9013
32	7111	7348	7585	7822	8059	9007	9244	9482
33	7517	7761	8006	8250	8494	9472	9717	9962
34	7933	8185	8437	8689	8941	9948	10200	10452
35	8361	8620	8880	9139	9398	10435	10694	10954
36	8800	9067	9333	9600	9867	10933	11200	11467
37	9250	9524	9798	10072	10346	11443	11717	11991
38	9711	9993	10274	10556	10837	11963	12244	12526
39	10183	10472	10761	11050	11339	12494	12783	13072
40	10667	10963	11259	11556	11852	13037	13333	13630
41	11161	11465	11769	12072	12376	13591	13894	14198
42	11667	11978	12289	12600	12911	14156	14467	14778
43	12183	12502	12820	13139	13457	14731	15050	15369
44	12711	13037	13363	13689	14015	15319	15644	15970
45	13250	13583	13917	14250	14583	15917	16250	16583
46	13800	14141	14481	14822	15163	16526	16867	17207
47	14361	14709	15057	15406	15754	17146	17494	17843
48	14933	15289	15644	16000	16356	17778	18133	18489
49	15517	15880	16243	16606	16968	18420	18783	19146
50	16111	16481	16852	17222	17592	19074	19444	19815
51	16717	17094	17472	17850	18228	19739	20117	20494
52	17333	17719	18104	18489	18874	20415	20800	21185
53	17961	18354	18746	19139	19531	21102	21494	21887
54	18600	19000	19400	19800	20200	21800	22200	22600
55	19250	19657	20065	20472	20880	22509	22917	23324
56	19911	20326	20741	21156	21570	23230	23644	24059
57	20583	21006	21428	21850	22272	23961	24383	24805
58	*21267*	*21696*	22126	22556	22985	24704	25133	25563
59	*21961*	22398	22835	23272	23709	25457	25894	26332
60	*22667*	*23111*	23556	24000	24444	26222	26667	27111

TABLE XXX.—CUBIC YARDS PER 100 FEET. SLOPES

Depth d	Base 12	Base 14	Base 16	Base 18	Base 20	Base 28	Base 30
1	52	59	67	74	81	111	119
2	119	133	148	163	178	237	252
3	200	222	244	267	289	378	400
4	296	326	356	385	415	533	56
5	407	444	481	519	556	704	74
6	533	578	622	667	711	889	93
7	674	726	778	830	881	1089	114
8	830	889	948	1007	1067	1304	13
9	1000	1067	1133	1200	1267	1533	16
10	1185	1259	1333	1407	1481	1778	18
11	1385	1467	1548	1630	1711	2037	21
12	1600	1689	1778	1867	1956	2311	24
13	1830	1926	2022	2119	2215	2600	2
14	2074	2178	2281	2385	2489	2904	3
15	2333	2444	2556	2667	2778	3222	3
16	2607	2726	2844	2963	3081	3556	3
17	2896	3022	3148	3274	3400	3904	4
18	3200	5333	3467	3600	3733	4267	4
19	3519	3659	3800	3941	4081	4644	4
20	3852	4000	4148	4296	4444	5037	
21	4200	4356	4511	4667	4822	5444	
22	4563	4730	4889	5052	5215	5867	
23	4941	5111	5281	5452	5622	6304	
24	5333	5511	5689	5867	6044	6756	
25	5741	5926	6111	6296	6481	7222	
26	6163	6356	6548	6741	6933	7704	
27	6600	6800	7000	7200	7400	8200	
28	7052	7259	7467	7674	7881	8711	
29	7519	7733	7948	8163	8378	9237	
30	8000	8222	8444	8667	8889	9778	
31	8496	8726	8956	9185	9415	10333	
32	9007	9244	9481	9719	9956	10904	
33	9533	9778	10022	10267	10511	11489	
34	10074	10326	10578	10830	11081	12089	
35	10630	10889	11148	11407	11667	12704	
36	11200	11467	11733	12000	12267	13333	
37	11785	12059	12333	12607	12881	13978	
38	12385	12667	12948	13230	13511	14637	
39	13000	13289	13578	13867	14156	15311	
40	13630	13926	14222	14519	14815	16000	
41	14274	14578	14881	15185	15489	16704	
42	14933	15244	15556	15867	16178	17422	
43	15607	15926	16224	16563	16881	18156	
44	16296	16622	16948	17274	17600	18904	
45	17000	17333	17667	18000	18333	19667	
46	17719	18059	18400	18741	19081	20444	
47	18452	18800	19148	19496	19844	21237	
48	19200	19556	19911	20267	20622	22044	
49	19963	20326	20689	21052	21415	22867	
50	20741	20711	21481	21852	22222	23704	
51	21533	21911	22289	22667	23044	24556	
52	22341	22726	23111	23496	23881	25422	
53	23163	23556	23948	24341	24733	26304	
54	24000	24400	24800	25200	25600	27200	
55	24852	25259	25667	26074	26481	28111	
56	25719	26133	26548	26963	27378	29037	
57	26600	27022	27444	27867	28289	29978	
58	27496	27926	28356	28785	29215	3093	
59	28407	28844	29281	29719	30156	3190	
60	29333	29778	30222	30667	31111	328	

TABLE XXX.—CUBIC YARDS PER 100 FEET. SLOPES 3 : 1.

Depth d	Base 12	Base 14	Base 16	Base 18	Base 20	Base 28	Base 30	Base 32
1	56	63	70	78	85	115	122	130
2	133	148	163	178	193	252	267	281
3	233	256	278	300	322	411	433	456
4	356	385	415	444	474	593	622	652
5	500	537	574	611	648	796	833	870
6	667	711	756	800	844	1022	1067	1111
7	856	907	959	1011	1063	1270	1322	1374
8	1067	1126	1185	1244	1304	1541	1600	1659
9	1300	1367	1433	1500	1567	1833	1900	1967
10	1556	1630	1704	1778	1852	2148	2222	2296
11	1833	1915	1996	2078	2159	2485	2567	2648
12	2133	2222	2311	2400	2489	2844	2933	3022
13	2456	2552	2648	2744	2841	3226	3322	3419
14	2800	2904	3007	3111	3215	3630	3733	3837
15	3167	3278	3389	3500	3611	4056	4167	4278
16	3556	3674	3793	3911	4030	4504	4622	4741
17	3967	4093	4219	4344	4470	4974	5100	5226
18	4400	4533	4667	4800	4933	5467	5600	5733
19	4856	4996	5137	5278	5419	5981	6122	6263
20	5333	5481	5630	5778	5926	6519	6667	6815
21	5833	5989	6144	6300	6456	7078	7233	7389
22	6356	6519	6681	6844	7007	7659	7822	7985
23	6900	7070	7241	7411	7581	8263	8433	8504
24	7467	7644	7822	8000	8178	8889	9067	9144
25	8056	8241	8426	8611	8796	9537	9722	9807
26	8667	8859	9052	9244	9437	10207	10400	10593
27	9300	9500	9700	9900	10100	10900	11100	11300
28	9956	10163	10370	10578	10785	11615	11822	12030
29	10633	10848	11063	11278	11493	12352	12567	12781
30	11333	11556	11778	12000	12222	13111	13333	13556
31	12056	12285	12515	12744	12974	13893	14122	14352
32	12800	13037	13274	13511	13748	14696	14933	15170
33	13567	13811	14056	14300	14544	15522	15767	16011
34	14356	14607	14859	15111	15363	16370	16622	16874
35	15167	15426	15685	15944	16204	17241	17500	17759
36	16000	16267	16533	16800	17067	18133	18400	18667
37	16856	17130	17404	17678	17952	19048	19322	19596
38	17733	18015	18296	18578	18859	19985	20267	20548
39	18633	18922	19211	19500	19789	20944	21233	21522
40	19556	19852	20148	20444	20741	21926	22222	22516
41	20500	20804	21107	21411	21715	22930	23233	23537
42	21467	21778	22089	22400	22711	23956	24267	24578
43	22456	22774	23093	23411	23730	25004	25322	25641
44	23467	23793	24119	24444	24770	26074	26400	26726
45	24500	24833	25167	25500	25833	27167	27500	27833
46	25556	25896	26237	26578	26919	28281	28622	28963
47	26633	26981	27330	27678	28026	29419	29767	30115
48	27733	28089	28444	28800	29156	30578	30933	31289
49	28856	29219	29581	29944	30307	31759	32122	32485
50	30000	30370	30741	31111	31481	32963	33333	33704
51	31167	31544	31922	32300	32678	34189	34567	34944
52	32356	32741	33126	33511	33896	35437	35822	36207
53	33567	33959	34352	34744	35137	36707	37100	37493
54	34800	35200	35600	36000	36400	38000	38400	38800
55	36056	36463	36870	37278	37685	39315	39722	40130
56	37333	37748	38163	38578	38993	40652	41067	41481
57	38633	39056	39478	39900	40322	42011	42433	42856
58	39956	40385	40815	41244	41674	43393	43822	44252
59	41300	41737	42174	42611	43048	44796	45233	45670
60	42667	43111	43556	44000	44444	46222	46667	47111

TABLE XXI.—USEFUL NUMBERS AND FORMULÆ.

Title.	Symbol.	Number.	Logarithm.
Ratio of circumference to diameter........	π	3.1415927	0.4971499
Reciprocal of same	$\frac{1}{\pi}$	0.3183099	9.5028501
Degrees in arc of length equal to radius....	$\frac{180°}{\pi}$	57.295780	1.7581226
Minutes " " " " "	$\frac{10800'}{\pi}$	3437.7468	3.5362739
Seconds " " " " "	$\frac{648000''}{\pi}$	206264.81	5.3144251
Length of 1° arc, radius unity...............	$\frac{\pi}{180°}$	.01745329	8.2418774
Length of 1′ arc, " "	$\frac{\pi}{10800}$	.00029089	6.4637261
Length of 1″ arc, " "	$\frac{\pi}{648000}$	.000004848	4.6855749
Radius by which 1 foot of arc = 1 degree.		57.295780	1.7581226
Radius " " $\frac{1}{10}$ " " = 1 minute.		343.77468	2.5362739
Radius " " $\frac{1}{100}$ " " = 10 seconds		206.26481	2.3144251
Factors for dividing a line into extreme and mean ratio...........................		0.6180340 0.3819660	9.7910124 9.5820248
Base of hyperbolic logarithms..............	ε	2.7182818	0.4342945
Modulus of common system of logs. = log ε	M	0.4342945	9.6377843
Reciprocal of same = hyp. log. 10...........	$\frac{1}{M}$	2.3025851	0.3622157
Length of seconds pendulum at New York in inches		39.11256	1.5923162
Length of seconds pendulum at New York in feet....................................		3.25938	0.5131350
Acceleration due to gravity at New York...	g	32.1688	1.5074347
Square root of same	$\sqrt{g}$	5.67175	0.7537173
Yards in 1 metre...........................		1.093623	0.0388676
Feet in 1 "		3.280869	0.5159889
Inches in 1 "		39.37043	1.5951701
Metres in 1 foot..............................		0.304797	9.4840111
Metres in 1 yard..............................		0.914392	9.9611394
Metres in 1 mile..............................		1609.330	3.20664

Title.	Symbol.	Number.	Logarithm.
Cubic inches in 1 U. S. gallon..............		231.	2.3636120
" " " 1 Imperial gallon..........		277.274	2.4429092
" " " 1 U. S. bushel.............		2150.42	3.3325233
Cubic feet in 1 U. S. gallon.................		0.133681	9.1260683
" " " 1 Imperial gallon............		0.160459	9.2053655
" " " 1 U. S. bushel.................		1.244456	0.0949796
Weight of 1 cub. foot of water, barom. 30 in. ther. 39°.83 Fah.; pounds..		62.379	1.7950384
" 62° " " ..		62.321	1.7946349
Weight in grains, 1 cubic inch, at 62° Fah..		252.458	2.4021892
No. of grains in 1 pound avoir...............		7000.	3.8450980
" " " 1 ounce "		437.5	2.6409781

r = radius of circular arc;
l = length of arc;
$a°$ = degrees in same arc.

$$\left.\begin{aligned} a° &= \frac{l}{r}\cdot\frac{180°}{\pi} \\ r &= \frac{l}{a°}\cdot\frac{180°}{\pi} \\ l &= a° r\cdot\frac{\pi}{180°} \end{aligned}\right.$$

Radius by which the length of chord c in feet $= \frac{a'}{10}$ in minutes;

$$r = \frac{\frac{1}{2}a'}{10 \sin \frac{1}{2}a'}$$

Hyp. log x = com. log $x \times \frac{1}{M}$, or

$$\text{com. log (hyp. log } x) = \text{com. log (com. log } x) + 0.3622157$$

Com. log $x = M \times$ hyp. log x; or

$$\text{com. log (com. log } x) = 9.6377843 + \text{com. log (hyp. log } x)$$

Circumference of circle (radius $= r$)................................ $2\pi r$

Area of circle .... πr^2

Area of sector (length of arc $= l$)................................ $\frac{1}{2}lr$

Area of sector (angle of arc $= a°$)................................ $\frac{a}{360}\pi r^2$

Approximate area of segment (chord = c, mid. ord. = m) .... $\frac{2}{3}c$

APPENDIX.

Verification of eq. (77).

Eq. (76) $$\rho = \frac{\sin\theta}{\sin\frac{\theta}{N}} = \sin\theta \,.\, \operatorname{cosec}\frac{\theta}{N}$$

$$\frac{d\rho}{d\theta} = \cos\theta \,.\, \operatorname{cosec}\frac{\theta}{N} - \frac{1}{N} \,.\, \sin\theta \,.\, \cot\frac{\theta}{N} \,.\, \operatorname{cosec}\frac{\theta}{N} \qquad (76\tfrac{1}{2})$$

$$\therefore \quad \frac{d\rho}{d\theta} = \rho\left(\cot\theta - \frac{1}{N}\cot\frac{\theta}{N}\right) \qquad (77)$$

Verification of eq. (81).

Differentiating eq. (76½)

$$\frac{d^2\rho}{d\theta^2} = -\sin\theta\operatorname{cosec}\frac{\theta}{N} - \frac{2}{N}\cos\theta\cot\frac{\theta}{N}\operatorname{cosec}\frac{\theta}{N} +$$

$$\frac{1}{N^2}\sin\theta\cot^2\frac{\theta}{N}\operatorname{cosec}\frac{\theta}{N} + \frac{1}{N^2}\sin\theta\operatorname{cosec}^3\frac{\theta}{N}$$

$$= -\rho - \frac{2\rho}{N}\cot\theta \,.\, \cot\frac{\theta}{N} + \frac{\rho}{N^2}\left(\cot^2\frac{\theta}{N} + \operatorname{cosec}^2\frac{\theta}{N}\right)$$

$$\therefore \quad \frac{d^2\rho}{d\theta^2} = \rho\left(-1 - \frac{2}{N}\cot\theta\cot\frac{\theta}{N} + \frac{1}{N^2}\left(2\cot^2\frac{\theta}{N} + 1\right)\right)$$

Now $$r = \frac{\left(\rho^2 + \frac{d\rho^2}{d\theta^2}\right)^{\frac{3}{2}}}{\rho^2 + 2\frac{d\rho^2}{d\theta^2} - \rho\,\frac{d^2\rho}{d\theta^2}}$$

in which substitute for $\frac{d\rho}{d\theta}$, and for $\frac{d^2\rho}{d\theta}$, and let

$$\cot\theta - \frac{1}{N}\cot\frac{\theta}{N} = -a$$

$$r = \frac{\left(\rho^2 + \rho^2(-a)^2\right)^{\frac{3}{2}}}{\rho^2 + 2\rho^2(-a)^2 - \rho^2\left(-1 - \frac{2}{N}\cot\theta\cot\frac{\theta}{N} + \frac{1}{N^2}(2\cot^2\frac{\theta}{N} + 1)\right)}$$

$$= \frac{\rho}{2}\cdot\frac{(1+a^2)^{\frac{3}{2}}}{1 + a^2 + \frac{1}{N}\cot\theta\,.\cot\frac{\theta}{N} - \frac{1}{N^2}\cot^2\frac{\theta}{N} - \frac{1}{2N^2}}$$

$$= \frac{\rho}{2}\cdot\frac{(1+a^2)^{\frac{3}{2}}}{1 - \frac{1}{2N^2} + a^2 + \frac{1}{N}\cot\frac{\theta}{N}\left(\cot\theta - \frac{1}{N}\cot\frac{\theta}{N}\right)}$$

$$= \frac{\rho}{2}\,\frac{(1+a^2)^{\frac{3}{2}}}{1 - \frac{1}{2N^2} + a\left(a - \frac{1}{N}\cot\frac{\theta}{N}\right)}$$

$$\therefore\ r = \frac{\rho}{2}\cdot\frac{(1+a^2)^{\frac{3}{2}}}{1 - \frac{1}{2N^2} - a\cot\theta} \qquad (8)$$